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Bifurcation Analysis of Pressure Drop Oscillations in Two-Phase Flow System Using Different Model

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Abstract – *Two-phase flow instabilities (e.g. pressure drop oscillations (PDOs)) can affect the performance of the system, leads to thermal oscillations, and in turn can result in premature critical heat flux. The linear and nonlinear stability analysis of PDOs in a two-phase flow system using two and three equations model is carried out by MATCONT software. For analysis of PDOs, steady state internal pressure drop of heater channel is obtained by polynomial fitting. After analyzing of PDOs, the following results are observed. The inlet mass flow into surge tank from main tank is almost constant during oscillations in three equation model; hence two-equation model is expected to be reasonably good for the analysis. The linear stability or local stability analysis for obtaining stable and unstable region shows that the results with the two models are essentially same. However, the non-linear stability analysis shows that the two-equation model is unable to capture the transition of the system from subcritical to supercritical Hopf bifurcation. It is seen that three equation model is able to capture the transition of the system from subcritical (hard) to supercritical (soft) Hopf bifurcation.*

I. INTRODUCTION

Thermal flow instabilities of two-phase flow system are observed in many industrial systems, heat exchanger of chemical industries, nuclear reactors, and thermal power plants. These kinds of instabilities are unfavorable for systems operation as they may cause several problems like thermal fatigue, premature critical heat flux, burnout of heater tube, mechanical vibration of system etc. [1]–[4]. Thermal flow instabilities (two-phase flow instabilities) broadly can be classified into two categories, namely, static where transients are not important but in the case of dynamic transients are quite important. Ledinegg instability is a static instability which is widely researched where flow excursion occurs. Whereas among dynamic instabilities, pressure drop oscillations (PDOs), density wave oscillations (DWO), and thermal oscillations are most widely observed and investigated experimentally as well as theoretically [1]–[5].

PDOs (times periods are very large compared to the fluid particle) have been first introduced by Stenning and Veziroglu [14]. They have been investigated that PDOs are observed in the two-phase flow systems in the presence of negative slope between pressure drop and mass flow rate into the heater section, and a compressible volume which may be present inside channel due to flexible hose or gas trapped inside channel [1], [4], [5]. It may also be placed at upstream of the heater section using a surge tank. During pressure drop oscillations, mass flow rate and channel pressure fluctuate in a regular manner with larger time periods and amplitudes between sub-cooled liquid and super-heated vapor at the exit of the system. These lead to vibration and thermal oscillation in the heater wall which may cause pipe break down and thermal fatigue [2].

A few investigations on nonlinear stability analysis (mainly bifurcation) of PDOs are carried out by some authors earlier. The first report on bifurcation analysis of PDOs is reported by Padki et al. using lumped parameter integral method [6]. Analytically; they have derived the criteria for the occurrence of PDOs and Ledinegg instability. The limit cycles (PDOs) occur after supercritical Hopf bifurcation where heat input to fluid is considered as a bifurcation parameter. Another study of bifurcation is reported by Liu et al. using lumped parameter planar model in which dynamical simulations are compared to experimental results [7]. They have found PDO limit cycles by varying bifurcation parameter (mass flow rate) at constant heat input. The earlier studies have used both two equations as well as three equation models [6], [7].

It is noted that in the earlier studies only one parameter is varied to carry out the bifurcation (nonlinear stability) analysis. It is also pointed out that the Liu. et. al. [7] have mentioned that the two-equation model is sufficient for the stability analysis. In the present work, two parameters have been simultaneously varied to identify the transition from subcritical to supercritical Hopf bifurcation in the parameter space. Such transition occurs at a point known as Generalized Hopf point and represents Bautin (or GH) bifurcation. Furthermore, two equation and three equation models are compared and it is observed that though linear stability analysis is unaffected by the choice of the model, the non-linear stability is affected quite significantly. It is pointed out that non-linear stability is important as linear stability analysis is valid only for infinitesimally small perturbation and nearer to stability boundary only.

