Proceedings of the Reactor Physics Asia 2015 (RPHA15) Conference
Jeju, Korea, Sept. 17-18, 2015

Estimation of Axial Peaking Factor Uncertainty by Fuel Fabrication Error using Multiple Correlation Sampling

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1. Introduction

The estimation of uncertainty for the nuclear core design parameter is very important in the nuclear design and safety analyses, code validation, and data evaluation. [1,2] The uncertainties in the nuclear core design parameters will be provided by experimental or code calculation results. However, it is difficult to conduct necessary experiments for a new nuclear design system by the lack of budget and experimental facilities. In such case, the Monte Carlo (MC) calculations would be used as an alternative. The accuracy of the continuous-energy MC calculation benefits from its ability to use continuous energy nuclear data and detailed geometric information. Meanwhile, there are various sources of the uncertainties in the nuclear design parameters, such as the cross section data, geometric information, and material composition data.

Recently, the nuclear fuel technology development group of KAERI (Korea Atomic Energy Research Institute) perform the tests for a new-type mixed metallic-uranium fuel rod. In the manufacturing process of the mixed metallic-uranium pellets, a pressing procedure is needed and may lead to the uncertainties in an axial uranium (U) mass distribution. It may affect the uncertainties in the nuclear core design parameter. This paper estimates the uncertainties of the neutronic design parameters such as peaking factor due to the uncertainties in the axial U mass distribution using MC calculations.

2. Methodology

2.1 Generation of Covariance Matrix in Axial U Mass

Figure 1 shows the relative axial U mass distribution obtained from raw fuel rod samples. [3] A single fuel rod is axially divided into 21 regions. The detail U mass for each axial region samples is provided from the raw data of 33 fuel rods. From Eq.(1) and (2), one can formally define the mean value, $\overline{Q^i}$ and variance, $\sigma^2(Q^i)$.

$$\overline{Q^i} \cong \frac{1}{K}\sum_{k=1}^{K} Q_k^i \qquad (1)$$

$$\sigma^2(Q^i) \cong \frac{1}{K-1}\sum_{k=1}^{K}(Q_k^i - \overline{Q^i})^2. \qquad (2)$$

where Q_k^i is the U mass of the *i*-th axial region for the *k*-th sample. Using a formal definition, the covariance and correlation coefficient matrix for the axial U mass distribution can be obtained from Eq. (3) and Eq. (4). $\mathrm{cov}[Q^i, Q^j]$ and $\rho[Q^i, Q^j]$ are the covariance and correlation coefficients between the U mass at *i*-th and *j*-th axial regions, respectively.

$$\mathrm{cov}[Q^i, Q^j] \cong \frac{1}{K-1}\sum_{k=1}^{K}(Q_k^i - \overline{Q^i})(Q_k^j - \overline{Q^j}). \qquad (3)$$

$$\rho[Q^i, Q^j] = \frac{\mathrm{cov}[Q^i, Q^j]}{\sigma(Q^i)\sigma(Q^j)}. \qquad (4)$$

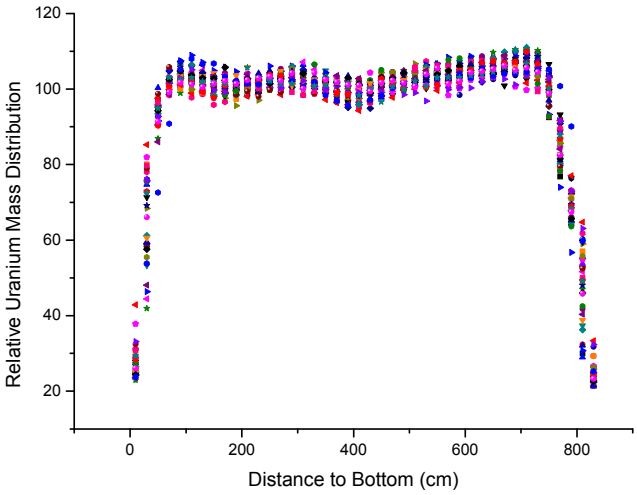

Fig. 1. Relative Axial Uranium Mass Distribution from fuel rod samples.

