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The new 3-D multigroup diffusion neutron noise solver of APOLLO3® and a theoretical discussion of fission-modes noise

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Abstract - *Traditional neutron noise analysis addresses the description of time-dependent flux fluctuations induced by small global or local perturbations of the macroscopic cross-sections, which may occur in nuclear reactors due to stochastic density fluctuations of the coolant, to vibrations of fuel elements, control rods or any other structures in the core. Neutron noise equations are obtained by assuming small perturbations of macroscopic cross-sections around a steady-state neutron field and by subsequently taking the Fourier transform in the frequency domain. In this work, we present the new 3-D multigroup diffusion neutron noise solver implemented in APOLLO3®, the new multi-purpose deterministic nuclear code under development in CEA. We illustrate the capacities of this new 3-D diffusion neutron noise solver by performing two neutron noise simulations in a large pressurized water reactor with heavy baffle in three dimensions: a cross-sections oscillation and a traveling perturbation. Moreover, we give a separate analysis of the neutron noise anomaly recently observed in KWU PWRs while proving the existence of steady-state like global noise modes in the low frequency spectrum.*

I. INTRODUCTION

Neutron noise appears as fluctuations of the neutron field induced by stochastic or deterministic changes in the macroscopic cross-sections [1]. The latter may result from vibrations of fuel elements, control rods or any other mechanical structures in the core, as well as from global or local fluctuations in the flow, density or void fraction of the coolant [2]. In power reactors, ex-core and in-core detectors can be used to monitor neutron noise with the aim of detecting possible anomalies and taking the necessary measures for continuous safe power production.

The general noise equations are obtained by assuming small perturbations around a steady-state neutron flux and by subsequently taking the Fourier transform in the frequency domain. The analysis is performed based on the neutron kinetic equations, including the coupling with neutron precursors. The outcome of the Fourier transform analysis is a fixed-source equation with complex operators for the perturbed neutron field, which can then be solved so as to predict noise measurements at detector locations. For each frequency, the neutron noise is a complex function having an amplitude and a phase.

Noise analysis relies upon the possibility of numerically simulating the behaviour of neutron noise and computing the changes in the neutron field produced by different representative sources of noise in reactor cores. These analysis are generally performed via analytical techniques [3, 4] or by resorting to diffusion theory with two energy groups [5, 6].

In this work, we present a new 3-D multigroup diffusion neutron noise solver implemented in APOLLO3®, the new multi-purpose deterministic nuclear code under development in CEA [7], and an analysis of the neutron noise anomaly recently observed in KWU PWRs. This paper is organized as follows. In Sec. II, the general neutron noise theory will be briefly introduced. In Sec. III, the new neutron noise solver developed in APOLLO3® will be described and we will analyze a cross-sections oscillation and a traveling perturbation in a large pressurized water reactor with heavy baffle in three dimensions.

Finally, in Sec. IV, we give a separate analysis of the neutron noise anomaly recently observed in KWU PWRs. We show that in the low frequency spectrum all frequency components of the noise equations can be neglected and that to leading order the equation reduces to a steady-state equation for the noise flux with given source. This suggests that global modes close to the steady state ones can be excited as these low frequencies and prompts us to provide a method to identify a mode which has the experimentally observed symmetries of the noise anomaly. The appendix summarizes the properties of some simple analytical solutions of the one-group diffusion equation that are used as examples in our analysis.

Conclusions will be presented in Sec. V.

II. NEUTRON NOISE THEORY

Here we summarize the general theory of neutron noise in transport theory and we detail the final equations in diffusion theory. In diffusion theory, in addition to the Fick's law application, fluctuations of the diffusion coefficients are disregarded and the term $\frac{1}{v}\partial_t J$ is neglected (with J the scalar current). Note that the zero power reactor noise (fluctuations inherent to the branching process) is neglected in power reactor noise theory [1].

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We assume small perturbations of the macroscopic cross-sections around the following critical steady state:

$$\mathcal{B}_0\psi_0(r,\Omega,E) = 0, \tag{1}$$

where ψ_0 is the steady-state angular flux and $\mathcal{B}_0 = \Omega\cdot\nabla + \Sigma_0 - \mathcal{H}_0 - \mathcal{P}_0$ the steady-state Boltzmann operator with Σ_0 the steady-state total cross-section, $\mathcal{H}_0$ the steady-state scattering operator and $\mathcal{P}_0$ the steady-state production operator. For the steady state, the effective multiplication factor is assumed to be $k = 1$. For a system in diffusion theory with one precursor group and one fissile isotope, the critical steady-state Boltzmann equation for each energy group g is (all notations are standard):

$$\Big(-\nabla\cdot D_g(r)\nabla + \Sigma_0^g(r)\Big)\Phi_{0,g}(r) = \sum_{g'}\Sigma_{0,s_0}^{g'\to g}(r)\Phi_{0,g'}(r) + \frac{\chi^g}{k}\sum_{g'}\nu^{g'}\Sigma_{0,f}^{g'}(r)\Phi_{0,g'}(r), \tag{2}$$

with Φ_0 the steady-state scalar flux.

We impose a temporal perturbation of the macroscopic cross-sections, which yields the kinetic equation:

$$\left[\frac{1}{\mathrm{v}}\partial_t + \mathcal{B}\right]\psi(r,\Omega,E,t) = 0, \tag{3}$$

where ψ is the angular flux, v the neutron velocity and $\mathcal{B} = \Omega\cdot\nabla + \Sigma - \mathcal{H} - \mathcal{P}$ the kinetic Boltzmann operator with Σ the total cross-section, $\mathcal{H}$ the scattering operator and $\mathcal{P}$ the production operator containing prompt and delayed neutron contributions. We impose a periodic perturbation of the kinetic operator with a period T_0, in the form:

$$\mathcal{B} = \mathcal{B}_0 + \delta\mathcal{B}. \tag{4}$$

This perturbation is supposed to start at time $t = -\infty$, so that we can reasonably assume that the asymptotic perturbation regime is attained. A similar decomposition is also used for the angular flux:

$$\psi(r,\Omega,E,t) = \psi_0(r,\Omega,E) + \delta\psi(r,\Omega,E,t), \tag{5}$$

where the perturbation term $\delta\psi$ is called "neutron noise". Finally, plugging expressions (4) and (5) into Eq. (3) leads to a kinetic source equation for the neutron noise:

$$\left[\frac{1}{\mathrm{v}}\partial_t + \mathcal{B}\right]\delta\psi(r,\Omega,E,t) = -\delta\mathcal{B}\psi_0(r,\Omega,E). \tag{6}$$

