Research on Monte Carlo Homogenization of a Small Prismatic HTGR

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1. Introduction

The High Temperature Gas-cooled Reactor (HGTR) has been well-known for its inherent safety and high coolant outlet temperature, which make it a great choice of versatile small modular reactor design. However, the double-heterogeneity of TRISO fuel elements used in the HTGR and the strong leakage and spatial coupling effects caused by the long neutron mean free path in graphite become challenging for conventional nuclear design codes, especially for the lattice codes. Therefore, the Monte Carlo codes have been extensively utilized to generate group constants for HTGR owing to its powerful geometry modelling capability and applicability for arbitrary energy spectrum. The Monte Carlo codes can obtain accurate energy spectrum without complex resonance calculation procedures as in deterministic lattice codes. In addition, they can be extended to explicitly model the coated fuel particles embedded in matrix. The remaining challenge, i.e., the strong leakage and spatial coupling effects, is the focus of this work.

In this paper, different homogenization schemes are proposed and compared for a small prismatic HTGR based on the Monte Carlo homogenization method.

2. Methodology

In this section, the homogenization methods, schemes and the codes used in this paper are described.

2.1 Monte Carlo group constants generation Method

Based on reaction rates conservation principle, the multigroup cross sections can be obtained with groupwise reaction rates and flux tallies, as shown in Eq. (1).

$$\Sigma_{k,g} = \frac{\int \int_{E_{g-1}}^{E_g} N_{i,k} \sigma_i(E) \Phi(E) dE dV}{\int \int_{E_{g-1}}^{E_g} \Phi(E) dE dV}$$
(1)

The code used in this study, RMC, is a versatile Monte Carlo code developed for reactor physics analysis [1,2] and is often used to provide a highfidelity solution for reference. In addition, a lot of researches about group constant generation have been carried out with RMC [3,4]. In this paper, the continuous-energy point-wise cross sections generated from ENDF/B-VII.1 libraries were used with RMC.

2.2 Leakage effect

In order to investigate the influence of leakage effect, different geometry models were proposed to generate group constants, which are described below.

1) Whole Core Model

The core model used in this paper is a small prismatic HTGR, as shown in Fig.1. All fuel blocks are of the same configuration except that fuel blocks in Zone1 positions have burnable poison (BP) rods to depress the central power. There are two sets of control rods (CR). One set is in the active core while the other set is in outer reflector adjacent to fuel blocks. Compared to the long neutron mean free path in the graphite, the core is small and the leakage effect plays an important part for group constants generation. Since the whole core model (WCM) has accounted for the real boundary conditions, the energy spectrum used to condense group constants is of the highest fidelity and thus the WCM is expected to be the most accurate. The WCM was rarely used for cross sections generation in the past mainly because of its massive calculation cost and the capability limit of conventional lattice codes.



Fig. 1. Whole core model

2) Single Assembly Model

The single assembly model (SAM) is universally used for fuel assembly homogenization while adopting the reflective boundary conditions. It was proposed based on infinite lattice assumption, which is mostly true for large commercial PWRs. It is much easier to generate homogenized group constants with this geometry model at low calculation cost.

3) Supercell Model

To take account for the leakage effect and the influence of surrounding assemblies, the Super Cell Model (SCM) has been commonly used to generate

cross sections for reflectors in PWR and various assemblies in fast reactors [5].

In this paper, two typical supercell models are selected for the core configuration shown in Fig.1. The CR01 model was used to generate group constants for the inner fuel and the CR assemblies while the CR2-Reflector model was used to generate group constants for the outer fuel, the CR, and the reflector blocks. Their radial configurations are shown in Fig. 2.



Fig. 2. CR01 model(left) and CR2-Reflector model(right)

2.3 SPH method

The Super Homogenization (SPH) method is one of the most common equivalence homogenization methods. SPH factors are calculated for each group and each region based on reaction rates conservation principle and used to correct group constants directly without extra parameters introduced in the homogenous calculations. The SPH factor is calculated by dividing the reference heterogeneous flux with the homogenous flux, as shown in equation (2), which is then used to correct the group constants as shown in Eq. (3):

$$\mu_{k,g}^{(n+1)} = \frac{\Phi_{k,g}}{\tilde{\Phi}_{k,g}^{(n)}}$$
(2)

$$\widetilde{\Sigma}_{k,g}^{(n+1)} = \mu_{k,g}^{(n+1)} \Sigma_{k,g} \tag{3}$$

In our implementation, multigroup cross sections and reference multigroup flux are provided by RMC heterogeneous calculation with continuous-energy point-wise cross sections while the iterative homogenous calculation can be conducted with RMC multigroup calculation or the SaraGR code [6].

