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DeCART Equivalent Group Constants Generation for Hexagonal TPEN Method

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1. Introduction

Nuclear code system is highly dependent on the capability of the lattice code which provides the homogenized group constants and assembly discontinuity factor. The conventional code system which is originally developed before the year 2000 with the insufficient computational resources, is based on two-dimensional lattice code using the ideal boundary condition. Recently, with the growth of the computing power, Sugimura proposed a hybrid core calculation system that uses a more realistic interface effect in the radial direction using the 2-D whole core transport capability of CASMO-4 [1]. Also, Cho proposed an innovative three dimensional nuclear analysis system, DeCART/CHORUS/MASTER using the 3-D whole core transport capability of the DeCART [2,3]. In that system, the assembly-wise equivalent group constants are generated and functionalized as the conventional code system through the whole core branch calculation. The merits of the this code system is in that it can remove the design uncertainty caused by the homogenization and unrealistic boundary condition and finally reduce the uncertainty level to the DeCART whole core transport calculation.

In this paper, the assembly-wise equivalent group constant generation capability of the DeCART code is extended to the hexagonal core. This capability is realized using the TPEN (Triangle-Based Polynomial Expansion Nodal) method [4] which is the basic nodal method of PARCS [5] and MASTER [6] codes for hexagonal core analysis. In chapter 2, the methodology to generate the equivalent group constants are explained. In Chapter 3, the capability is examined for the hexagonal 2-D and 3-D test problems.

2. Methodology

The equivalent group constants consist of the homogenized few group constants (HGC) and the flux discontinuity factors (DFs) which is defined as the ratio of the heterogeneous surface flux to the homogeneous flux. The HGC can easily be generated by using the heterogeneous flux and cross section through the traditional procedure. To generate the DF, however, the homogeneous flux, the flux shape in the homogenized node, is required. In DeCART, the SENM for the rectangular geometry [3] and the TPEN method for the hexagonal geometry are implemented to generate the DFs. For each homogenized node, homogenized cross-sections, node average flux, and the surface average net currents are given as constraints with the multiplication factor as shown in Fig. 1 and the intra-nodal homogeneous flux distribution can be determined with these constraints.

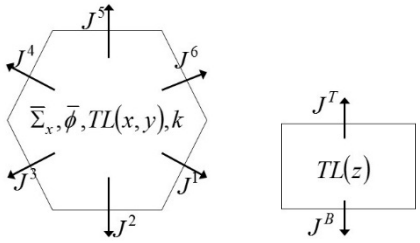


Fig. 1. Given Conditions in Radial and Axial Directions.

The surface flux DF (SDF) can be determined as the ratio of the surface average heterogeneous flux to the surface average homogeneous flux. Unlike the transverse-integrated nodal methods for rectangular geometry including the SENM, most of the nodal methods for hexagonal geometry including TPEN use the corner point flux as a basic unknown and they require the corner flux DF (CDF) in addition to the SDF. However, unlike the SDF, the CDF can be defined arbitrary for example 1.0 or the heterogeneous to homogeneous corner flux. How to define the CDF affect the flux shape along the surface, but not affect the average net current. Undoubtedly, the CDF definition should be consistent with the nodal code. In this paper, the CDF is approximated by the average of the neighboring SDFs as the MASTER code. And the heterogeneous corner fluxes are determined by solving the corner point balance (CPB) equation with the CDFs approximated.

In TPEN method, the two radial and the axial equations are coupled through the transverse leakages, and MASTER solves the coupled equation using analytic solution. However, when applying the given conditions of Fig. 1, the two equation can be decoupled because the transverse leakages are given. By using the 7 transverse leakages shown as Fig. 2, the transverse leakage shape in the center fuel assembly is approximated as following:

$$TL(x,y) = c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 u^2 + c_5 p^2 + c_6 xup \quad (1)$$

where

$$u = -\frac{1}{2}x - \frac{\sqrt{3}}{2}y,$$

$$p = -\frac{1}{2}x + \frac{\sqrt{3}}{2}y$$

The 7 coefficients can be expressed in terms of the 7 transverse leakages in Fig. 2. The transverse leakages in 6 triangles in the center assembly can be calculated by integrating Eq. (1) for each triangular node.