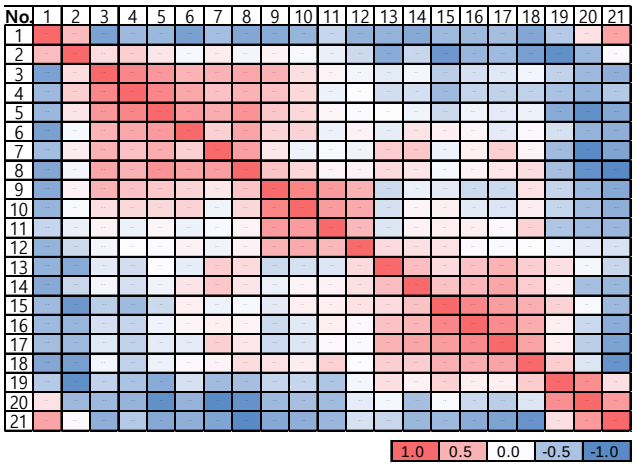

Fig. 2. Correlation Coefficients Matrix for Axial Uranium Mass Distribution

Figure 2 shows the 21x21 correlation coefficient matrix calculated by Eq.(3) and (4) from the raw data of the 33 samples. It was noted that there is a very strong negative correlation between the edge regions and the other regions by the pressing procedure.

2.2 Direct Sampling Method

To consider the multiple correlation as shown in Fig 2, the direct sampling method (DSM) [4], often called the brute force method, was adopted to determine the uncertainties of the nuclear core design parameter. Suppose that $\mathbf{C}_Q$ is the covariance matrix defined by $\mathrm{cov}[Q^i, Q^j]$ and that a lower triangular matrix $\mathbf{B}$ is known through the *Cholesky decomposition* of $\mathbf{C}_Q$ as:

$$\mathbf{C}_Q = \mathbf{B} \cdot \mathbf{B}^T \tag{5}$$

where $\mathbf{B}^T$ is the transpose matrix of $\mathbf{B}$. Then one can sample the axial U mass distribution set by:

$$\mathbf{X}^\kappa = \overline{\mathbf{X}} + \mathbf{B} \cdot \mathbf{Z} \tag{6}$$

$\overline{\mathbf{X}}$ is the mean vector defined by the mean value using Eq.(1). $\mathbf{Z}$ is a random normal vector which can be calculated directly from a random sampling of the standard normal distribution. In this study, $\mathbf{Z}$ vector is calculated using the *Box-Muller* method. The capabilities for the DSM calculations are implemented into MIG code [5].

3. Numerical Results

3.1 Uncertainty Estimation of Sampled Peaking Factor

In order to examine the effect of the axial U mass distribution uncertainty on the mixed metallic fuel pressing process, a pin cell test problem is considered. The height of the active region is 80 cm while the height of the top and bottom reflector regions is 30 cm. The 21x21 covariance matrix $\mathbf{C}_Q$ from the axial U mass distribution as shown in Fig 2 can be generated. After that, the random variate sampling scheme is used to sample 100 McCARD [6] input sets for the MC calculations. In order to generate the sampled inputs for MC calculations, we assume the following condition:

a) The radius of fuel pellet region and cladding region is fixed.
b) The density of uranium at the fuel pellet region depends on the sampled uranium mass.

For a reference calculation, the MC calculation for the fuel pin problem with the axially uniform U mass distribution is performed. All the MC calculations are performed with 50 inactive cycles and 1,000 active cycles of 100,000 neutron histories per cycle with the ENDF/B-VII.1 [7] library. In the calculations, the T/H feedback was not considered and the temperature at all the region was set as 600K. Figure 3 shows normalized fission power distributions from 100 McCARD calculations with the 100 randomly sampled McCARD input sets. The gray line in Fig 3 indicates the average fission power distribution calculated by the 100 results. Figure 4 shows a distribution of the peaking factors calculated by the 100 sample inputs while Table 1 shows the mean and standard deviation of peaking factor for sampling calculations as well as the reference calculation. The mean and standard deviation by DSM, $\overline{\mathbf{X}_S}$ and $\sigma^2(\mathbf{X}_S)$ can be calculated by:

