Reply to reviewer comments, RPHA2023

Paper No. 23A-086

Title: Investigation of the SP3 Method in Fast Rector Physics Analysis

Comment 1:

It is recommended for readers to show the detail specification of problems in the presentation.

Reply:

Sincerely thank you for the comment. My apology for the detail about reactors used in this work is not included. In the upcoming RPHA2023, I will introduce the details about problem specifications and method in the poster session.

Comment 2:

The authors provide interesting topics and conclusions. The reviewer recommends checking the contents of Table 2.

Reply:

Sincerely thank you for this comment.

Table 2 is about the maximum bias on power distribution calculation. The maximum bias in both diffusion and SP_3 calculations occur in the region near control absorber or near reflector (this can be confirmed with Figures 13 and 14). Besides, the value of maximum biases for diffusion and SP_3 calculations is accidentally identical (-5.63% in MET1000) with each other.

In the upcoming RPHA 2023, I will show more details on this.

Additional information

Dear committee:

Sincerely sorry for the inconvenience.

In this paper, the reference data used for comparison were set as S_N (P1-S4) results previously. In this version of revision, the reference was changed to S_N (P0-S4) results, out of comparing with S_N P0-S4 is more reasonable. This is because that the term of 'P0' stands for 'transport-corrected P0 approximation' which is commonly adopted by diffusion and SP_3 calculations as well. Therefore, it is more intuitive to make comparisons between SN (P0-S4), diffusion and SP_3.

Besides, a mistake in the reference results calculation was found and revised recently. So that some figures about reactivity were updated in this revision.

Conclusion of this paper is not changed. The advantage of SP3 solver becomes more obvious according to current paper, consistent biases can be found in both 2D-RZ and 3D-XYZ calculations.

Best Regards

Jun-Shuang FAN.

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Investigation of the SP_3 Method in Fast Rector Physics Analysis

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1. Introduction

The Simplified-P_N theory was proposed by Gelbard [1] initially in 1960. As an approximate approaching to solve the transport equations, the SP_3 theory was developed without solid theory basis in the very beginning, but Larsen et al. [2] confirmed that the SP_N theory is valid through practical application.

The advantage of the SP_3 method in general type of fast reactor physis analysis was observed in the authors' works [3][4]. In these two studies, four sodium-cooled fast reactor concepts were modeled as two-dimensional cylindrical (2D-RZ) geometry. The SP_3 method calculation showed obvious advantage in effective neutron multiplication factor k_{eff} [3], the computing time of the SP_3 method was shorter than the transport method with different computing options, and the results accuracy were better than the diffusion solver results. It was also confirmed the prediction accuracy on medium-wise power level of the SP_3 method was better [3]. Distribution of the error on medium-wise power level is flattened with the SP_3 method. Meanwhile, the SP_3 method exhibited advantage on non-leakage component sodium void reactivity $\Delta\rho_{void}$ [4], in comparison with the diffusion method result. As well as the dominance of non-leakage component reactivity is scattering component, the observed feature of the SP_3 method can be regarded as an advantage on scattering component reactivity prediction, and this feature was observed with three designed void pattern problems.

To confirm that the advantage of the SP_3 method we have found is a universal feature, not an exclusive phenomenon under 2D-RZ calculation, a similar investigation is undertaken in the present study with three-dimensional Cartesian (3D-XYZ) geometry model. The diffusion, transport (S_N) and SP_3 three calculation methods are employed. The same parameters are focused on, which are k_{eff}, medium-wise power level and sodium void reactivity $\Delta\rho_{void}$. Particularly, the component-wise reactivity is discussed.

The structure of this paper is organized as follows: the code system, calculation models, and methods are introduced in Section 2. Section 3 presents a summary and discussion of the results. Conclusion is presented in Section 4.

2. Methods

The scope of this study is four sodium-cooled fast reactors concepts provided by the OECD/NEA benchmark report [5]. Four sodium-cooled fast reactors named as MET-1000, MOX-1000, MOX-3600 and CAR-3600, respectively, were defined in this benchmark. It can be known from the name of reactors that they applied metallic fuel, MOX fuel, MOX fuel and carbide fuel, respectively. Power output of two middle-sized cores is 1000 MWth (MET-1000 and MOX-1000), and 3600 MWth for two large-sized cores (MOX-3600 and CAR-3600).