II. MATHEMATICAL FORMULATION

In the present study, PDOs are numerically investigated in a heated channel with different heat inputs as well as mass flow rates. The data used here is same as that used by Kakac et al. [8]. Since periods of PDOs is large (~1-2 minutes), the flow is assumed to be quasi-steady and each point of steady characteristics curve of the heated channel corresponds to pressure oscillations. The lumped parameter model is used for analysis of PDOs in the heated channel with different heat inputs. The schematic diagram for analysis of PDOs using three equation model is shown in Fig. 1 and the inset shows the two equation model diagram. A mathematical model of PDOs are constructed with assumptions described in the literature [6], [7], [9]:

(i) System exit pressure Pe remains constant.

(ii) Surge tank temperature is constant during oscillations.

(iii) The inlet temperature is constant and

(iv) Instant mass flow rate between the main tank to surge tank and surge tank and exit of the system is constant.

1. Three-Equation Model of the System

Continuity equation of surge tank (ST) with the presence of ideal gas in surge tank,

$$\frac{dP_S}{dt} = P_S^{\,2}\,\frac{(M-m)}{(P_O V_O \rho_l)}. \tag{1}$$

Momentum equation between surge tank and main tank,

$$\frac{dM}{dt} = \frac{A}{L_1}\left(P_i - P_S - \frac{K_i M^2}{\rho_l A^2}\right). \tag{2}$$

and momentum equation between surge tank and exit of the heater,

$$\frac{dm}{dt} = \frac{A}{L}\left(P_S - P_e - (P_S - P_e)_S\right). \tag{3}$$

where $\left(P_S - P_e\right)_S$. is steady state pressure drop or internal pressure drop of the heated channel from surge tank (heater inlet) to system exit. In the present study, $\Delta P = \left(P_S - P_e\right)_S$. is determined by polynomial fitting which is a function of independent variables, heat input (H) and operating mass flow (M_O) with form:

$$\Delta P(M_O, H) = a + bM_O + cH + dM_O^2 + eM_O H + fH^2 + gM_O^3 + hM_O^2 H + iM_O H^2 + jM_O^4. \tag{4}$$

where a, b, c, d, e, f, g, h, i, and j are constant coefficients of the polynomial which are obtained by MATLAB.

2. Two-Equation Model of the System

Several authors reported about inlet mass flow into surge tank from the main tank is almost constant (maximum variation being ±5 % of its operating value) which leads to the two-equation model [7] [10]. The schematic leading to this model for analyzing of a system for PDOs is shown in inset of Fig. 1. Hence, equation (2) can be removed from the three equation model, leading to a two equation model as follows:

Continuity equation,

$$\frac{dP_S}{dt} = P_S^{\,2}\,\frac{(M-m)}{(P_O V_O \rho_l)}. \tag{5}$$

Momentum equation,

$$\frac{dm}{dt} = \frac{A}{L}\left(P_S - P_e - (P_S - P_e)_S\right). \tag{6}$$

3. Non-Dimensionalization

For analyzing the stability of the system, generally, the equations are considered in their non-dimensionalized form. The steady state mass flow ($M_O = M = m$) and P_O of surge tank are taken as reference point for non-dimensionalization.

For three equations model, non-dimensional equations are obtained as:

$$\frac{d\overline{P}}{d\tau} = (\overline{P}+1)^2(\overline{M} - \overline{m}). \tag{7}$$

$$\frac{d\overline{M}}{d\tau} = REu\left(-\overline{P} - K_i M_o^{\,2}\,\frac{(2\overline{M} + \overline{M}^2)}{\rho_l P_O A^2}\right). \tag{8}$$

$$\frac{d\overline{m}}{d\tau} = rEu\left(\overline{P} - \Delta\overline{P}(M_O) - \Delta\overline{P}(M_O + M_O\overline{m}, H)\right). \tag{9}$$

And in the two equation model, as follows:

$$\frac{d\overline{P}}{d\tau} = -(\overline{P}+1)^2\,\overline{m}. \tag{10}$$

$$\frac{d\overline{m}}{d\tau} = rEu\left(\overline{P} - \Delta\overline{P}(M_O) - \Delta\overline{P}(M_O + M_O\overline{m}, H)\right). \tag{11}$$

Note that the equilibrium point is at the origin ($\overline{P}$, $\overline{M}$, $\overline{m}$) = (0, 0, 0) or ($\overline{P}$, $\overline{m}$)=(0, 0), for three equation and two equation model, respectively. Non-dimensional parameters are given in Appendix.