$$\overline{\mathbf{X}_S} \cong \frac{1}{100}\sum_{k=1}^{100} X_k \tag{7}$$

$$\sigma^2(\mathbf{X}_S) \cong \frac{1}{99}\sum_{k=1}^{100} (X_k - \overline{\mathbf{X}_S})^2. \tag{8}$$

where X_k is the peaking factor by a *k*-th input set. $\overline{\mathbf{X}_R}$ and $\sigma^2(\mathbf{X}_R)$ mean the results by the reference calculation and its statistical standard deviation. Meanwhile, the Upper Tolerance Limit (UTL) of the sampled peaking factors can be calculated by:

$$\mathbf{X}_{S,U} = \overline{\mathbf{X}_S} + k_s^{95\%,95\%} \times \sigma(\mathbf{X}_S) \tag{7}$$

$k_s^{95\%,95\%}$ is determined so that the intervals cover at least 95% of the population with 95% confidence [8]. The difference between $\overline{\mathbf{X}_R}$ and $\overline{\mathbf{X}_S}$ is 3.2% while that between $\overline{\mathbf{X}_R}$ and $\mathbf{X}_{S,U}$ is 4.3%.

Table I. Peaking Factor Results by McCARD

Case	Peaking Factors	Standard Deviation	Differ* (%)
Reference($\mathbf{X}_R$)	1.234	0.04%	-
DSM($\mathbf{X}_S$)	1.274	0.53%	3.2%
DSM($\mathbf{X}_{S,U}$)	1.287	-	4.3%

*Diff=$(\mathbf{X}_S - \mathbf{X}_R)/\mathbf{X}_R \times 100$

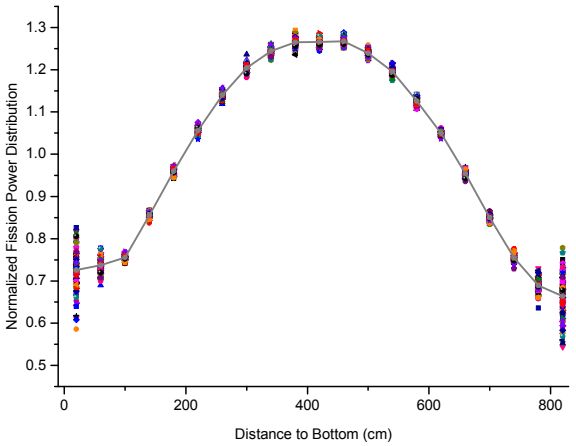

Fig. 3. Normalized Fission Power Distribution in Fuel Pin Problem

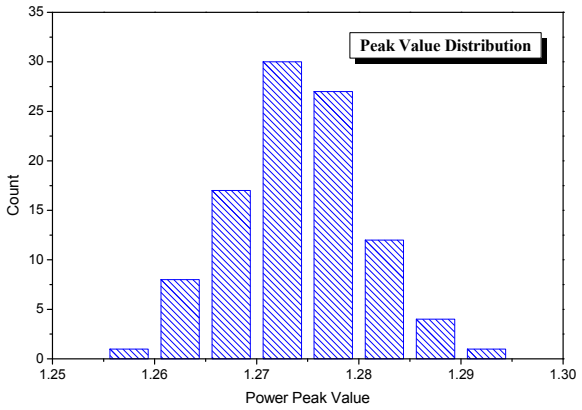

Fig. 4. Distribution of Peaking Factors

4. Conclusions

In this paper, we successfully adopted the DSM method as a way to quantify uncertainties of nuclear design parameters such as peaking factors through MC calculations. From the results by the DSM method, the uncertainty of the axial peaking factor could be calculated as the basic information to perform the safety analysis. Meanwhile, the DSM method needs a lot of computing time and a sophisticated sampling processing. Due to these drawbacks, one may adopt the sensitivity and uncertainty (S&U) analysis or the perturbation-based method as an alternative way to do so. In the near future, the same uncertainty quantification analysis will be tested using the perturbation technique.

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