The second order term $\delta\mathcal{B}\delta\psi$ will be neglected, so that we obtain the traditional linearized kinetic equation:

$$\left[\frac{1}{\mathrm{v}}\partial_t + \mathcal{B}_0\right]\delta\psi(r,\Omega,E,t) = -\delta\mathcal{B}\psi_0(r,\Omega,E). \tag{7}$$

We want to determine the unique periodic solution of this equation. We apply the Fourier transform and we obtain the noise equation in the usual form:

$$\mathcal{B}_{0,\omega}\delta\psi(r,\Omega,E,\omega) = -\delta\mathcal{B}_\omega\psi_0(r,\Omega,E), \tag{8}$$

where $\mathcal{B}_{0,\omega} = i\frac{\omega}{\mathrm{v}} + \Omega\cdot\nabla + \Sigma_0 - \mathcal{H}_0 - \mathcal{P}_{0,\omega}$ is a modified (complex) Boltzmann operator, i the imaginary unit and $\omega = 2\pi f$ the angular frequency. The right hand side of Eq. (8) represents a (known) "noise source".

For a system in diffusion theory with one precursor group and one fissile isotope, the noise equation is:

$$\begin{aligned}\Big(-\nabla\cdot D_g(r)\nabla + \Sigma_0^g(r)\Big)\delta\Phi_g(r,\omega) &= -i\frac{\omega}{\mathrm{v}_g}\delta\Phi_g(r,\omega)\\ &+ \sum_{g'}\Sigma_{0,s_0}^{g'\to g}(r)\delta\Phi_{g'}(r,\omega) + \frac{\chi_p^g}{k}\sum_{g'}\nu_p^{g'}\Sigma_{0,f}^{g'}(r)\delta\Phi_{g'}(r,\omega)\\ &+ \frac{\chi_d^g}{k}\sum_{g'}\nu_{d,\omega}^{g'}\Sigma_{0,f}^{g'}(r)\delta\Phi_{g'}(r,\omega) + S_g(r,\omega),\end{aligned} \tag{9}$$

with $\nu_{d,\omega}^g = \left(\frac{\lambda^2}{\lambda^2+\omega^2} - i\frac{\lambda\omega}{\lambda^2+\omega^2}\right)\nu_d^g$ and S_g the noise source defined by:

$$S_g(r,\omega) = -\delta\Sigma^g(r,\omega)\Phi_{0,g}(r) + \sum_{g'}\delta\Sigma_{s_0}^{g'\to g}(r,\omega)\Phi_{0,g'}(r) + \frac{1}{k}\sum_{g'}\Big(\chi_p^g\nu_p^{g'} + \chi_d^g\nu_{d,\omega}^{g'}\Big)\delta\Sigma_f^{g'}(r,\omega)\Phi_{0,g'}(r), \tag{10}$$

where $\delta\Sigma_x^g(r,\omega)$ is the Fourier transform of the perturbed term of the macroscopic cross-section $\Sigma_x^g(r,t) = \Sigma_{0,x}^g(r) + \delta\Sigma_x^g(r,t)$.

Thus, because of the delayed neutrons, the production operator $\mathcal{P}_{0,\omega}$ depends on the frequency. The real and imaginary components of the neutron noise are coupled by two terms: $i\frac{\omega}{\mathrm{v}}$ and the modified production operator $\mathcal{P}_{0,\omega}$.

III. APOLLO3® 3-D MULTIGROUP DIFFUSION NEUTRON NOISE SOLVER: DESCRIPTION AND EXAMPLES

APOLLO3® is the new multi-purpose deterministic nuclear code under development in the frame of the neutronics simulation project of the Nuclear Energy Division (DEN) of the CEA and with financial support from AREVA and EDF [7]. In this section we will present the new 3-D multigroup diffusion neutron noise solver developed in this new generation deterministic code. Then, we will detail two neutron noise simulations in a large pressurized water reactor with heavy baffle in three dimensions: a cross-sections oscillation and a traveling perturbation.

1. Description of the 3-D multigroup diffusion neutron noise solver

One of the diffusion solvers implemented in APOLLO3® is a 3-D multigroup nodal diffusion solver based on the classical Nodal Expansion Method [8, 9]. In order to solve Eq. (9) (which is an equation of a fixed-source problem with a coupling between the real and imaginary parts of $\delta\Phi$), we choose to work with this diffusion solver: we apply the same iteration loops to the fission source (but the production operator is now complex) and to the scattering source as customary, and we add an iteration loop between the real and imaginary parts of