Diversity on fuel type and core size provides universality of this work. All calculations are undertaken by a general-purpose reactor physics code system CBZ [6].

Motivation of this work is that the characteristic of the SP_3 method can be observed in 3D-XYZ geometry model calculations as well. Details about calculations and results will be explained in the following subsections.

2.1 2D-RZ Model

The reactors are modeled as 2D-RZ geometries, with a visual representation presented in Fig. 1, which is using the MET-1000 reactor as an example.

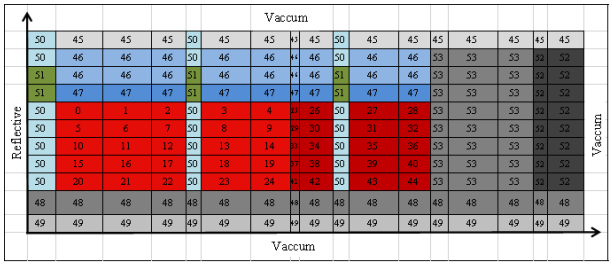

Fig. 1. Schematic cross-section of MET-1000 2D-RZ core model.

In this visual representation, the y-axis corresponds to the axial direction of the mesh position, while the x-axis denotes the mesh position in radial direction. Number within each mesh represents the medium. For instance, the red color regions whose medium number are 0 to 44 belong to active fuel region.

2.2 3D-XYZ Model

The 3D-XYZ core model is constructed by the following steps,

1) assemblies are defined,
2) the defined assemblies are filled into the core according to the actual core configuration, which means that the core is modeled as hexagonal-Z geometry at first,
3) the CBZ code system will transform the core model into XYZ geometry automatically (Fig. 2).

It is notable that the periphery region is filled with

fictitious assemblies that have no medium information.

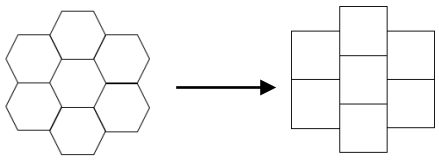

Fig. 2. Schematic of radial configuration of transformation from Hexagonal-Z geometry to XYZ geometry.

2.3 Calculation Methods

Calculations in this work are carried out with the diffusion, transport (S_N), and SP_3 methods.

There are two arguments, P_N and S_N, to control the order of the S_N method in CBZ. P_N indicates the maximum order of the Legendre polynomial for the anisotropic scattering cross-section expansion. S_N is the Discrete Ordinates Method, and its order determines number of discrete points over entire angle. N×(N+2) angles in total are chosen. For instance, 24 angles (three angles in each quadrant) are chosen if N=4.

Sodium void reactivity is calculated by the perturbation theory. CBZ has perturbation calculation capability corresponding to the diffusion, transport and SP_3 solvers, respectively. The perturbation theory could provide details about the contribution of each of the different physical quantities to reactivity. Such detailed information cannot be acknowledged by direct calculation from the k_{eff} values.

To exhibit the advantage of the SP_3 method, a straightforward comparison is employed. Reference results are given by the S_N (P_0-S_4) since the transport-cooP0 is more appropriate to compare with the diffusion and SP_3 methods. The diffusion and SP_3 results are compared to references accordingly.

3. Results

3.1 Comparison in k_{eff}

The relative percent difference between three methods on k_{eff} analysis in the cases of 2D-RZ and 3D-XYZ model is shown in Figs. 3 and 4, respectively.

In both 2D-RZ and 3D-XYZ geometry model calculations, the SP_3 method exhibits better predication on keff. The improvement of keff prediction is significant with the SP_3 method. It cans be found that the relative difference shifted in the positive direction by about 0.1% to 0.2% for both diffusion and SP_3 methods when applying the 3D-XYZ model.

