#### Transient Characteristic Analysis of Nuclear Metallic Fuel System

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**Abstract** –A transient code STAC was developed based on one-point kinetic equation, unsteady heat transfer equation and linear elasticity equation for the purpose of analyzing nuclear metallic fuel system. Calculation was conducted with STAC code to simulate the change of fission rate, reactivity and temperature during prompt super-critical experiment on Godiva-I device. The simulation results are consistent with the experiment results on Godiva-I device, which verifies the accuracy of STAC code.

## I. INTRODUCTION

Throughout the life cycle, nuclear fuel may exist in a variety of states, such as solid metal, powder, and solution. It is of great significance to know more about its critical and transient characteristics.

This paper focused on the solid metal state of the nuclear fuel, developed a methodology to simulate the process of super-critical accidents happened on solid nuclear fuel system and got the change of power (or fission rate), reactivity and temperature over time.

### **II. DESCRIPTION OF THE ACTUAL WORK**

In the manufacturing process of nuclear fuel pellets, the enriched fissile material often exists as solid metal. Critical accidents may occur in any moving situation during the transportation and storage of metallic nuclear fuel<sup>[1,2]</sup>. It is of great significance to evaluate the system power (or fission rate), reactivity and other parameters after the critical accidents.

In this paper, the STAC code is developed based on the physical and material properties equation and is used to simulate the change of fission rate during the prompt supercritical experiment on Godiva-I device to verify the accuracy of STAC code.

### 1. Basic Theory

When the environment changes and introduces positive reactivity to the nuclear metallic fuel system, which brings the system to supercritical state, the power (or fission rate) rises rapidly at the beginning of reactivity introduction. Due to the accumulation of fission heat, the volume of the system begins to expand, the density becomes smaller, the neutron leakage increases, and the negative reactivity feedback is introduced into the system. In order to simulate the whole process, we need to combine one-point kinetic equation, unsteady heat transfer equation and linear elasticity equation.

In this paper, all the equations were solved using numerical discretization and the author had solved the equations in different coordinate systems so that they can be applied to systems of different shapes, such as cuboid, cylinder and sphere.

#### 1.1 Point kinetic equation

Since the time of power burst process is short, the shape function of the flux will not change much. Therefore, the amplitude function and the shape function of the whole neutron flux density can be separated. Therefore, point kinetic equation can be used to simulate the change of power. The cubic Hermit interpolation method is used in the numerical solution of the point kinetic equation.

$$\frac{dP}{dt} = \frac{\rho - \beta_{eff} - \Delta \rho}{\Lambda} P + \sum_{i=1}^{6} \lambda_i C_i + S$$

$$\frac{dC_i}{dt} = \frac{\beta_{ieff}}{\Lambda} P - \lambda_i C_i \quad i = 1, 2, \dots 6$$
(1)

### 1.2 Unsteady heat transfer equation

The form of the unsteady heat transfer equation is shown in eq. (2).

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c} \nabla^2 T + \frac{Q_{fission}}{\rho c}$$
(2)

Because the time of power burst is short, and the heat convection between metal and air is not strong, the boundary condition between the metal system and the boundary is assumed to be adiabatic.

$$\nabla T \big|_{surface} = 0$$

### 1.3 Linear elasticity equation

In the event of a transient accident in a nuclear metallic fuel system, the material within the system undergoes expansion or contraction due to the accumulation and release of heat and the temperature gradient<sup>[3]</sup>. It is assumed that the deformation occurring in the solid metal system is within the elastic deformation range of the material. M&C 2017 - International Conference on Mathematics & Computational Methods Applied to Nuclear Science & Engineering, Jeju, Korea, April 16-20, 2017, on USB (2017)

Therefore, the linear elasticity equation is used to solve the displacement of the material in the system.

The form of the unsteady heat transfer equation is shown in eq. (3).

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla \left[ \mu \nabla u + \mu (\nabla u)^T + \lambda Itr(\nabla u) + (3\lambda + 2\mu) \alpha T \right]$$
(3)

In eq. (3), the *Lame'* coefficients  $\mu$  and  $\lambda$  are calculated as follows.

$$\mu = \frac{E}{2(1+\nu)}$$
$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

The center of the metal material is with fixed boundary conditions, and it is assumed that the material surface is without external force.