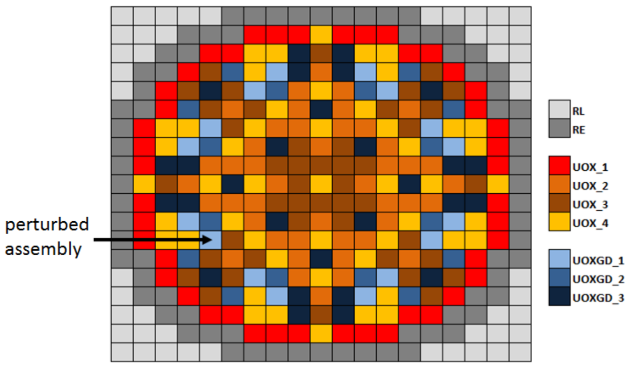
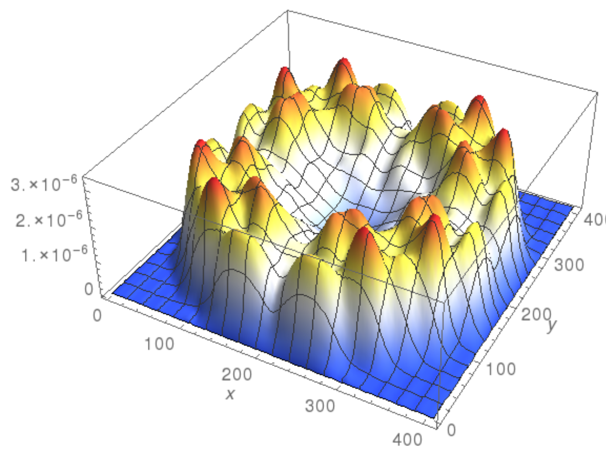
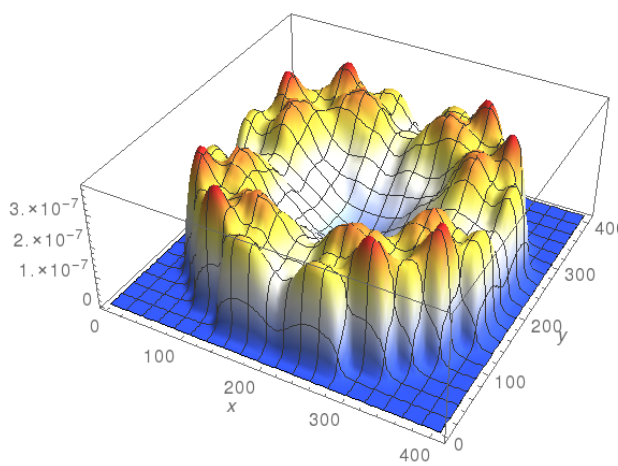


Fig. 1. Fuel loading map of the studied 3-D core. RE refers to homogeneous heavy baffle assembly, RL to homogeneous quasi-void assembly, UOX_N to homogeneous UOX fuel assembly and UOXGD_N to homogeneous UOX fuel assembly with Gadolinium control rods (with N the number of cycles passed in core).

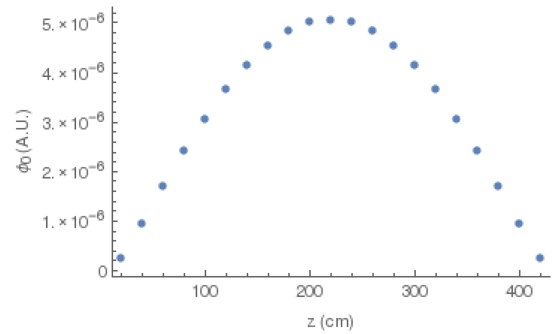


(a) Fast steady-state flux.

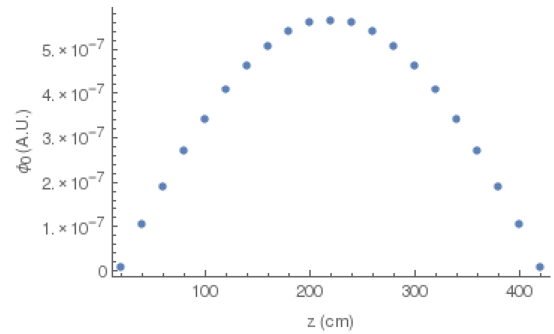


(b) Thermal steady-state flux.

Fig. 2. Steady-state flux of the studied 3-D core (numerical results of the 5th axial plane starting from the bottom).



(a) Fast steady-state flux.



(b) Thermal steady-state flux.

Fig. 3. Steady-state flux of the studied 3-D core in the perturbed assembly in function of the axial position z.

the neutron noise equation to the solution of the one-group problem. Thus, we can use the same one-group diffusion solver as customary.

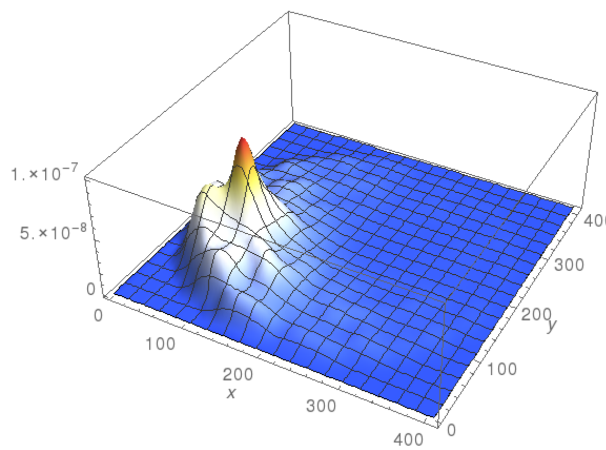
For the moment, the new 3-D multigroup diffusion neutron noise solver developed in APOLLO3® can be applied to homogeneous cartesian geometries.

This new solver was verified by comparaison with some analytical results in a homogeneous infinite medium and in an infinitely-long homogeneous cylinder exactly as in [10].

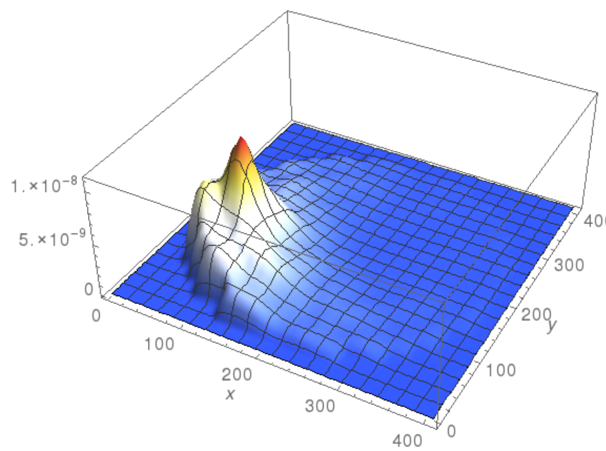
2. Description of the 3-D core

In order to illustrate the capacities of the new 3-D multigroup diffusion neutron noise solver, we have performed neutron noise simulations in a large pressurized water reactor with heavy baffle in three dimensions. This 3-D core is composed of 17×17 homogeneous assemblies (x- and y-dimensions of each assembly are 21.5×21.5 cm) and its axial height is 420 cm. The 3-D core is decomposed into 21 axial planes of 20 cm: the first and the last planes are only composed by homogeneous heavy baffle assemblies (noted RE in Fig. 1) and all other planes are set by the same fuel loading map described in Fig. 1.