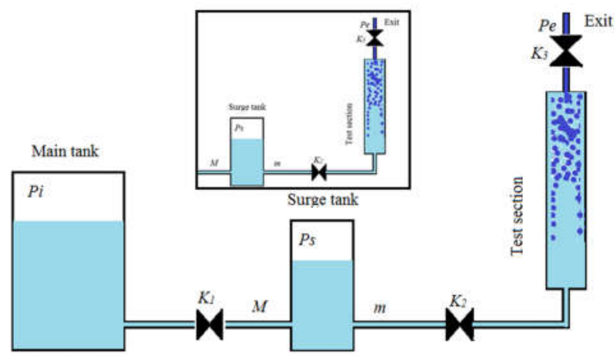


Fig. 1. Schematic diagram for analysis of PDOs using three equation model and the inset shows the schematic for two equation model.

4. Stability Analysis of the System

Stability analysis of a dynamical system corresponds to a perturbation to equilibrium point which may be due to external or internal disturbance. An equilibrium point of a system is considered to be stable if the system returns to its original equilibrium point after the introduction of a small perturbation. In the case of an unstable system, perturbation grows with respect to time and reaches a new operating point. In 2-D parameter space; stable region and the unstable region is divided by a boundary which is known as stability boundary (SB).

In the present study, the linear stability analysis of PDOs in the system, either equations 1-3 or equations 5-6; is carried out to obtain stability boundary (SB). The stability boundary (SB) is derived by linearizing the system of equations (either Eqs. 7-9) or (Eqs. 10-11) depending on the model used, with respect to the equilibrium point (0, 0, 0) or (0, 0), respectively. The stability behavior of PDOs of the system is identified by characteristics of the eigenvalues of the Jacobian matrix. Analysis of PDOs in the system is said to be stable if all the eigenvalues are having a negative real part corresponding to an equilibrium point. On the other hand, analysis of PDOs in the system is said to be unstable if at least one eigenvalue is having positive real part related to a certain equilibrium point.

III. RESULTS AND DISCUSSIONS

1. Validation of Results

Kakac et al. [8] carried out the investigation of PDOs and thermal oscillations using the operating fluid as Freon-11 with constant inlet temperature (20°C). The experiment was carried out by varying mass flow rate while keeping rest of the parameters constant. The experiments were repeated with varying heat input while keeping other parameters constant. It was observed that the negative slope region had become steeper at higher heat input and the system had become more unstable. The correlated internal pressure drop of heated channel and stability boundary of the system is validated with their experimental results.

A. Parity Plot for Internal Pressure Drop

Parity plot of the system between experimental pressure drop and correlated pressure drop is shown in Fig. 2. The internal pressure drop of the heater channel is obtained by a polynomial fitting whose functional form is given by equation (4) and compared with experimental results. The correlated internal pressure drop of the heater channel lies between ± 15% of experimental internal pressure drop as can be seen in Fig. 2.

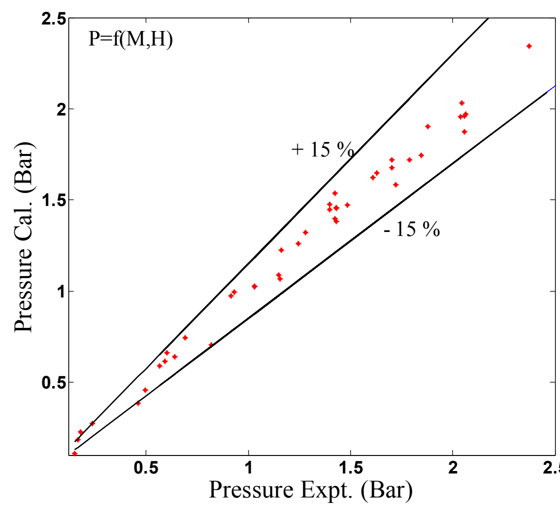


Fig. 2. Parity plot of the system is plotted between experimental internal pressure drop against correlated internal pressure drop of the heater channel.