2.4 The SaraGR Core Code

The SaraGR core code solves multigroup transport equation on 3D core with homogenized assembly geometry using the SN nodal method [7]. Microscopic cross sections for different nuclides are pre-generated and included in the group constant libraries, and microdepletion model is adopted to account for the spectral history effect. The temperature and burnup dependencies are the main factors for the cross section parameterization.

3. Homogenization Results and Analysis

In this section different homogenization schemes were investigated and compared.

3.1 Energy structure and leakage effect

Based on the WCM, several energy structures including 2-group, 4-group, 8-group, 16-group, 25-group, 40-group and 70-group energy structures from the CASMO code [8] have been used to generate group constants. The keff discrepancy of the corresponding core calculations are shown in Fig. 3. Conventional 2-group energy structure used in PWR core calculations produced a keff discrepancy of more than 3000 pcm and is unsatisfactory for small reactors with strong leakage. As the number of energy groups increase, the keff discrepancy decreases rapidly. When the number of energy groups excels 25, the gain of keff precision is negligible. Therefore, the 25-group energy structure is adopted for further investigations.



Fig. 3. k_{eff} discrepancy for different group structures

In order to investigate leakage effect, five cases were constructed and described as follows:

- Case 1: group constants for all assemblies with WCM.
- Case 2: group constants for Zone 1 fuel blocks with SAM, others with WCM.
- Case 3: group constants for Zone 1/2/3/4 fuel blocks with SAM, others with WCM.
- Case 4: group constants for Zone 1/2 fuel blocks with CR01 model, others with WCM.
- Case 5: group constants for Zone 1/2 fuel blocks with CR01 model, those for Zone 3/4 fuel blocks with CR2-Reflector model, others with WCM.

The keff results of all the cases are listed in Table I. Compared with Case 1, Case 2/3 replace fuel block group constants with those from single assembly models and thus introduce an extra keff error of about 700pcm. Case 4/5 replace fuel block group constants with those from the CR01 model and the CR2-Reflector model, produce almost the same keff results with Case 1.

	$k_{e\!f\!f}$	$\varDelta k_{e\!f\!f}$
Case 1	1.02873±0.00023	0.00738
Case 2	1.03525 ± 0.00024	0.01390
Case 3	1.03565 ± 0.00025	0.01430
Case 4	1.02884 ± 0.00023	0.00691
Case 5	1.02708 ± 0.00024	0.00749

Table I: *k_{eff}* results of different models

Since group constants of Zone 1 fuel blocks generated with different geometry models introduce the largest error, the energy spectrum used to condense group constants are compared and shown in Fig. 4. It can be seen that energy spectrum of CR01 model is almost the same with WCM while that of SAM is remarkably harder than that of WCM. As BP rods are located around the boundary in Zone1 fuel blocks, the surrounding blocks have a significant impact on the energy spectrum of BP rods. Therefore, the harder spectrum in SAM results in a decline of thermal absorption cross section and thus a higher keff.



Fig. 4. Energy spectrum for Zone 1 fuel block in different models

3.2 SPH method

Since keff discrepancy is higher than 500 pcm for both WCM and SCM, SPH method is considered to further reduce the error.

In terms of SCM, the CR01 and CR2-Reflector models conduct the SPH calculations separately. The results are shown in Table II. Keff discrepancies are reduced from 450~750 pcm to 100~200 pcm with SPH corrected group constants.

Table II: k_{eff} results for d	fferent geometry models
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	CR01 model		CR2-Reflector model	
	$k_{e\!f\!f}$	Δk_{eff}	$k_{e\!f\!f}$	Δk_{eff}
CE	1.19219	-	0.77666	-
MG	1.19693	0.00474	0.78413	0.00747
MG-SPH	1.19427	0.00208	0.77532	-0.00134

The results of core homogeneous calculations with group constants generated from WCM and SCM plus SPH corrections are listed in Table III.

Table III: *k_{eff}* results of core calculations with SPH corrected group constants

	$k_{e\!f\!f}$	$arDelta k_{e\!f\!f}$
CE	1.02135 ± 0.00024	-
WCM	1.02464 ± 0.00023	0.00329
SCM	1.02277±0.00023	0.00142

After SPH correction, both WCM and SCM are able to reduce the keff discrepancy to less than 500 pcm. SCM gives a relatively smaller keff discrepancy than WCM because the outer fuel blocks in CR2-Reflector model have much stronger leakage than WCM. It can also be seen from the relative errors (RE) of the assembly power distributions shown in Fig. 5. The power discrepancy for WCM is smaller than that of SCM in general. Because fuel blocks in CR01 model with reflective boundary take no consideration of the core leakage while fuel blocks in CR2-Reflector model with vacuum boundary have stronger leakage, the power of inner fuel blocks are relatively higher and that of outer fuel blocks are lower in the case with SCM.