All simulations have been performed in diffusion theory with 2 energy groups (fast and thermal groups), 8 precursors groups, a quartic approximation of the volume flux and a corrected scattering anisotropy P_0^* (i.e., the diffusion coefficients are defined by $1/[3(\Sigma_{0,a}^g + (\Sigma_{0,s_0}^g)^*)]$ where all macroscopic selfscattering cross-sections are corrected by $(\Sigma_{0,s_0}^{g\to g})^* = \Sigma_{0,s_0}^{g\to g} - \sum_{g'} \Sigma_{0,s_1}^{g\to g'}$ with g the energy group).

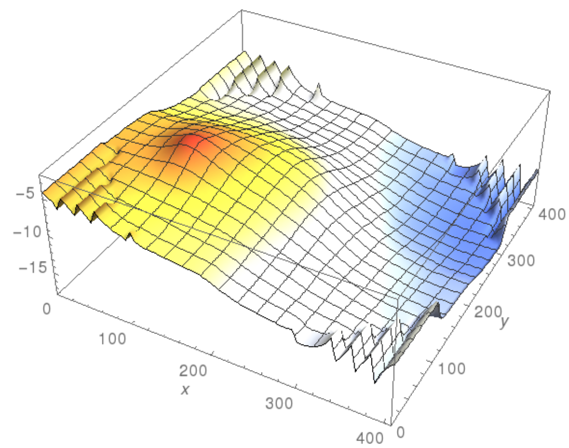


(a) Fast neutron noise modulus.

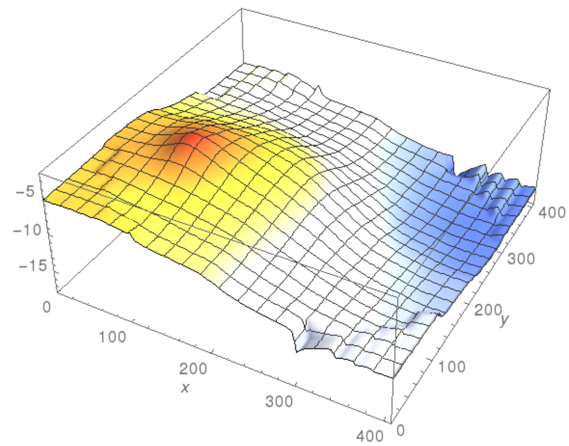


(b) Thermal neutron noise modulus.

Fig. 4. Moduli of the fast and thermal neutron noises at 3 Hz induced by a cross-sections oscillation in the UOXGD assembly in position (5,7) (numerical results of the 5th axial plane starting from the bottom).



(a) Fast neutron noise phase.



(b) Thermal neutron noise phase.

Fig. 5. Phases (in degree) of the fast and thermal neutron noises at 3 Hz induced by a cross-sections oscillation in the UOXGD assembly in position (5,7) (numerical results of the 5th axial plane starting from the bottom).

Figure 2 presents the fast and thermal steady-state fluxes obtained in diffusion theory with APOLLO3®. The eigenvalue of this system is close to 1 ($k = 0.99674$). Figure 3 details the steady-state flux the UOXGD assembly in position (5,7) (see Fig. 1, it is the future perturbed assembly) in function of the axial position z. As expected, the axial steady-state distribution follows the sine shape.

3. Analysis of a cross-sections oscillation

We present in this section the neutron noise induced by a cross-sections oscillation of the UOXGD assembly in position (5,7) (see Fig. 1). All macroscopic cross-sections of this homogeneous assembly are periodically perturbed on the whole height of the assembly (380 cm, the first and the last planes are excluded) around their steady-state value $\Sigma_{0,x}^g$ with an amplitude of 1% and a frequency of 3 Hz. Thus, the perturbed term $\delta\Sigma_x^g$ is defined by:

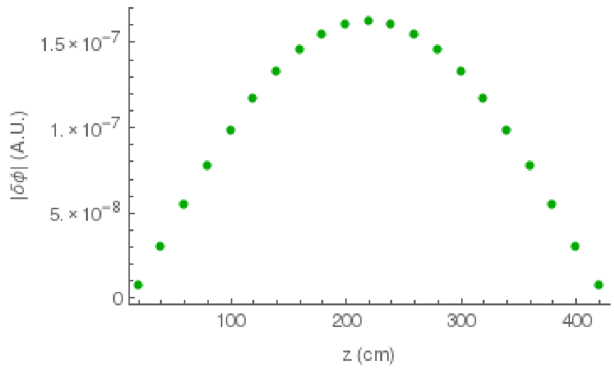
$$\delta\Sigma_x^g(r,t) = 0.01 \times \Sigma_{0,x}^g(r)\cos(\omega_0 t), \qquad (11)$$

with ω_0 the angular frequency of the perturbation ($\omega_0 = 6\pi$ rad/s). Note that the noise source is monochromatic so the neutron noise is also monochromatic.

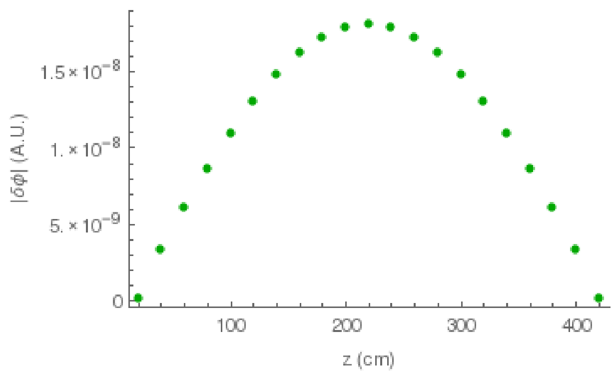
Figures 4 and 5 detail the fast and thermal neutron noises moduli and phases at 3 Hz of the 5th axial plane. Observe that the neutron noise is localized around the perturbed assembly and the phases are almost constant in the whole core (the axial distribution of the phases is almost constant). Figure 6 presents the fast and thermal neutron noises moduli in the perturbed assembly in function of the axial position z. As expected, these axial distributions follow the sine shape as the steady-state distribution.