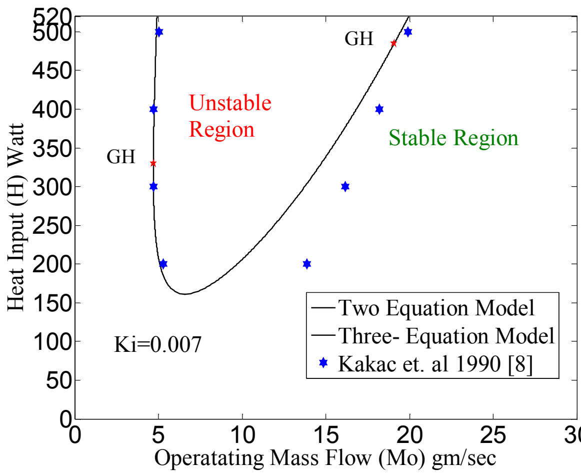


Fig. 3. Stability Boundary of system plotted between operating mass flow (M_o) vs. heat input (H). The two equation and three equation results overlap and are not distinguishable.

B. Stability Map of the System

The obtained stability boundary is compared with experimental data as shown in Fig. 3 and it can be seen that experimental and numerical results match quite well. The stability boundary of the given system is obtained using two-equation model and three-equation model. It is shown from stability map that stability boundaries from these two models are almost overlapping each other. For a given heat input to the system there are two stability boundary points, one at lower mass flow which is defined as LHS boundary and another at higher mass flow which is defined as RHS boundary. The quantitative comparison of the stability boundary in Table I shows that the difference in the two boundaries is not significant. Hence, either two-equation model or three equation model can be used to detect stability boundary of the system in the analysis of PDOs.

Table I. Stability Boundary comparison between two-equation model and three-equation model

Heat input to heater channel (Watt)	LHS boundary (gm/sec)		RHS boundary (gm/sec)	
	Two Eqn.	Three Eqn.	Two Eqn.	Three Eqn.
200	5.0850	5.0852	9.7139	9.7138
250	4.7844	4.7846	11.9185	11.9184
300	4.7058	4.7060	13.7471	13.7470
350	4.7094	4.7096	15.3637	15.3637
400	4.7515	4.7517	16.8342	16.8342
450	4.8132	4.8134	18.1967	18.1966
500	4.8849	4.8851	19.4766	19.4766

2. Investigation of the Stability Map

Fig. 3 shows the stability boundary or threshold boundary of the system for the present analysis. From this figure, the stability of the system can be analyzed. The stability map shows that for a constant heat supplied to the system, stability increases with increasing mass flow into heater channel. And for a constant mass flow into heater channel, the stability of the system decreases with increasing heat input to the channel. This kind of behavior of a system for the PDOs is reported earlier [1].

2. Characteristics of the Stability Boundary

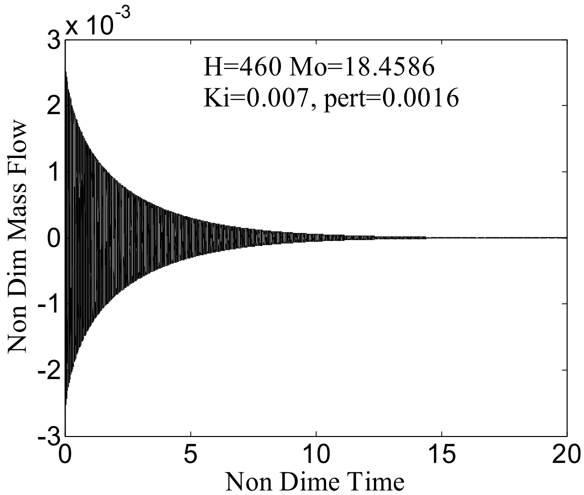
Table II shows the First Lyapunov coefficient (l_1) of stability boundary at different heat inputs with two and three equations model, respectively. For ($K_i = 0.007$), the three equations model shows both positive and negative (l_1). At lower heat input to system, it shows that for both lower (LHS) as well as higher (RHS) mass flow stability boundary is having a negative First Lyapunov coefficient. The l_1 changes sign as heat input is increased. Hence, in three equations model both, subcritical (characterized by positive l_1) and supercritical (characterized by negative l_1) Hopf bifurcations are observed [11], [12]. In case the of two equations model, the (l_1) values are negative irrespective of heat input to fluid, hence only supercritical behavior is seen which is reported earlier[6], [7]. Therefore, it is clear that while for linear stability analysis both the models give almost same results (Fig. 3), the results for non-linear stability analysis are quite different.