3.3 geometry simplification

In this part, several geometry modifications are introduced. First, the radial reflector boundary is modified from rings to hexagonal grids, which is consistent with fuel assembly grids and is easier to construct the homogeneous geometry in the SaraGR core calculations. Second, the radial core geometry is simplified to a sixth of the core due to its symmetry. Third, since fuel assemblies are the same in different axial layers, only one fuel layer was modeled here and the final geometry model is close to a 2D model with a radial combination of the CR01 and CR2-Reflector models, as shown in Fig. 4.



Fig. 4. A sixth of whole core with assembly heigh

Since axial leakage effect was omitted in this model, the final homogeneous calculations with fuel block group constants generated from this model and others from WCM give a higher k_{eff} and the keff discrepancy is about 400 pcm. Nevertheless, the axial leakage effect is not so prominent and the k_{eff} discrepancy introduced is acceptable. Therefore, this model is adopted to perform burnup calculations and generate group constants for fuel blocks at different burnup and temperature points for downstream homogenous core calculations.

3.4 Cases with control rod inserted

Based on WCM and SPH correction, cases with CR1 inserted or CR2 inserted were calculated and the results are listed in Table IV.

Table IV: <i>k_{eff}</i> results of core calculations with	SPH
corrected group constants	

CR1 inserted	CE	MG-SPH
<i>k_{eff}</i>	0.78538	0.78723
$\Delta k_{e\!f\!f}$	-	0.00185
CR worth /pcm	26271	26036
CR worth RE	-	-0.9%
CR2 inserted		
$k_{e\!f\!f}$	0.90837	0.92676
$\varDelta k_{e\!f\!f}$	-	0.01839
CR worth /pcm	11723	9719
CR worth RE	-	-17.1%

In CR1 inserted case, the multigroup homogeneous calculation with SPH corrected group constants give a satisfactory result and the keff discrepancy is within 200 pcm. The relative error of the integral worth of CR1 is within 1%.

In CR2 inserted case, although the SPH corrected group constants are used, the keff discrepancy of the homogeneous calculation is up to 1839 pcm and the relative error of the integral worth of CR2 is about 17%. The most remarkable difference between CR1 and CR2 is that CR2 rods are eccentrically located in hexagonal blocks. Because they are closer to fuel blocks, the absorption rates are higher. When the CR2 block is

divided into more than 1 material zone, group constants for the zone containing CR rod could be significantly different from those for other zones and the precision of the homogeneous calculation could be improved. Another feasible solution is to adjust the absorption cross section of CR2 blocks to keep keff constant, which requires no modification for the core code. Hence, an adjustment of the CR2 block absorption cross sections is adopted in this paper. Similarly, the group constants for CR2 block with CR out were also adjusted by a factor to account for the neutron stream leaked from void regions and make keff consistent with the reference solution.

4. Core Homogenization Calculation Results

Based on the group constants generated from the above homogenization schemes, the SaraGR core code is used to conduct the transport-depletion calculations. The results of two cases are shown in Fig. 5. Multigroup libraries with no adjust factor are used in Case 1 (SaraGR1) while libraries with adjusted CR2 block group constants are used in Case 2 (SareGR2). Compared with the reference solution provided by RMC calculations with continuous-energy point-wise cross sections, the keff discrepancy of both cases were within 500pcm for all burnup points. The discrepancies between Case 1 and Case 2 are almost constant within the whole lifetime, which indicates that the adjust factors only need to be calculated once and work for different burnup points.

In addition, the assembly power distributions for beginning of lifetime (BOL), middle of lifetime (MOL) and end of lifetime (EOL) are compared with reference solutions for Case 2. It can be seen from Fig.6 that the maximum power relative errors are within 1.5%.



Fig. 5. Keff variations with burnup for different cases



5. Conclusions

Different homogenization schemes for a small prismatic HTGR have been investigated based on Monte Carlo homogenization method in this paper. It can be concluded that the homogenization scheme with 25-group energy spectrum, whole core model and SPH correction gives the satisfactory homogeneous calculation precision for small HTGRs with strong leakage. As for eccentrically located CR blocks, an adjustment of absorption cross sections to keep keff constant was proven to be feasible and reliable. The next step is to refine the material mesh in those blocks to improve precession for strong absorbers and void regions.

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