C5G7-TD Benchmark for Time-Dependent Heterogeneous Neutron Transport Calculations

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Abstract - *A new benchmark for time-dependent neutron transport calculations without spatial homogenization has been created to facilitate the development and assessment of numerical methods for solving the space-time neutron kinetics equations. The benchmark has been named the OECD/NEA C5G7-TD, and has been divided into three phases each corresponding to a different stage of the multi-physics transient analysis of the nuclear reactor core. This paper provides a brief introduction of the benchmark specification of Phase I, known as the "kinetics phase", including the geometry description, supporting neutron transport data, transient scenarios in both two-dimensional (2-D) and three-dimensional (3-D) configurations, as well as the required output parameters from the participants. Preliminary results for selected 2-D transient benchmark problems that have been obtained using Surface Harmonic Method (SHM) are also presented.*

I. INTRODUCTION

In recent years increasing efforts have been made in the development of numerical methods in space-time transient modelling of nuclear reactors, which requires solving the time-dependent Boltzmann equation considering delayed neutrons. In order to ensure the accurate and reliable modelling of neutron physics within a state-of-the-art transient code, the neutron kinetics part of such a method should be derived for the full-scale calculation of the space-time neutron kinetics equations without incorporating the diffusion approximation and spatial homogenization. Such advanced approaches require the evaluation of neutron kinetics program modules through the cross-verification of computational codes, which are employed to calculate thoroughly defined test problems, or the benchmarks.

However, existing benchmark problems cannot satisfy the demand for verifying numerical methods for performing homogenization-free time-dependent transport calculations. On one hand, many of them are simplified diffusion problems, where the computational domain consists of a number of homogeneous regions characterized by few-group diffusion macroscopic cross sections [1, 2, 3, 4]. Although these test problems are useful in assessing the performance of codes at the initial stage of their development, they fail to provide opportunities to verify computer codes that are not built upon spatial homogenization and diffusion approximation. On the other hand, another set of benchmark problems are available whose computational domain is significantly more heterogeneous than those of the first category and the material specification was provided with respect to isotopic concentrations [5, 6, 7]. It is inevitable that the corresponding numerical solutions are influenced by additional uncertainties stemming from the nuclear data, the group constants preparation procedure, as well as computational method in another physics domain (e.g., thermal-hydraulics), which makes it difficult to reveal methodical errors of space-time neutron kinetics codes.

The main objective of the proposed benchmark is to specify a series of space-time neutron kinetics test problems with heterogeneous domain description for solving the time-dependent group neutron transport equation. This benchmark, named as C5G7-TD, has been approved by Organization for Economic Cooperation and Development Nuclear Energy Agency (OECD/NEA) Nuclear Science Committee (NSC) [8]. This paper summarizes the development and specification of Phase I (kinetics phase) of the C5G7-TD benchmark, where no feedback is meant to be accounted in the modelling effort.

II. BENCHMARK SPECIFICATION

1. Core Description

The reactor model is based on the well-studied steady-state C5G7 benchmark problems, which were developed to test the capabilities of radiation transport codes that do not utilize spatial homogenization above the fuel pin level [9, 10, 11]. It is a miniature light water reactor (LWR) with sixteen fuel assemblies: eight uranium oxide (UO_2) assemblies and eight mixed oxide (MOX) assemblies, surrounded by a water reflector. The 2-dimensional (2-D) configuration of the current benchmark is exactly the same as that of the C5G7 problem, which features a quarter-core radial symmetry, as depicted on the left of Fig. 1. The four assemblies in this representation are numbered 1-4 for the convenience of the following specification. The 3-dimensional (3-D) configuration of the reactor core, as can be seen on the right on of Fig. 1, has reflectors residing on the top and bottom of the active core with equal height [12].