Table II. Stability Boundary character comparison

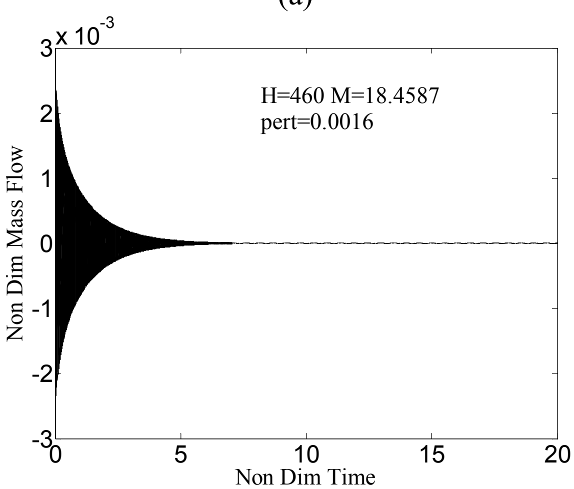
Heat Input (Watt)	First Lyapunov coefficient: Three equations model K_i=0.007		Two equations model	
	LHS	RHS	LHS	RHS
500	8.77e4	1.49e3	-5.20e4	-1.02e5
450	6.23e4	-0.47e4	-5.39e4	-9.57e4
400	3.74e4	-2.85e4	-5.59e4	-9.07e4
350	1.32e4	-4.05e4	-5.77e4	-8.66e4
300	-9.99e3	-5.11e4	-5.94e4	-8.32e4
250	-3.17e4	-6.05e4	-6.11e4	-8.03e4
200	-5.16e4	-6.81e4	-6.31e4	-7.67e4

3. Numerical Simulations

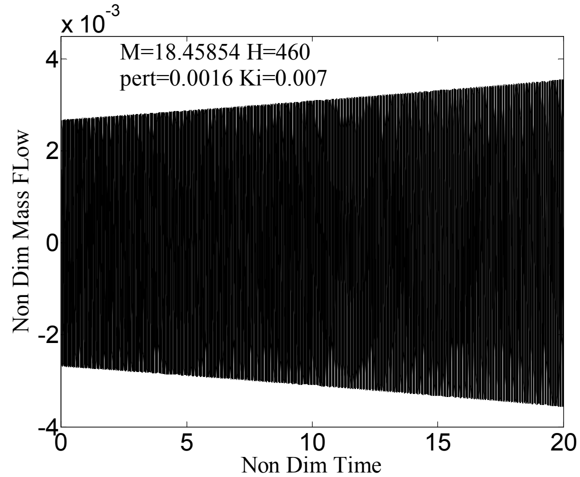
The linear stability analysis is a local stability analysis and is valid only for small perturbation and near to stability boundary [13]. To verify the nonlinear characteristics of the system for large perturbation as well as away from the stability boundary predicted by MATCONT, simulation is carried out by 4th order Runge-Kutta method for the Eqns. 6-8 and Eqns. 9-10. The Fig. 4(a-f) shows the time evolution of non-dimensional mass flow rate ($\overline{m}$) of heater channel from surge tank and non-dimensional mass flow ($\overline{M}$) into surge tank from main tank. Fig. 4(a) shows time evolution of mass flow ($\overline{m}$) in the presence of small perturbation while the system is in stable region close to stability boundary. The perturbation dies out with respect to time and the system goes back to the equilibrium point. Similar results are also found for two equation model in stable region near to SB which is shown in Fig. 4(b). The panel (c) of the same figure shows that time evolution of mass flow ($\overline{m}$) in unstable region very close to stability boundary. The perturbation grows with respect to time very slowly and moves away from the equilibrium point of the system. Again, Fig. 4(d) shows the time evolution of mass flow ($\overline{m}$) in the presence of small disturbance to the equilibrium point of the system in unstable region far from the stability boundary. The perturbation grows with respect to time very rapidly and then settles to the large constant oscillations of mass flow in heater channel.