4. Analysis of a traveling perturbation

We briefly analyze in this section a traveling perturbation in the 3-D core described previously. We perturb the same assembly as in the previous section (see Fig. 1) by a periodically



(a) Fast neutron noise modulus.



(b) Thermal neutron noise modulus.

Fig. 6. Moduli of the fast and thermal neutron noises at 3 Hz in the perturbed assembly in function of the axial position z.

traveling perturbation with an amplitude of 1% defined by:

$$\delta\Sigma_x^g(z,t) = \delta\Sigma_x^g(z = z_i, t - \frac{z}{v_e})$$
$$= 0.01 \times \Sigma_{0,x}^g(z = z_i)\cos\left(\omega_0(t - \frac{z}{v_e})\right), \qquad (12)$$

where z is the axial position, z_i the beginning axial position of the perturbation ($z_i = 20$ cm, i.e., the beginning of the fuel assembly), ω_0 the angular frequency of the perturbation and $\mathrm{v}_e = 190$ cm/s the perturbation axial velocity (supposed constant whatever is the axial position z).

Here we only present how we can find the perturbation axial velocity v_e thanks to the numerical results of the phase shift between two axial positions. Figure 7 details the phase shift versus frequency between axial positions $z_a = 60$ cm and $z_b = 140$ cm included in the perturbed assembly. According to the perturbation definition, the neutron noise in z_b is phase shifted by $-\omega\tau$ compared to the neutron noise in z_a, with $\tau = (z_b - z_a)/\mathrm{v}_e$ the propagating time of the perturbation. So, according to Fig. 7, $\tau \approx 0.421$ s and thus we find $\mathrm{v}_e \approx 189.9$ cm/s.

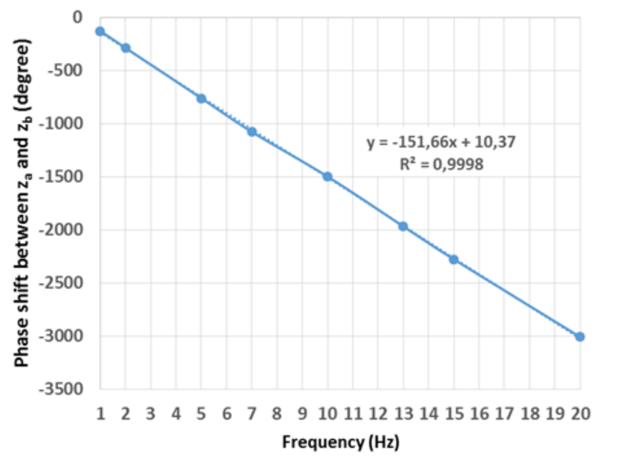


Fig. 7. Phase shift versus frequency between axial positions $z_a = 60$ cm and $z_b = 140$ cm included in the perturbed assembly. Equation $y = -151.66x + 10.37$ refers to a linear regression with $R^2 = 0.9998$ the coefficient of determination.

IV. A COMMENT REGARDING ABNORMAL NOISE IN KWU PWRS

Recently, an anomaly of the Low-Frequency neutron noise Signals (LFNS) was observed in KWU PWRs [11]. The anomaly consisted of a period of increase and then decrease of the magnitude of the LFNS in the Von-Konvoi plants. A comparison of measurements done in Von-Konvoi and Konvoi plants showed that the auto-power spectral densities (APSD) at ex-core detectors were similar in shape and that in the LFSN domain $\omega \in [0.5, 10]$ Hz this shape showed a white noise behavior in $1/\omega^2$.

More to the object of this discussion, measurements in inner and outer detectors revealed that along the axial direction the neutron noise at the top and bottom of the core was in phase, while it had opposite phase across opposite quadrants. This global noise response suggests that the perturbations excited one of the steady-state core modes which the same symmetries and, in particular, the simplest mode with the higher eigenvalue. Modes with opposite radial behavior have been calculated for a 2D PWR reactor [12] (see, for instance, modes 1, 2, 6, 7, 13 and 14 in Fig. 1 of [12]). A more compelling example is provided by analytical expressions for the one-group, homogeneous diffusion theory. For instance, the modes for a bare cylindrical core of radius R and height H are

$$\Phi_{n_r,m,n_z}(\rho,\varphi,z) = J_{n_r}(j_{n_r,m}\frac{\rho}{R})\cos(n_r\varphi + \varphi_0)\sin(n_z\pi\frac{z}{H}), \quad (13)$$

where $j_{n_r,m}$ is the m-th positive zero of the Bessel function of integer order J_{n_r} and $\varphi_0 \in [0, 2\pi)$. The mode with the appropriate symmetries and highest eigenvalue is given by $n_r = m = 1$ and $n_z = 1$ and for a critical core its eigenvalue $\lambda_{1,1,1} = 1/(1 + \Gamma)$, where $\Gamma = (D/\nu\Sigma_f)(j_{1,1}^2 - j_{0,1}^2)/R \ll 1$, is very close to 1. Because the geometry is invariant under axial rotations this mode is degenerated with a continuum of states, as given by the angle φ_0, so one must assume that only one of the modes is excited. For a bare core with the shape of a rectangular prism, there is also a double degeneracy of this mode and again one has to postulate that only one of the two modes are excited. For the reader convenience, a detailed discussion of these elementary diffusion results is given in the appendix.