Both UO_2 and MOX fuel assemblies follow the 17×17 design, consisting of 264 fuel pins, 24 guide tubes for control rods and one instrument tube for a fission chamber in the center grid-cell. All pin cells have a pin radius of 0.54 cm with a pitch of 1.26 cm. The pin cell layout for the south-east quadrant is depicted in Fig. 2.

This benchmark adopts the transport corrected few-group cross sections that is directly available from the original C5G7 benchmark. In the transport calculations, standard flux weighting was used to collapse cross sections to seven energy groups and to homogenize fuel, gap, and cladding materials

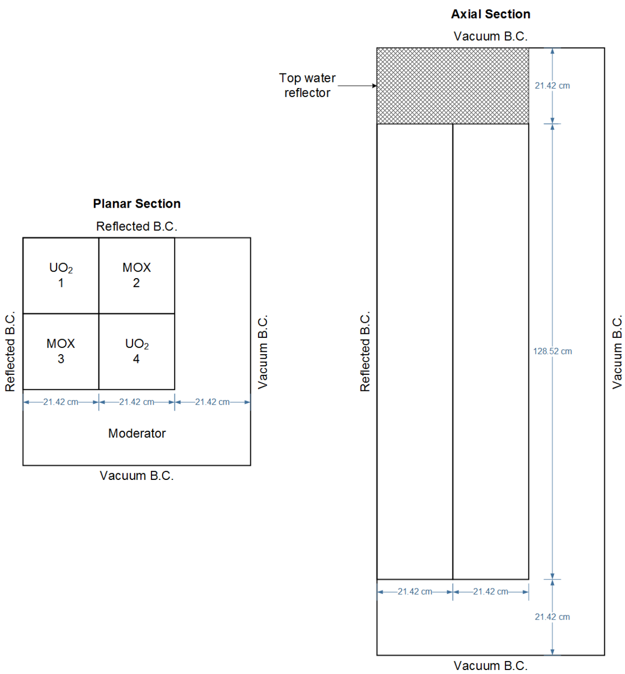

Fig. 1. Planar and axial view of the benchmark problem.

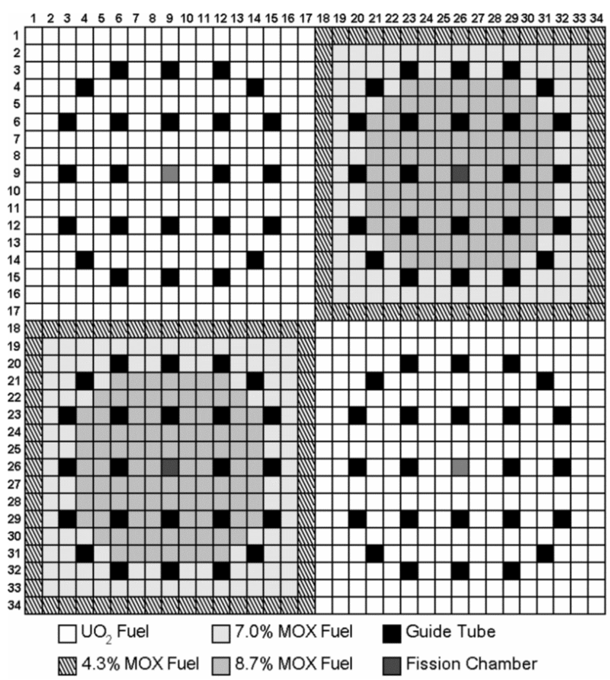

Fig. 2. C5G7 fuel pin compositions and numbering scheme.

into homogenized fuel compositions for the UO_2 and MOX fuel pins (three enrichments of 4.3%, 7.0%, and 8.7%). The group constants of the moderator, homogenized guide tube, control rod, and fission chamber were obtained using the UO_2 fuel spectrum. The homogenization is depicted as processes *A* and *B* in Fig. 3, respectively, for the fuel and non-fuel cells. In other words, cross section data were provided for all the pin cells in a simplified 2-region geometry, where Zone 2 represents the moderator outside the outer tube and Zone 1 refers to the mixture of all medium surrounded by Zone 2.