(a)



(b)



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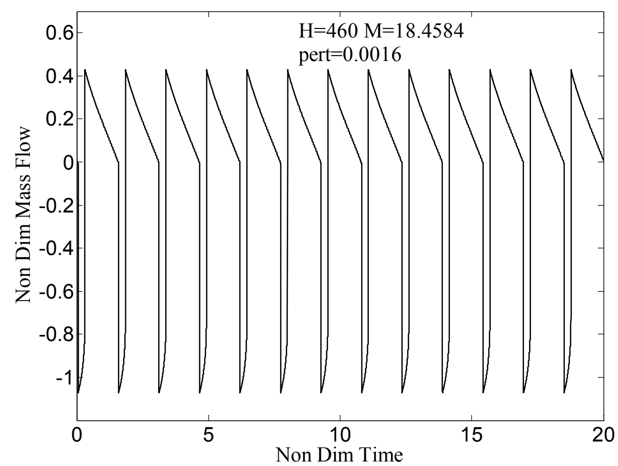
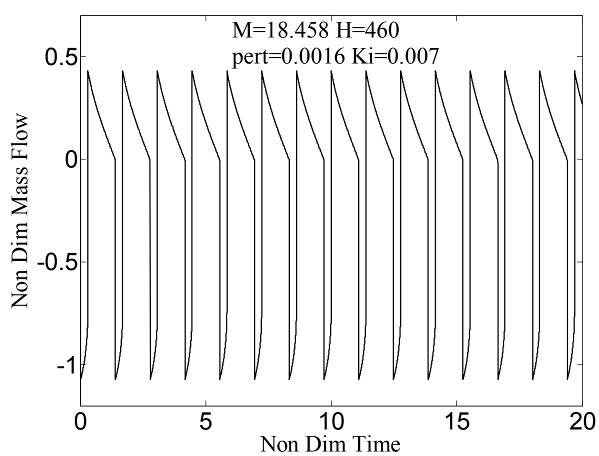
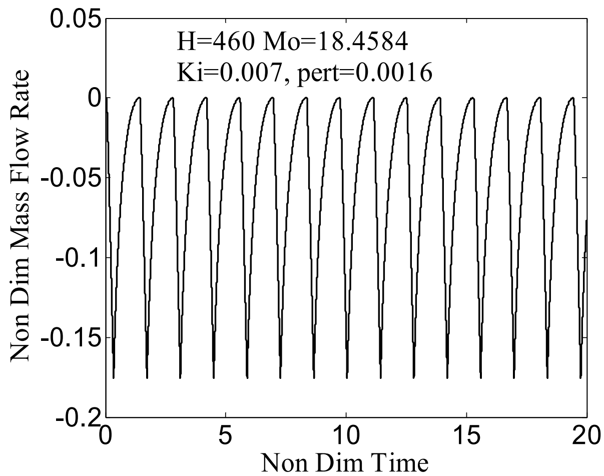


(d)



(e)



(f)

Fig. 3. Time evolution of non-dimensional mass flow (a, b) $\bar{m}$ of three equation model and two equation model respectively in stable region, (c) $\bar{m}$ of three equation model very close to stability boundary; (d, e) $\bar{m}$ of three and two equation model respectively far from stability boundary; and (f) $\bar{M}$ of three equation model in unstable region.

IV. CONCLUSIONS

The linear and nonlinear stability analysis of PDOs in two-phase flow system using two and three equations model is carried out by MATCONT software. For analysis of PDOs, steady state pressure drop of heater channel is obtained by polynomial fitting. After analyzing of PDOs, the following results are observed. The inlet mass flow into surge tank is almost constant during oscillations; hence two-equation model is expected to be reasonably good for the analysis of PDOs. The linear stability analysis shows that the results with the two models are essentially same. However, the non-linear stability analysis shows that the two-equation model is unable to capture the transition of the system from subcritical to supercritical Hopf bifurcation.

In the case of nonlinear stability analysis, time evolution is shown the physically acceptable oscillations of mass flow which not possible to capture by linear stability analysis.