3.2 Comparison in $\Delta\rho_{void}$

The void pattern setting for sodium void reactivity calculation is whole-core void, which is the same void pattern used in the benchmark. Normally, reactivity can be decomposed into several components according to the causes: yield, scattering, absorption and leakage components, and the sum of yield, scattering and absorption components can be regarded as non-leakage component. The sum of non-leakage and leakage component reactivity are net reactivity. Discrepancy in $\Delta\rho_{void}$ will be discussed regarding the component-wise reactivity as well.

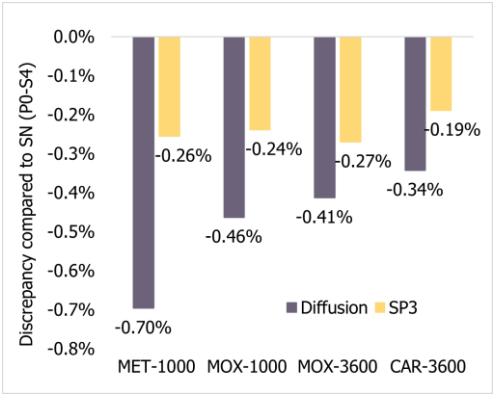

Fig. 3. Relative differences on k_{eff} given by three methods in 2D-RZ geometry calculations.

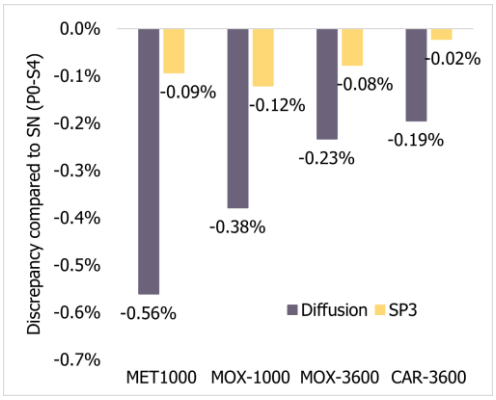

Fig. 4. Relative differences on k_{eff} given by three methods in 3D-XYZ geometry calculations.

Relative difference of results on net $\Delta\rho_{void}$ prediction is summarized in Figs. 5 and 6.

Obvious advantage can be found in two middle-sized cores which are MET-1000 and MOX-1000 among 2D-RZ calculations (Fig. 5). As for two large-sized cores, the SP_3 and diffusion methods give similar results on net $\Delta\rho_{void}$. In 3D-XYZ calculations, the SP_3 method only gives better prediction than the diffusion method on MET-1000 (Fig. 6). Similarly, it can be observed that the relative difference moves towards positive direction simultaneously in both the diffusion and SP_3 methods when applying the 3D-XYZ geometry. More importantly, it can be observed that the SP_3 method overestimated the net $\Delta\rho_{void}$ while the diffusion method underestimated the net $\Delta\rho_{void}$.

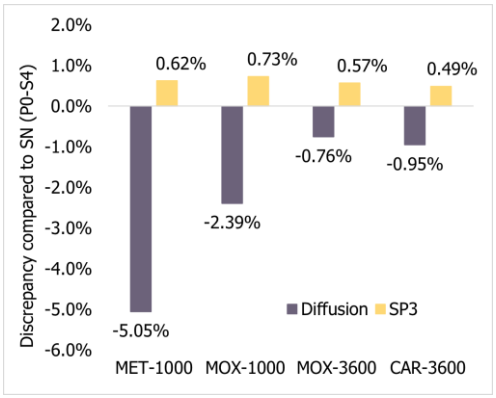

Fig. 5. Relative differences on net $\Delta\rho_{void}$ given by three methods in 2D-RZ geometry calculations.

To further understand the performance of the SP_3 method on $\Delta\rho_{void}$ prediction, results on component-wise reactivity is summarized in the following figures, in which Figs. 7 and 8 are about non-leakage component reactivity, and Figs. 9 and 10 are about leakage component reactivity. Figures. 11 and 12 are about scattering component reactivity.