In this section we give theoretical support to our assumption. We start with the transport (or diffusion) noise equation that we write as

$$(\frac{i\omega}{v} + \mathcal{B}_0 - \widehat{\mathcal{P}}_{0,\omega})\delta\psi_\omega = -\delta\mathcal{B}_\omega\psi_0, \tag{14}$$

To obtain this equation we have Fourier transformed the classical kinetic equations with delayed neutrons. Then we have introduced a reference steady-state critical transport operator $\mathcal{B}_0 = \mathbf{\Omega} \cdot \nabla + \Sigma_0 - \mathcal{H}_0 - \mathcal{P}_0$ (the diffusion case is obtained by the replacement $\mathbf{\Omega} \cdot \nabla \to -\nabla \cdot D\nabla$), where $\Sigma_0, \mathcal{H}_0$ and $\mathcal{P}_0$ are the total cross section and the scattering and production operators, respectively, and

$$\widehat{\mathcal{P}}_{0,\omega} f = i\omega \sum_d \frac{\chi_d}{\lambda_d + i\omega} \sum_f \beta_{d,f}(\upsilon\Sigma^f_{f,0}, f) \tag{15}$$

is the a component of the contribution of delayed neutrons to the neutron production term. Also, in the last equation $\beta_{d,f}$ $(10^{-4} - 10^{-3})$ is the fission yield of fissile isotope f for precursors in group d and λ_d and χ_d denote the decay constant and the fission spectrum for delayed neutrons.

To derive Eq. (14) we have followed the traditional technique and used the critical flux ψ_0, associated to the reference steady-state operator $\mathcal{B}_0$, to introduce a perturbation formula so that $\mathcal{B}_\omega = \mathcal{B}_0 + \delta\mathcal{B}_\omega$ and $\psi_\omega = \psi_0 + \delta\psi_\omega$, where $\delta\mathcal{B}_\omega$ and $\delta\psi_\omega$ are assumed small perturbations and, finally, we have neglected products of two small quantities. Also, in Eq. (14) $\delta\mathcal{B}_\omega = \delta\Sigma_\omega - \delta\mathcal{H}_\omega - \delta\mathcal{P}_\omega$ contains the Fourier transforms of the perturbations of the cross sections about their reference values in $\mathcal{B}_0$.

We note that for the LFNS the term in ω/v can be safely neglected in Eq. (14). Moreover, the production operator $\widehat{\mathcal{P}}_{0,\omega}$ can also be neglected as compared with the steady-state production term $\mathcal{P}_0$. The resulting equation for the LFNS noise,

$$\mathcal{B}_0\delta\psi_\omega \simeq -\delta\mathcal{B}_\omega\psi_0, \tag{16}$$

has a degenerated operator on the left-hand-side and, according to Fredholm's alternative, a solution exist iff $\delta\mathcal{B}_\omega\psi_0$ is orthogonal to ψ_0: $(\psi_0^\dagger, \delta\mathcal{B}_\omega\psi_0) = 0$, where $\psi_0^\dagger$ is the adjoint flux and $(\cdot,\cdot)$ denotes the scalar flux. Granted that this condition is satisfied, the solution of Eq. (16) can be written as

$$\delta\psi_\omega = z_0\psi_0 + \psi_\perp,\ z_0 \in \mathbb{Z}, \tag{17}$$

where the latter flux component is orthogonal to ψ_0: $(\psi_0^\dagger, \psi_\perp) = 0$. Note that both z_0 and $\psi_\perp$ have to depend on the frequency ω.

Our approach to neutron noise equations for power reactors accounts for the fact that the averaged power is kept constant via the action of built in mechanisms or control rods.[13] This leads to a construction of operator $\mathcal{B}_0$ so the iff condition $(\psi\dagger_0,\ \delta\mathcal{B}_\omega\psi_0) = 0$ is automatically fulfilled with $z_0 = 0$ in Eq. (17). Therefore, we set $z_0 = 0$ and use an expansion of the orthogonal term over the eigenfunctions $\{\lambda_n, \psi_n\}$ of the steady-state problem[2]

$$\delta\psi_\omega = \sum_{n>0} z_n\psi_n,\ z_n \in \mathbb{Z}. \tag{18}$$

We notice again that the z_n ought to depend on ω. At this point we use the experimental evidence to assume that this dependence must be in $1/\omega^2$. To simplify our notation hereafter we shall omit this common dependence on the frequency and concentrate on the spatial dependence. Hence, we shall consider Eq. (18) with the z_n as simple complex constants.

Finally, use of this expansion in Eq. (16) leads to a final expression for the noise source $\delta S_\omega = -\delta\mathcal{B}_\omega\psi_0$:

$$S_\omega = \sum_{n>0} z_n\lambda_n\psi_n,\ z_n \in \mathbb{Z}. \tag{19}$$

Since the detailed spatial behavior of the neutron noise ψ_ω is not known, our approach consists of assuming that all the modes in Eq. (18) are very small except the mode ψ_* which is the simplest mode that reproduces all the experimentally observed symmetries with exclusion of other behavior. Among the modes which exhibit axial symmetry and radial antisymmetry this is the one with the highest eigenvalue. Hence, we assume that the source is

$$S_\omega = \frac{1}{\omega^2}\psi_*(\mathbf{r}), \tag{20}$$

where we have put back the frequency dependence. It remains, however, to characterize the original source of the noise, i.e., the $\delta\mathcal{B}$. The explanation given in [11] points to inlet temperature fluctuations. Because the anomaly in the noise flux was coincidental with a change of fuel elements which grids that increased turbulence, we think that one important source of noise is due to the turbulence generated at the grids, which could generate a volume-distributed noise source as the one in Eq. (20) and lead to the excitation of the flux steady-state mode $\psi_*(\mathbf{r})$. To confirm or disapprove this thesis it remains, thus, to effectuate the pertinent numerical experimentation and this is not in the scope of this work. The observed increase of noise with burnup could provide a led by investigating first which are the components of $\delta\mathcal{B}_\omega$ which increases the most with burnup.

V. CONCLUSIONS

In this paper we have presented a new 3-D multigroup diffusion neutron noise solver implemented in the new generation deterministic code APOLLO3® developed at CEA. We illustrated the capacities of the new 3-D diffusion neutron noise solver by performing two neutron noise simulations in a large pressurized water reactor with heavy baffle in three dimensions. Future work will concern the development of a new 3-D transport neutron noise solver in APOLLO3® and the comparison with neutron noise simulations performed with

[2] this expansion should also include a contribution from the continuum spectrum which might be important at very high frequencies

the reference Monte Carlo code TRIPOLI-4® developed at CEA [14, 15].