### 1.4 Calculation of reactivity feedback

According to the results of MCNP calculation, the neutron energy spectrum in metal nuclear fuel is very hard and the neutron energy is above 1MeV. Meanwhile, the temperature rise of the metallic system is not great. Therefore, the influence of Doppler effect on reactivity can be neglected<sup>[4]</sup>. The feedback is mainly caused by the volume changes due to temperature. The single group perturbation theory is used to calculate the reactivity feedback of metal nuclear fuel system. The temperature feedback coefficient can be calculated by the following eq. (4).

$$\alpha_{T} = -\frac{\alpha}{\beta_{eff}} \begin{cases} \left(1 - \frac{\sum_{a}}{v \sum_{f}}\right) \frac{V \int_{V} B(r)^{3} dV}{\int_{V} B(r)^{2} dV \int_{V} B(r) dV} + \\ \frac{1}{3v \sum_{s} \sum_{f}} \frac{V \int_{V} B(r) [\nabla B(r)]^{2} dV}{\int_{V} B(r)^{2} dV \int_{V} B(r) dV} \end{cases}$$
(4)

#### 2. Code verification

In order to verify the correctness of the STAC code, the code simulation results are compared with experiment results on the Godiva-I transient experimental device<sup>[5]</sup>.

The Godiva-I device is a non-reflective, highly enriched uranium metal ball. The core assembly of the device is a uranium metal sphere with a  $U^{235}$  enrichment of 93.8% and a density of 18.75g/cm<sup>3</sup>. The diameter is about 17.145cm.

The changes of fission rate and temperature of Godiva-I device were simulated by STAC program, and the kinetic parameters were calculated in Table I. Using the singlegroup perturbation theory, combined with the MCNP statistical cross-section data, the parameters used to calculate the temperature coefficient are listed in Table II. The temperature coefficient calculated from the parameters in Table II is shown in Table III and in comparison with the reference. It can be seen from Table III that the relative error between the results of the single-group perturbation theory and the benchmark experiment results is 3.02%, which indicates that using the single-group perturbation theory to evaluate temperature coefficient of the metal nuclear fuel is reasonable.

Table I. Kinetic parameter for Godiva- I

i	$eta_{ ext{ieff}}$	$\lambda_i/\mathrm{s}^{-1}$
1	2.31×10 <sup>-4</sup>	1.28×10 <sup>-2</sup>
2	1.37×10 <sup>-3</sup>	3.19×10 <sup>-2</sup>
3	1.26×10 <sup>-3</sup>	$1.18 \times 10^{-1}$
4	2.70×10 <sup>-3</sup>	3.18×10 <sup>-1</sup>
5	9.11×10 <sup>-4</sup>	1.51
6	1.32×10 <sup>-4</sup>	5.32

Table II.	Parameters	of	calculating	temperature	coefficient

for Godiva- I using one group perturbation method

$\Sigma_{\rm a}/{\rm cm}^{-1}$	$\Sigma_{\rm s}/{\rm cm}^{-1}$	$\Sigma_{\rm f}/{ m cm}^{-1}$	V	α
6.31×10 <sup>-2</sup>	3.28×10 <sup>-1</sup>	5.65×10 <sup>-2</sup>	2.599	1.39×10 <sup>-5</sup>

Table III. Temperature coefficient calculation result

compared with benchmark	
Source	$\alpha_{\rm T}/(\$ \cdot ^{\circ}{\rm C})$
Benchmark	4.30×10 <sup>-3</sup>
Single-group perturbation theory	4.43×10 <sup>-3</sup>

Table IV. Fission rate calculation result compared with benchmark

Period	Benchmark	Simulation	Relative error
/s	/fission ⋅ s <sup>-1</sup>	/fission ⋅ s <sup>-1</sup>	/%
29.5	$2.67 \times 10^{19}$	$2.71 \times 10^{19}$	1.50
90	$2.54 \times 10^{18}$	$2.62 \times 10^{18}$	3.15
160	$1.07 \times 10^{18}$	$0.95 \times 10^{18}$	-11.2
320	2.63×1017	$2.62 \times 10^{17}$	-0.38

The STAC code was used to simulate the experiments with the system period  $T_s$  of 29.5, 90, 160 and 320µs in the Godiva-I benchmarks. The comparison of the fission rate with the experiment is shown in Fig. 1.

