Feasibility Study on Subcritical Rod Worth Measurement for UTR-KINKI

T. Endo [a*], G. Chiba [b], K. Watanabe [c], C. H. Pyeon [d], G. Wakabayashi [e]
[a] *Nagoya Univ., Furo-cho, Chikusa-ku, Nagoya-shi, Aichi, Japan 464-8603*
[b] *Hokkaido Univ., Kita-ku, Sapporo, Japan 060-8628*
[c] *Kyushu Univ., 744 Motooka, Nishi-ku, Fukuoka, Japan 819-0395*
[d] *Kyoto Univ., Asashiro-Nishi, Kumatori-cho, Sennan-gun, Osaka, Japan 590-0494*
[e] *Kindai Univ., 3-4-1 Kowakae, Higashiosaka-shi, Osaka, Japan 577-8502*
[*] *Corresponding author: t-endo@ energy.nagoya-u.ac.jp*

1. Introduction

In this study, we investigate the feasibility of the subcritical rod worth measurement (SRWM) [1] to UTR-KINKI (University Training Reactor, Kindai University, [2]) through a virtual numerical experiment.

To train human resources for the future nuclear field, we have been developing an advanced educational program for reactor physics experiments by utilizing Japanese educational reactors with recent measurement techniques. The control rod calibration measurement is one of the important reactor physics experiments, to convert from the control rod position to the negative reactivity worth and to confirm the shutdown margin. For this purpose, after a target core maintains a critical state, the period method and the rod drop method are typically utilized. Note that these methods require a long measurement time, although the experimental period is limited during the reactor physics experiment program.

To reduce the experimental time for the control rod calibration measurement, we aim to reuse the experimental result of the "approach to criticality experiment using the control rod," which is firstly conducted for a source-driven subcritical core. In addition, we try to estimate the negative reactivity worth based on an improved method of simplest reactivity estimator (SRE) [3], even if the external neutron source strength Q and the point kinetics parameters (the neutron generation time Λ and the effective delayed neutron fraction β_{eff}) are unknown in advance.

2. Methodology

2.1 Improved Simplest Reactivity Estimator

In a subcritical core with an external neutron source (e.g., Pu-Be neutron source), the point kinetics equations are described as follows:

$$\frac{dP}{dt} = \frac{\rho(t) - \beta_{\mathrm{eff}}}{\Lambda} P(t) + \sum_i \lambda_i C_i(t) + \varepsilon Q, \tag{1}$$

$$\frac{dC_i}{dt} = -\lambda_i C_i(t) + \frac{a_i \beta_{\mathrm{eff}}}{\Lambda} P(t), \tag{2}$$

where $P(t)$ and $C_i(t)$ represent neutron count rate and delayed neutron precursor; λ_i and a_i are decay constant and relative yield ($\sum_i a_i = 1$) for the ith group of the delayed neutron; and ε means the detection efficiency. By dividing Eqs. (1) and (2) by $\beta_{\mathrm{eff}}/\Lambda$ (i.e., prompt neutron decay constant at a critical state) and neglecting the time derivative of $P(t)$, the following equations for the improved SRE are obtained:

$$0 = \left(\frac{\rho(t)}{\beta_{\mathrm{eff}}} - 1\right) P(t) + \bar{D}(t) + S, \tag{3}$$

$$\frac{dD_i}{dt} = -\lambda_i D_i(t) + \lambda_i P(t), \tag{4}$$

$$D_i(t) \equiv \frac{\lambda_i \Lambda C_i(t)}{a_i \beta_{\mathrm{eff}}}, \tag{5}$$

$$\bar{D}(t) \equiv \sum_i a_i D_i(t), \tag{6}$$

$$S \equiv \frac{\varepsilon \Lambda Q}{\beta_{\mathrm{eff}}}, \tag{7}$$

where $\bar{D}(t)$, $D_i(t)$ and S are introduced for convenience. Based on Eqs. (3), (4), and (6), we improved SRE to estimate $\rho(t)/\beta_{\mathrm{eff}}$ without prior knowledge of Λ and β_{eff} for the target subcritical core. Note that the values of λ_i and a_i are already known because the major fission reaction in UTR-KINKI is due to ^{235}U induced by thermal neutrons. When the time series data of neutron count P_n are successively measured for the nth time steps during a counting gate width Δt, the procedures of our improved SRE are described as follows:

1. Each $D_{i,0}$ value is initialized as $D_{i,0} = P_0$ by assuming the steady state.
2. $D_{i,n}$ is recurrently estimated based on the analytical solution of Eq. (4) with the linear approximation for the measured time variation in $P(t)$ during Δt.

$$D_{i,n} \approx D_{i,n-1} \mathrm{e}^{-\lambda_i \Delta t} + \left(\frac{1 - \mathrm{e}^{-\lambda_i \Delta t}}{\lambda_i \Delta t} - \mathrm{e}^{-\lambda_i \Delta t}\right) P_{n-1} + \left(1 - \frac{1 - \mathrm{e}^{-\lambda_i \Delta t}}{\lambda_i \Delta t}\right) P_n. \tag{8}$$

3. Using Eq. (3) with measured P_n and estimated $\bar{D}_n$, reactivity in dollar units can be inversely estimated:

$$\frac{\rho_n}{\beta_{\mathrm{eff}}} = 1 - \frac{\bar{D}_n + S}{P_n}. \tag{9}$$

2.2 Calibration of External Source Strength in SRE

In the improved SRE of Eq. (9), the external source strength S should be appropriately calibrated to determine the reactivity. To calibrate S by a data-driven approach, we apply the least squares method for measured P_n and estimated $\bar{D}_n$ [4]. Let us assume that P_m is successively measured during a transient period after the control rod withdrawal at the last step (i.e., the shallowest subcritical state of ρ_{last}) in the approach to criticality. Then, Eq. (3) can be transformed into the following linear relationship between P_m and $\bar{D}_m$:

$$P_m = (\bar{D}_m + S)\Big/\left(1 - \frac{\rho_{\text{last}}}{\beta_{\text{eff}}}\right). \qquad (10)$$

Therefore, by fitting Eq. (10) for the dataset of $(\bar{D}_m, P_m)$, the unknown S can be obtained as the intercept value.