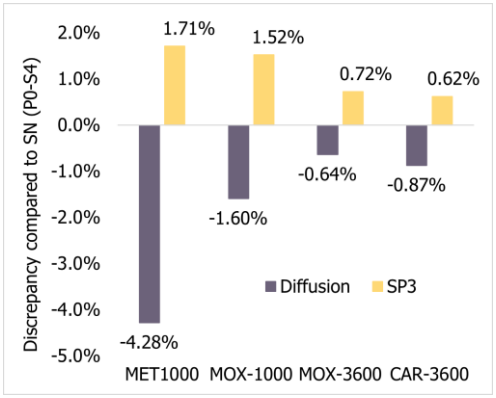

Fig. 6. Relative differences on net $\Delta\rho_{void}$ given by three methods in 3D-XYZ geometry calculations.

Based on the observations from Fig. 7, it is reasonable to conclude that the SP_3 method exhibits advantage over the diffusion method, since the discrepancy in the SP_3 method results is systematically less than it for the diffusion method results. Results in Fig. 8 reveal the fact that the SP_3 method predicts non-leakage component reactivity better than the diffusion method in general, especially for large-sized reactor system.

Regarding the leakage component reactivity, Figs. 9 and 10 illustrate that SP_3 yields worse accuracy prediction than the diffusion method in the 2D-RZ geometry calculations. However, advantage of the SP_3 method on net $\Delta\rho_{void}$ is kept out of the non-leakage component is dominant component.

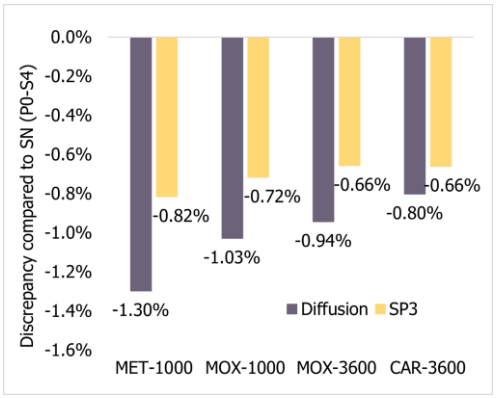

Fig. 7. Relative differences in non-leakage component $\Delta\rho_{void}$ given by three different methods in 2D-RZ geometry calculations.

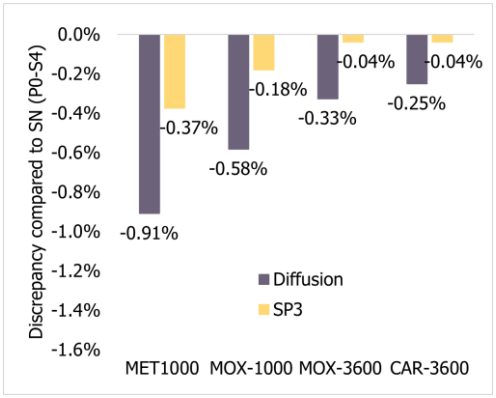

Fig. 8. Relative differences on non-leakage component $\Delta\rho_{void}$ given by three methods in 3D-XYZ geometry calculations.

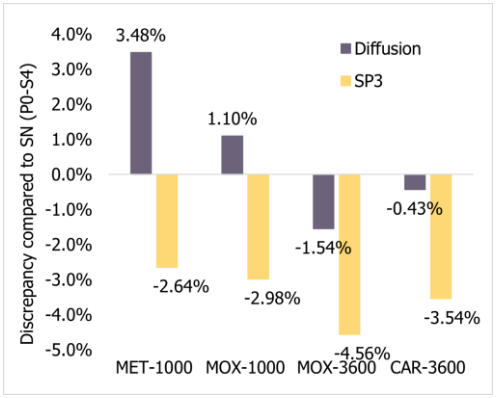

Fig. 9. Relative differences in leakage component $\Delta\rho_{void}$ given by three methods in 2D-RZ geometry calculations.

On the scattering component reactivity, the discrepancy trend is almost identical with the results on non-leakage component (Figs. 7 and 8) since the dominant part of non-leakage component reactivity is scattering component reactivity. Hence, the advantage

of the SP_3 method on scattering component prediction in both 2D-RZ and 3D-XYZ calculations is confirmed.