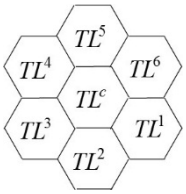


Fig. 2. Neighboring Axial Transverse Leakages for Source Approximation in Center Assembly.

In the MASTER code, the CDF is approximated by using the SDFs of the neighboring surfaces as:

$$CDF^i = \frac{1}{2}\left(SDF^i + SDF^{i+1}\right) \quad (2)$$

The above CDF approximation is suggested to consider the simplified equivalence theory (SET). In SET, the SDF is generated by the single assembly model using the ideal boundary condition, and it can be used to the nodal method by explicit treatment or implicit treatment. In the implicit treatment, the homogenized group constants divided by the SDF are used in the nodal calculation. When applying the explicit treatment, the use of the above CDF approximation guarantee the same solution with the implicit treatment.

DeCART performs the P3 transport calculation for the axial direction [3], and generates the 3rd angular flux moment additionally with the 1st moment. From the angular flux moments, the surface flux in the axial direction is calculated as:

$$\phi_s = \frac{8}{5}\left\{\left(j_1^+ + j_1^-\right) - \left(j_3^+ + j_3^-\right)\right\} \quad (3)$$

To generate the equivalent group constants for hexagonal TPEN method, the following procedure is implemented to the DeCART code. First the SDFs for the radial direction are obtained by the following procedure.

(1) Determine the axial leakages for 6 triangular nodes for all assemblies by using Eq. (1).
(2) Assume the homogeneous surface and corner fluxes by the heterogeneous solutions, and the triangular fluxes by the assembly average flux.
(3) Determine the CDF by the SDFs of the neighboring surfaces as Eq. (2).
(4) Solve the CPB equation and update the heterogeneous corner flux. Hear 'heterogeneous corner flux' means just the solution of CPB equation using the CDF.
(5) Solve the one-node TPEN equation for the radial direction using the given conditions of Fig. 1 and the homogeneous corner fluxes determined by the heterogeneous flux in step (4) and the CDF in step (3). In this step, the surface fluxes and the triangle averaged flux and moments are updated.
(6) Check the convergence of the surface flux. If not converged, go to step (3).

Next, the SDFs for the axial direction are obtained by the following procedure.

(1) Determine the radial leakages for the axial NEM solution.
(2) Assume the homogeneous surface fluxes by the heterogeneous solutions.
(3) Solve the one-node NEM equation for the axial direction using the given conditions of Fig. 1 and update the surface fluxes and the axial flux moments.
(4) Check the convergence of the surface flux. If not converged, go to step (3).

MASTER obtains the TPEN solution based on one-node kernel which uses the homogeneous incoming partial currents at the assembly boundaries as the given condition. Also MASTER stores the heterogeneous outgoing partial currents at the assembly boundaries. Therefore, MASTER generates the homogeneous partial currents using the heterogeneous ones before calling the one-node TPEN kernel, and the heterogeneous partial currents using the homogeneous solutions after TPEN kernel calculation.

The two surface conditions of the surface flux discontinuity and the net current continuity conditions can be expressed as:

$$\phi_s^{het} = 2\left(j_s^{het,+} + j_s^{het,-}\right) = 2f_s\left(j_s^{ho,+} + j_s^{ho,-}\right) = f_s\,\phi_s^{ho} \quad (4)$$

$$J_s^{het} = j_s^{het,+} - j_s^{het,-} = j_s^{ho,+} - j_s^{ho,-} = J_s^{ho} \quad (5)$$