In a separated analysis we have portrayed the low-frequency neutron noise signal in some German PWRs as resulting from the excitation of a high-order steady-state like transport mode with the symmetries observed in the noise measurements. This leads to an inverse problem for the perturbations of the cross sections or to a trial and error search for the perturbations with guidance from the symmetries of the observed neutron noise. A result of this analysis is that large core modes can contribute to the LFNS noise with frequencies as high as 10 Hz.

VI. ACKNOWLEDGMENTS

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APPENDIX: ONE-GROUP DIFFUSION RESULTS FOR A BARE HOMOGENEOUS CORE

For the one-group homogeneous core with vacuum boundary conditions elementary diffusion theory gives the dispersion equation:

$$\lambda = \frac{k_\infty}{1 + L^2 B^2}, \tag{21}$$

where $k_\infty = \nu\Sigma_f/\Sigma_a$ is the infinite medium multiplication factor and $L^2 = D/\Sigma_a$ is the square of the diffusion length. The values of the so-called material bucklings B^2 are the eigenvalues of the Laplacian equation

$$-\Delta\Phi = B^2\Phi, \tag{22}$$

where Δ is the Laplacian operator and Φ is the flux associated to eigenvalue λ.

It is well known that the latter problem has a real point spectrum and that the maximum eigenvalue is simple and its eigenfunction is nowhere negative. The λ eigenvalues increases with decreasing buckling. Hence, the dominant eigenvalue λ_0 corresponds to the minimum value B_0^2 of B^2. We shall assume that the reactor is critical so that $\lambda_0 = 1$. Therefore, for any other eigenvalue λ one has

$$\lambda = \lambda_0 - \frac{\Gamma}{1+\Gamma} = \frac{1}{1+\Gamma},$$

where

$$\Gamma = \frac{L^2}{k_\infty}\delta B^2$$

with $\delta B^2 = B^2 - B_0^2$.

Cylindrical core

We consider a homogeneous cylinder of height H and radius R. In cylindrical coordinates (ρ, φ, z) the Laplacian reads

$$\Delta = \partial_\rho^2 + \frac{1}{\rho}\partial_\rho + \frac{1}{\rho^2}\partial_\varphi^2 + \partial_z^2. \tag{23}$$

We want to solve eigenvalue problem (22) with vacuum boundary conditions:

$$\begin{aligned} \Phi(R,\varphi,z) &= 0, && 0 < z < H,\ 0 \le \varphi < 2\pi, \\ \Phi(\rho,\varphi,0) = \Phi(\rho,\varphi,H) &= 0, && 0 \le \rho < R,\ 0 \le \varphi < 2\pi. \end{aligned} \tag{24}$$

Because the boundary conditions are independent of the azimuthal angle φ and apply separately to the radial $\rho = R$ and the two horizontal $z = 0$ and $z = H$ surfaces, we can write the general solution of problem (22) as the product of three functions of a single variable, each one or which depending on a different spatial coordinate:

$$\Phi(\rho,\varphi,z) = R(\rho)F(\varphi)Z(z).$$

Substitution of this anszat in Eq. (22) gives the solution:

$$\Phi_{\nu,m,n_r,n_z}(\rho,\varphi,z) = J_\upsilon(j_{\nu,m}\frac{\rho}{R})\cos(n_r\varphi + \varphi_0)\sin(n_z\pi\frac{z}{H}).$$

where J_ν is the Bessel function of order ν, $j_{\nu,m}$ is the m-th root (excluding zero) of J_ν, $n_r \ge 0$ and $n_z > 0$ are integers and $\varphi_0 \in [0, 2\pi)$. Under replacement in (22) and use of the boundary conditions (24) we find that the constraint

$$\frac{\nu^2}{\rho^2} - \frac{j_{\nu,m}^2}{R^2} - \frac{n_r^2}{\rho^2} - (n_z\frac{\pi}{H})^2 = -B^2,$$

requires to select $\nu = n_r \ge 0$ integer and results in the final relation

$$B^2 = \frac{j_{\nu,m}^2}{R^2} + (n_z\frac{\pi}{H})^2 \tag{25}$$

with the final eigenmodes of the form

$$\Phi_{n_r,m,n_z}(\rho,\varphi,z) = J_{n_r}(j_{n_r,m}\frac{\rho}{R})\cos(n_r\varphi + \varphi_0)\sin(n_z\pi\frac{z}{H}).$$

The critical mode $\lambda_0 = 1$ corresponds to $n_r = 0$, $m = 1$ and $n_z = 1$. This yields the criticality condition

$$\frac{j_{0,1}^2}{R^2} + (\frac{\pi}{H})^2 = \frac{k_\infty - 1}{L^2},$$

which implies $k_\infty \ge 1$, and a critical positive flux

$$\Phi_{0,1,1}(\rho,z) = J_0(j_{0,1}\frac{\rho}{R})\sin(\pi\frac{z}{H}).$$

All others λ eigenvalues corresponds to $n_z > 0$ and either $n_r = 0$ and $m > 1$ or $n_r > 0$ and $m > 0$ and we have

$$\delta B^2 = \frac{j_{n_r,m}^2 - j_{0,1}^2}{R^2} + (n_z^2 - 1)(\frac{\pi}{H})^2.$$

We are interested in high λ modes which have opposite variations in opposed quadrants, $\Phi(\rho, \varphi + \pi, z) = -\Phi(\rho,\varphi,z)$, and no opposition along z. Hence we must have n_r odd and n_z odd. The highest λ results from the choice $n_r = 1$, $m = 1$ and $n_z = 1$ and gives

$$\Gamma = \frac{1}{R^2}\frac{L^2}{k_\infty}(j_{1,1}^2 - j_{0,1}^2)$$

with $j_{1,1}^2 - j_{0,1}^2 = 8.899$. It follows that for a typical LWR this eigenvalue is very close to 1.