APPENDIX: NON-DIMENSIONALIZATION

Non-dimensional mass flow into surge tank from the main tank:

$$\overline{M} = \frac{(M - M_O)}{M_O}.$$

Non-dimensional mass flow into heater channel from surge tank:

$$\overline{m} = \frac{(m - M)}{M_O}.$$

Non-dimensional pressure of surge tank:

$$\overline{P} = \frac{(P_S - P_O)}{P_O}.$$

Non-dimensional time:

$$\tau = \frac{t}{V_O \rho_l / M_O}.$$

The ratio of compressible volume of surge tank to the inner volume of channel tube between the main tank and surge tank:

$$R = \frac{V_O}{AL_1}.$$

The ratio of compressible volume of surge tank to the inner volume of test heater channel:

$$r = \frac{V_O}{AL}.$$

NOMENCLATURE

A = inner surface area of heater (m^2)
H = heat input to fluid (Watt)
K_i =loss coefficient
L = heater length (m)
L_1 = length between surge tank to main tank (m)
m = mass flow out into heater tube (gm. /sec)
$\bar{m}$ = Non dimensional mass flow out of surge tank
M = mass flow into surge tank (gm. /sec)
M_o = Operating or steady state mass flow (gm. /sec)
$\bar{M}$ = Non dimensional mass flow into sure tank
P_e = exit pressure (N/m^2)
P_o = steady state pressure of surge tank (N/m^2)
P_S = surge tank pressure (N/m^2)
$\bar{P}$ = Non dimensional pressure of surge tank
t = time (sec)
V_o = steady state compressible volume of surge tank (m^3)
ρ_l = liquid density (kg/m^3)
τ = Non dimensional time
Eu = Euler number, $\Delta P(M)/(M^2/A^2\rho_l)$

REFERENCES

1. S. KAKAC and B. BON, "A Review of two-phase flow dynamic instabilities in tube boiling systems,'' *Int. J. Heat Mass Transf.*, 51, 399 (2008).
2. L. C. RUSPINI, C. P. MARCEL, and A. CLAUSSE, "Two-phase flow instabilities: A review," *Int. J. Heat Mass Transf.*, 71, 521 (2014).

3. L. TADRIST, "Review on two-phase flow instabilities in narrow spaces," *Int. J. Heat Fluid Flow*, 28, 54 (2007).
4. J. A. BOURE, A. E. BERGLES, and L. S. TONG, "Review of two phase flow instabilty," *Nucl. Eng. Des.*, 25, 165 (1973).
5. E. MANAVELA CHIAPERO, M. FERNANDINO, and C. A. DORAO, "Review on pressure drop oscillations in boiling systems," *Nucl. Eng. Des.*, 250, 436 (2012).
6. M. M. PADKI, K. PALMER, S. KAKAÇ, and T. N. VEZIROĞLU, "Bifurcation analysis of pressure-drop oscillations and the Ledinegg instability," *Int. J. Heat Mass Transf.*, 35, 525 (1992).
7. H. T. LIU, H. KOKAC, and S. KAKAC, "Dynamical analysis of pressure-drop oscillations with a planar model," *Int. J. Multiph. Flow*, 21, 851 (1995).
8. S. KAKAÇ, T. N. VEZIROĞLU, M. M. PADKI, L. Q. FU, and X. J. CHEN, "Investigation of thermal instabilities in a forced convection upward boiling system," *Exp. Therm. Fluid Sci.*, 3, 191 (1990).
9. M. M. PADKI, H. T. LIU, and S. KAKAC, "Two-phase flow pressure-drop type and thermal oscillations," *Int. J. Heat Fluid Flow*, 12, 240 (1991).
10. H. GÜRGENCI, T. N. VEZIROGLU, and S. KAKAÇ, "Simplified nonlinear descriptions of two-phase flow instabilities in vertical boiling channel," *Int. J. Heat Mass Transf.*, 26, 671 (1983).
11. G. C. LAYEK, *An Introduction to Dynamical Systems and Chaos*, p. 226, Springer India, New Delhi (2015).
12. J. GUCKENHEIMER and P. HOLMES, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, p. 152, J. F. MARSDEN, and L. SIROVICH, 1st Ed., Springer-Verlag, New York (1983).
13. A. A. KARVE, RIZWAN-UDDIN, and J. J. DORNING, "Stability analysis of BWR nuclear-coupled thermal-hydraulics using a simple model," *Nucl. Eng. Des.*, 177, 155 (1997).
14. A. H. STENNING, and T. N. VEZIROGLU, "Flow oscillations modes in forced-convection boiling,"*In: Preceedings of the 1965 Heat Transfer and Fluid Mechanics Institute*, Starndford Unviversity Press, (1965).