It can be seen from Table IV that the relative error of the fission peak calculated by STAC code is 1.50%, 3.15%, -11.2% and -0.38%, when the period is 29.5, 90, 160 and  $320\mu$ s, respectively. The maximum relative error can be controlled within 12%, indicating that the STAC code can predict the fission peak accurately. The STAC code is more accurate in simulating the rise phase, while the simulation value is always smaller than the experimental result in the power decline phase of the system.

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It is believed that the reflected neutrons are re-involved in the nuclear reaction due to the reflection of the wall around the Godiva-I device. In other words, the delayed neutrons are not only from the delayed precursor, but also the neutrons reflected from the surrounding walls. However, the target nucleus can not distinguish between the two, so the experimental results are larger than the STAC code simulation results. In addition, treating the temperature coefficient as a fixed value is a certain approximation. The error may also come from this approximation.

For nuclear metallic fuel, when the positive reactivity is introduced, since the neutron generation time of the metal system is usually very short and the delayed neutrons have not yet produced, the effect of the prompt neutrons will make the power (or fission rate) exponentially increase. The rapid increase in power, the accumulation of fission heat, and the rise of system temperature, result in the expansion of the system. The expansion of the system makes the volume of the system increase, the density decline, the macro section decrease, and the leakage rate of the neutron increase, which introduces a negative feedback to the system. Because the time of the power burst is very short, the state between the system and air is close to the adiabatic. Consequently, the heat will continue accumulating in the system and the negative feedback effect keeps on increasing, which makes the speed of the system power growth slow down gradually.

Figure 2 shows the change of the reactivity. It can be seen from Fig. 2 that when the reactivity of the system is reduced to 1\$, the power comes to the maximum.

## **III. RESULTS**

Based on one-point kinetic equation, unsteady heat transfer equation and linear elasticity equation, the STAC code for transient analysis of nuclear metallic fuel system was developed and verified by the Godiva-I device. The maximum relative error of the fission rate peak can be controlled within 12%, which indicates that the STAC code can well simulate the transient characteristics of nuclear metallic fuel system. This procedure has laid a theoretical foundation for the critical safety analysis and transient experiment of nuclear fuel.



Period: a--29.5 μs; b--90 μs; c--160 μs; d--320 μs Fig. 1. Fission rate comparison with benchmark.

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Period: a--29.5  $\mu$ s; b--90  $\mu$ s; c--160  $\mu$ s; d--320  $\mu$ s Fig. 2. Change of the reactivity with time.



Period: a--29.5  $\mu s;$  b--90  $\mu s;$  c--160  $\mu s;$  d--320  $\mu s$  Fig. 3. Change of temperature with time.

# NOMENCLATURE

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One-point kinetic equation P = power density t = time  $\rho = \text{reactivity}$   $\Delta \rho = \text{feedback reactivity}$   $\beta_{ieff} = \text{delayed neutron fraction of the i-th group}$   $\beta_{eff} = \text{delayed neutron fraction}$   $\Lambda = \text{prompt neutron lifetime}$   $\lambda_i = \text{decay constant of ith precursor group}$   $C_i = i$ th delayed group precursor concentration S = neutron source

Unsteady heat transfer equation T = temperature k = thermal conductivity  $\rho =$  density of the material c = specific heat

 $Q_{fission}$  = fission heat production rate

Linear elasticity equation u = displacement  $\mu \text{ or } \lambda = Lame' \text{ coefficient}$  $\alpha = \text{thermal expansion coefficient}$ 

Calculation of reactivity feedback

 $\alpha_T$  = temperature feedback coefficient

 $\Sigma_a$  = macro absorption cross section

 $\Sigma_{f}$  = macro fission cross section

 $\Sigma_s$  = macro scattering cross section

V = mean number of neutrons released per fission

V = volume of the system

B(r) = shape function of the flux

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