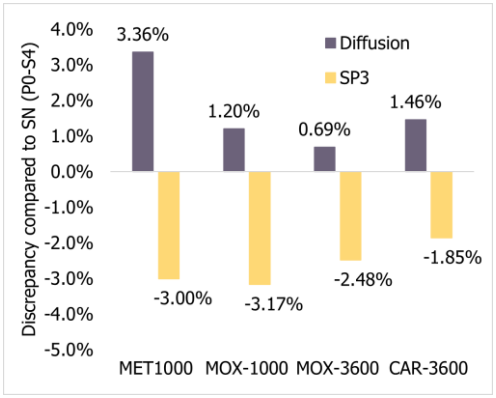

Fig. 10. Relative differences on leakage component $\Delta\rho_{void}$ given by three methods in 3D-XYZ geometry calculations.

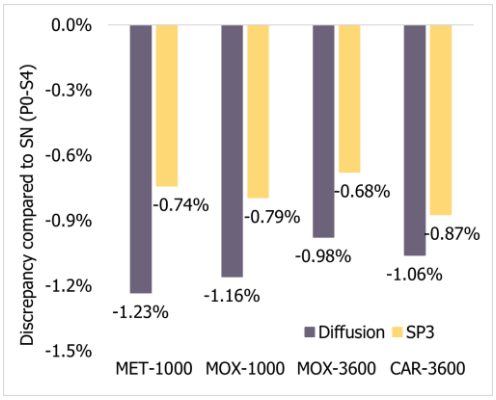

Fig. 11. Relative differences on scattering component $\Delta\rho_{void}$ given by three methods in 2D-RZ geometry calculations.

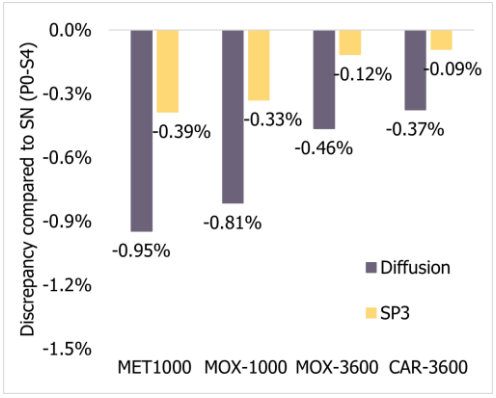

Fig. 12. Relative differences on scattering component $\Delta\rho_{void}$ given by three methods in 3D-XYZ geometry calculations.

In general, advantage of the SP_3 method in reactivity calculation is confirmed with the 3D-XYZ calculations, especially on the scattering and non-leakage components.

3.3 Comparison in medium-wise power level

The characteristic of the SP_3 method on power distribution calculation is more interesting in comparison with the diffusion method. This comparison inspired the authors to investigate the SP_3 method with 3D-XYZ geometry calculations aiming to prove the advantage of the SP_3 method is a general feature.

Figures 11 and 12 exhibit the discrepancy on power distribution results obtained with the diffusion and SP_3 methods in the 2D-RZ calculation (taking MET-1000 as an example).

Table 1. RMSE of power level.

	Diffusion	SP_3
2D-RZ geometry calculations		
MET-1000	1.81%	1.15%
MOX-1000	1.78%	1.16%
MOX-3600	0.75%	0.77%
CAR-3600	0.71%	1.13%
3D-XYZ geometry calculations		
MET-1000	0.53%	0.35%
MOX-1000	0.46%	0.38%
MOX-3600	0.27%	0.52%
CAR-3600	0.32%	0.84%

In Figs. 11 and 12, the x-axis represents mesh position in radial direction, the y-axis represents mesh position in axial direction. the X-Y plane is actually the longitudinal section of 2-D model ofMET-1000. The z-axis is the discrepancy in comparison with the S_N method result. Only the power level of active core region (the red color regions in Fig. 1) is discussed in these two figures.

It is intuitive to noticed that the SP_3 solver gives less bias prediction on power distribution, although in the top of the active core regions both methods give similar discrepancies (these regions are the edge of active core, besides, the larger biases occur in regions near control rod absorber).