3. Virtual Numerical Experiment

3.1 Calculation Conditions

Using the virtual console of UTR-KINKI, typical digital data of shim safety rod (SSR) positions z_n were recorded in an experiment of approach to criticality, where the SSR was withdrawn step-by-step from the fully inserted state to the near-critical state. A reference value of SSR worth $\Delta\rho_{\text{ref}}(z)$ was virtually given for $0 \leq z \leq 100$ based on the following empirical formula:

$$\Delta\rho_{\text{ref}}(z) = \Delta\rho_{\text{ref}}(100)\frac{1-\cos(2Bz)}{1-\cos(200B)}, \qquad (11)$$

where we assumed that $\Delta\rho_{\text{ref}}(100) = 0.74$ \$ and $B = 0.014$ in this study; and $z = 0$ and 100 correspond to the fully inserted and withdrawn SSR positions, respectively. Then, the target core was assumed to become a critical state at the SSR position $z = 90$. By applying the frequency transform and Crank-Nicolson methods with $\Delta t = 0.2$ s to Eqs. (1) and (2), the time series data of P_n were numerically calculated using Eq. (11) with recorded time series data z_n and. Finally, the virtual experimental data of P_n were generated by adding statistical noises based on the Poisson distribution as shown in Fig. 1.

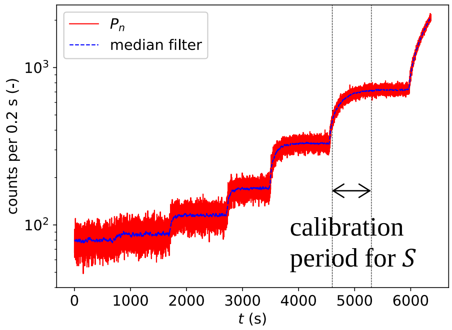

Fig. 1. Virtual experimental data simulating the approach to criticality.

3.2 Numerical Results

As can be seen from Fig. 1, the statistical error of the neutron count becomes relatively larger as the subcriticality $-\rho$ deepens, because the magnitude of the neutron count decreases inversely proportional to $-\rho$. In order to mitigate the statistical variation of P_n, the median filter technique [5] was applied to the time series data of P_n. After that, as shown in Fig. 2, the external source term S was calibrated by the least square fitting of Eq. (10) within a period after inserting SSR at $z = 80$.

Figure 3 shows the estimated SSR worth using the improved SRE with the calibrated S. Consequently, we demonstrated that the estimated results by the improved SRE were in good agreement with the reference values using Eq. (11). Note that the estimated rod worth statistically fluctuates due to the statistical errors of P_n. Therefore, some kind of smoothing procedure is necessary to obtain the rod worth curve, e.g., the least squares method using a fitting formula (e.g., Eq. (11)).

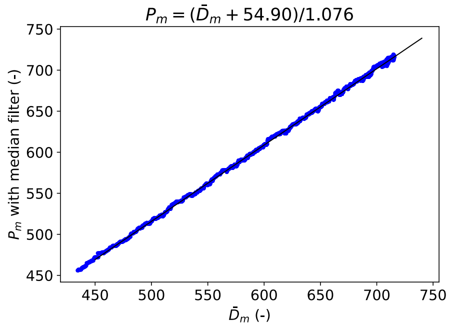

Fig. 2. Calibration of S by the least squares method.

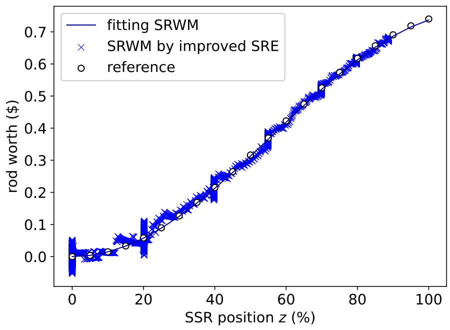

Fig. 3. Estimation results of SSR rod worth.

4. Conclusions

Through this virtual numerical experiment simulating the approach to criticality in UTR-KINKI, we suggested that the experimental data of the approach to criticality is also available for the SRWM using our proposed method. Future research topics are to analyze the actual experimental data measured at UTR-KINKI and to investigate the spatial higher mode effect on the SRWM.

REFERENCES

[1] A. Kauchi, M. Nakano, J. Kimura, Y. A. Chao, Development of Sub-critical Rod Worth Measurement, Proceedings of 2004 Fall Meetings of Atomic Energy Society of Japan, Sep. 15–17, 2004, Kyoto, Japan.

[2] G. Wakabayashi, T. Yamada, T. Endo, C. H. Pyeon, Introduction to Nuclear Reactor Experiments, Springer, Singapore, 2023.

[3] Y. Shimazu, A Simple Procedure to Estimate Reactivity with Good Noise Filtering Characteristics, Annals of Nuclear Energy, Vol.73, pp. 392–397, 2014.

[4] J. E. Hoogenboom, A.R. van der Sluijs, Neutron Source Strength Determination for On-Line Reactivity Measurements, Annals of Nuclear Energy, Vol.15, pp. 553–559, 1988.

[5] G. R. Arce, Nonlinear Signal Processing: A Statistical Approach, Wiley-Interscience, NJ, 2004.