The homogeneous incoming partial current expressed by the heterogeneous ones can be obtained by eliminating the homogeneous outgoing partial current in the above equations as:

$$j_s^{ho,-} = \frac{1}{2}\left\{\frac{1}{f_s}\left(j_s^{het,+} + j_s^{het,-}\right) - \left(j_s^{het,+} - j_s^{het,-}\right)\right\} \quad (6)$$

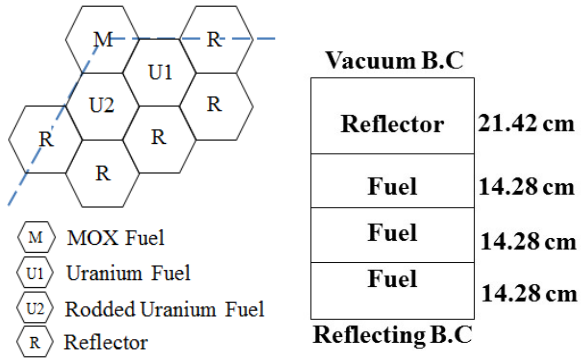
The heterogeneous outgoing partial current can be obtained from Eq. (4) as:

$$j_s^{het,+} = j_s^{het,-} + j_s^{ho,+} - j_s^{ho,-} \quad (7)$$

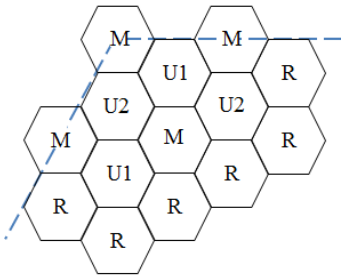
3. Numerical Results

The implemented equivalence TPEN method is examined for the 2-D and 3-D hexagonal problems as shown in Fig. 3. The 19, 37 and 61 assembly problems consists of 7 fuel and 12 reflector assemblies, and 19 fuel and 18 reflector assemblies, and 37 fuel and 24 reflector assemblies, respectively. In the 3-D problem, there are 3 fuel planes with the size of 14.28 cm and 1 reflector plane with 21.42 cm. The reflecting boundary

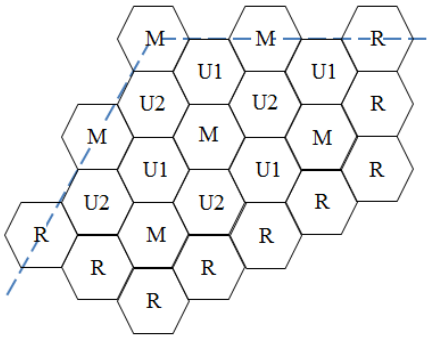
condition is applied at the bottom surface, and the vacuum condition at the radial and the top surfaces. The assembly configuration and the cross sections from the C5G7 hexagonal variation problem [2] are used in DeCART reference calculation.