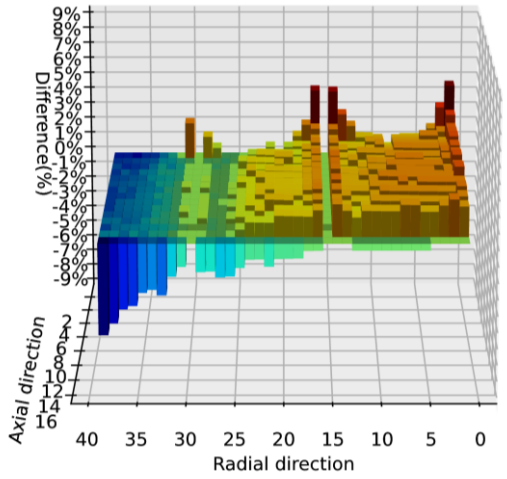

Fig. 13. Relative difference on power distribution in 2D-RZ calculations, diffusion method, MET-1000.

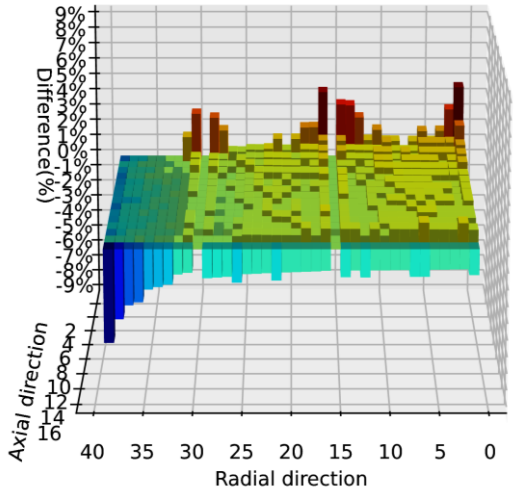

Fig. 14. Relative difference on power distribution in 2D-RZ calculations, SP_3 method, MET-1000.

Then, the RMS of errors on medium-wise power level for four reactors in both 2D-RZ and 3D-XYZ calculations are summarized in Table 1. Particularly, the target medium with 3D-XYZ geometry calculations is set as half of a fuel assembly (corresponding to a coarse mesh in XY plane). Although the target medium in 2D-RZ and 3D-XYZ calculations are different, we can draw a conclusion qualitatively by this comparison that the SP_3 method gives more accurate predictions for two middle-sized cores only in both 2D-RZ and 3D-XYZ calculations.

Table 2. Maximum error on power distribution.

	Diffusion	SP_3
2D-RZ geometry calculations		
MET-1000	-5.63%	-5.63%
MOX-1000	8.0%	8.0%
MOX-3600	4.83%	5.52%
CAR-3600	6.15%	7.14%
3D-XYZ geometry calculations		
MET-1000	-1.24%	0.89%
MOX-1000	-1.36%	0.89%
MOX-3600	-1.09%	1.17%
CAR-3600	-1.47%	2.09%

Then, the maximum error of power level is summarized into Table 2. Based on Table 1 and Table 2, we can say that the SP_3 method can only give less discrepancy result for two middle-sized reactors in both 2D-RZ and 3D-XYZ geometry calculations. This phenomenon indicates the advantage of the SP_3 method is more obvious in small size systems. In other words, the advantage of the SP_3 method is more obvious in a system which neutron leakage impact is more obvious.

4. Conclusions

In this work, four fast reactor concepts were used to investigate the characteristic of the SP_3 method in general type of fast reactor analysis. In comparison with the diffusion solver, the SP_3 method predicted the k_{eff} and non-leakage $\Delta\rho_{void}$ as well as scattering component reactivity with less discrepancy for all four reactors in both 2D-RZ and 3D-XYZ geometry model calculations. Particularly, the trend of biases of the SP_3 and diffusion results on all parameters were consistent, this means the advantage of the SP_3 method is general. Besides, the comparison on power distribution indicated the SP_3 method is more appropriate for smaller size fast reactor analysis. This point was confirmed with both 2D-RZ and 3D-XYZ geometry model calculations.

This study proved the advantage of the SP_3 method we have found is a general characteristic. Consequently, from the viewpoint of practice, the SP_3 method is a favorable choice for general type of fast reactor analysis, and it is more appropriate to be applied in relatively small-sized systems.

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