Rectangular prism core

For the simplest case of a bare rectangular prism-shaped core with square cross section the general eigenfunction is

$$\Phi(x,y,z) = \sin(n_x\pi\frac{x}{a})\sin(n_y\pi\frac{y}{a})\sin(n_z\pi\frac{z}{H}),$$

with the eigenvalue

$$B^2 = (n_x^2 + n_y^2)(\frac{\pi}{a})^2 + (n_z\frac{\pi}{H})^2$$

and $n_x, n_y, n_z > 0$ integers.

The dominant λ eigenvalue corresponds to $n_x = n_y = n_z = 1$ and gives the criticality condition

$$(\frac{2}{a^2} + \frac{1}{H^2})\pi^2 = \frac{k_\infty - 1}{L^2},$$

where for higher modes ones have

$$\delta B^2 = (n_x^2 + n_y^2 - 2)(\frac{\pi}{a})^2 + (n_z^2 - 1)(\frac{\pi}{H})^2.$$

The highest modes with opposite variation at opposite quadrants, $\Phi(a-x, a-y, z) = -\Phi(x,y,z)$, require either n_x odd and n_y even or n_x even and n_y odd. Therefore the highest mode is a degenerate eigenvalue with two possible states with $n_z = 1$ and either $n_x = 1$ and $n_y = 2$ or $n_x = 2$ and $n_y = 1$. This gives

$$\Gamma = 3\frac{L^2}{k_\infty}(\frac{\pi}{a})\cdot$$

Validity of approximation (16) **in the LFNS**

Finally, in view of the results given by simplified diffusion theory, we analyze here the range of frequencies where one can neglect operator $\widehat{\mathcal{P}}_{0,\omega}$ in the noise equation. We note that the fact that the fission precursor yields are small as compared to unity leads to the ordering $O(\widehat{\mathcal{P}}_{0,\omega}) \sim \varepsilon_\beta(\omega/\sqrt{\lambda_p^2 + \omega^2})O(\mathcal{P}_0)$, where typically $\varepsilon_\beta = O(\beta_p) \sim 0.001$. Let ε be the order required for the linear noise equation, i.e. $O(\delta\psi_\omega) \sim \varepsilon O(\delta\psi_0)$. This condition entails $\omega^2 \leq r^2\lambda_p^2/(1-r^2)$, where $r = \varepsilon/\varepsilon_\beta$. Hence, if $\varepsilon \geq \varepsilon_\beta$ we might neglect $\widehat{\mathcal{P}}$ as compared to $\mathcal{P}_0$ for all ω's or, otherwise, we can discard $\widehat{\mathcal{P}}$ only in the range $\omega \leq r\lambda_p/\sqrt{1-r^2}$.

REFERENCES

1. PÁZSIT I. and DEMAZIÈRE C., *Noise Techniques in Nuclear Systems*, vol. 3, Handbook of Nuclear Engineering, Ed. D. G. Cacuci, Springer Verlag (2010).
2. FRY D. N., MARCH-LEUBA J., and SWEENEY F. J., "Use of Neutron Noise of Diagnosis of In-vessel Anomalies in Light-Water Reactors," *ORNL/TM-8774* (1984).
3. PÁZSIT I. and ANALYTIS G. TH., "Theoretical Investigation of the Neutron Noise Diagnostics of Two-Dimensional Control Rod Vibrations in a PWR," *Annals of Nuclear Energy*, **7**, 171–183 (1980).
4. JONSSON A., TRAN H.N., DYKIN V., and PÁZSIT I., "Analytical investigation of the properties of the neutron noise induced by vibrationg absorber and fuel rods," *Kerntechnik*, **77**, 371–380 (2012).
5. DEMAZIÈRE C., "CORE SIM: A multi-purpose neutronic tool for research and education," *Annals of Nuclear Energy*, **38**, 2698–2718 (2011).
6. MALMIR H., VOSOUGHI N., and ZAHEDINEJAD E., "Development of a 2-D 2-group neutron noise simulator for hexagonal geometries," *Annals of Nuclear Energy*, **37**, 1089–1100 (2010).
7. SCHNEIDER D., DOLCI F., GABRIEL F., PALAU J-M., GUILLO M., and POTHET B., "APOLLO3: CEA/DEN Deterministic multi-purpose code for reactor physics analysis," *Unifying Theory and Experiments in the 21st Century (PHYSOR 2016), Sun Valley, Idaho, United States, May, 1-5.* (2016).
8. DANTE G., *Piecewise Assembly Homogenization via Flux Discontinuity Factors*, M.Sc, Politecnico di Torino (2012).
9. GUPTA N.K., "Nodal methods for three-dimensional simulators," *Progress in Nuclear Energy*, **7**, 127–149 (1981).
10. YAMAMOTO T., "Monte Carlo method with complex-valued weights for frequency domain analyses of neutron noise," *Annals of Nuclear Energy*, **58**, 72–79 (2013).
11. SEIDL M., KOSOWSKI K., SCHÃIJLER U., and BELBLIDIA L., "Review of the historic neutron noise behavior in German KWU built PWRs," *Progress in Nuclear Energy*, **85**, 668–675 (2015).
12. CARNEY S., BROWN F., KIEDROWSKI B., and MARTIN W.R., "Theory and applications of the fission matrix method for continuous-energy Monte Carlo," *Annals of Nuclear Energy*, **73**, 423–431 (2014).
13. SANCHEZ R., "Some comments in neutron noise theory," *Annals of Nuclear Energy*, **86**, 88–98 (2015).
14. BRUN E., DAMIAN F., DIOP C.M., DUMONTEIL E., HUGOT F.X., JOUANNE C., LEE Y.K., MALVAGI F., MAZZOLO A., PETIT O., TRAMA J.C., VISONNEAU T., and ZOIA A., "TRIPOLI-4®, CEA, EDF and AREVA reference Monte Carlo code," *Annals of Nuclear Energy*, **82**, 151–160 (2015).
15. ROUCHON A., ZOIA A., and SANCHEZ R., "A new Monte Carlo method for neutron noise calculations in the frequency domain," *Annals of Nuclear Energy*, **102**, 465–475 (2017).