(a) 19 assembly problem



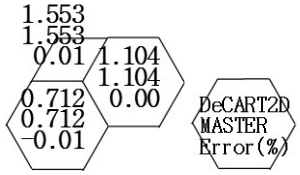
(b) 37 Assembly Problem



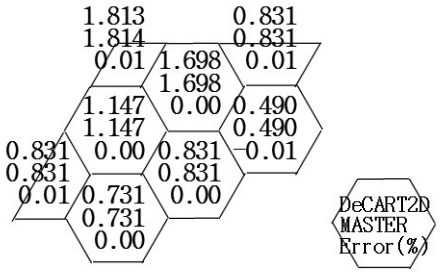
(c) 61 Assembly Problem

Fig. 3 Test Problems

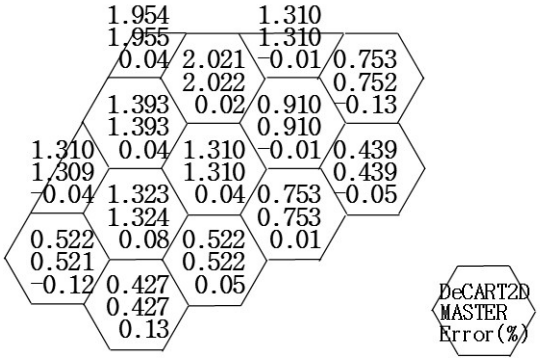
Fig. 4 shows the axially integrated assembly power distribution and errors. MASTER showed essentially the reproducibility of the DeCART. Table I summarizes the errors of the MASTER nodal calculations. MASTER showed some trivial errors mainly due to the treatment of the discontinuity factors and the boundary conditions. The unrealistic discontinuity factors such as negative or very low or high vales at the boundary surfaces are neglected in the MASTER code to exclude for the convergence stability. Also, while the DeCART code uses the reentering neutron model at the non-flat boundary surface, the MASTER code uses the no reentering model. Considering those error sources, the results from the two codes are practically identical. These results mean that the equivalence theory to the whole core transport calculation for the TPEN method explained in Chapter 2 is well implemented to the DeCART code and works soundly.



(a) 19 assembly problem



(b) 37 Assembly Problem



(c) 61 Assembly Problem

Fig. 4. Axially Integrated Power Distribution and Errors for 3-D Test Problems

Table I. MASTER Errors

Problems[1]		ε_k (pcm)[2]	$\varepsilon^n_{p,\max}$ (%)[3]
2-D	19 Assembly	4	0.05
	37 Assembly	1	0.03
	61 Assembly	0	0.02
3-D	19 Assembly	6	0.19
	37 Assembly	4	0.15
	61 Assembly	4	0.18-

1) Eigenvalues are 1.01758, 1.11534 and 1.15092 for the 2-D 19, 37 and 61 assembly problems, and 0.98392, 1.07645 and 1.11000 for the 3-D 19, 37 and 61 assembly problems.

2) $\varepsilon_k = 10^5 \times \left(k_{MASTER} - k_{DeCART}\right)$

3) Maximum Relative Node Power Error

4. Conclusions

This paper proposed an equivalent group constant generation methodology using the whole core transport calculation for the hexagonal TPEN method. The proposed methodology is implemented to the DeCART code and examined for the 3 2-D and 3-D test problems. The test calculation showed that the MASTER code can reproduce essentially the same result of the DeCART code. From these results, it was concluded that the equivalence theory was well implemented to the DeCART code and worked soundly. In the future, this capability is more examined for the realistic hexagonal problems through the DeCART/CHORUS/MASTER system. Also, this code system will be applied to an equivalent calculation for the pin power distribution.

References

1. N. SUGIMURA et al., "Development of Hybrid Core Calculation System using Two-Dimensional Full-Core Heterogeneous Transport Calculation and Three-Dimensional Advanced Nodal Calculation," *J. of Nuc. Sci. and Tech.*, **43**, No. 7, 731 (2006)
2. Cho, Jin Young, et al., 2008. Whole Core Transport Calculation Employing Hexagonal Modular Ray Tracing and CMFD Formulation, *J. of Nucl. Sci. and Tech.*, **45**, No. 8, 740 (2008)
3. Cho, Jin Young et al., "Three Dimensional Nuclear Analysis System DeCART/CHORUS/MASTER," *ANS Annual Meeting*, Atlanta, June 16-20 (2013)
4. Cho, Jin Young, et al., 2001. Hexagonal CMFD Formulation Employing Triangle-based Polynomial Expansion Nodal Kernel, *M&C 2001*, Salt Lake City, USA (2001)
5. T. Downar, Y. Xu, V. Seker, "PARCSv3.0 Theory Manual," UM-MERS-09-001, October (2009)
6. Cho, Byong Oh et al., 2002. MASTER-3.0: Multi-purpose Analyzer for Static and Transient Effects of Reactors, KAERI/TR-2061/2002, Korea Atomic Energy Research Institute (2002)
7. Cho, Jin Young, et al., 2007b. Axial SPN and Radial MOC Coupled Whole Core Transport Calculation, *J. of Nuc. Sci. and Tech.*, **44**, No. 9, 1156 (2007)