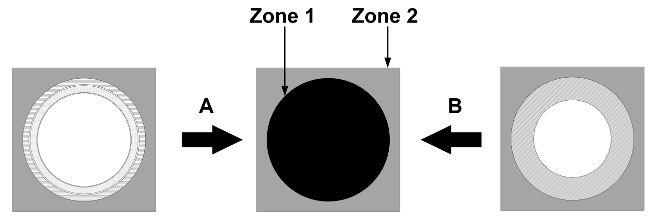

Fig. 3. C5G7 fuel pin compositions and numbering scheme.

2. Preparation of Kinetics Parameters

The kinetics parameters are prepared in support of the benchmark calculation, including delayed neutron fractions, delayed neutron precursor decay constants, delayed neutron group spectra, and group neutron velocities. Transport calculations using the WINS-D code with the 69-group energy structure were performed to obtain the space and energy dependent neutron flux for each pin cell models [13]. The resulting kinetics parameters are given in the 8-group delayed neutron representation owing to its advantages relative to the traditional 6-group representation, that is, in additional to using a single set of precursor half-lives, it also allows for a more consistent description of delayed neutron emission from the longest-lived precursors so as to avoid distortions in the reactivity measurement analysis [14].

3. 2-D Transient Exercises

Two sets of transient problems are considered in the C5G7-TD benchmark corresponding to the 2-D or 3-D configuration of the core separately. The 2-D transient problems consist of 4 exercises, including exercises TD0, TD1, TD2 and TD3.

Exercises 0, 1 and 2 of this benchmark, referred to as TD0, TD1 and TD2, are focused on the simulation of the postulated control rods movements. All control rods (one rod bank per fuel assembly) are at fully withdrawn position at $t = 0$ and the transient is initiated by a control rods insertion, continues with the rod extraction beginning at $t = 1$ s, and completes with all rods returning to their initial positions at the end of 2 s. Different types of the control rods movement have been specified for TD0 and TD1/2, as depicted in Fig. 4 for the relative depth of the insertion. Each exercise includes multiple test problems with different selection on locations where the control rods movements occur, as shown in Table I, so that the spatial effect imposed on the reactor core behavior can be revealed.

The postulated transient event can be interpreted in 2-D calculations as the material composition change, i.e., the replacement of the moderator-filled guide tube material by the control rod material in Zone 1 of all affected cells. For example, the cross sections mixing in TD1 can be described

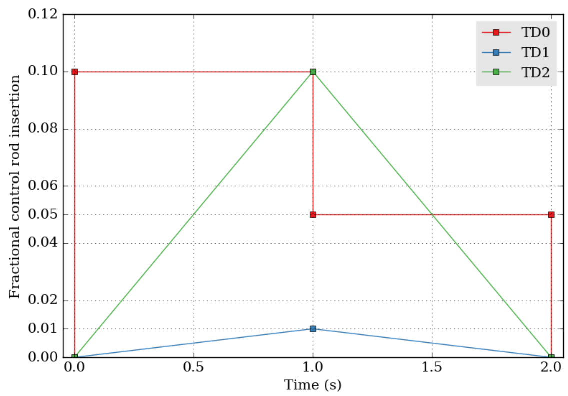

Fig. 4. Control rod movement in exercises TD0/1/2.

TABLE I. Scenarios of control rods movements defined in TD0, TD1 and TD2 exercises.

Test cases	TD0	TD1	TD2
1	Bank 1	Bank 1	Bank 1
2	Bank 3	Bank 3	Bank 3
3	Bank 4	Bank 4	Bank 4
4	Bank 1, 3 and 4	Bank 1, 3 and 4	–
5	All banks	All banks	–

by the linearly increasing and decreasing functions:

$$\begin{aligned} \Sigma_x(t) &= \Sigma_x^{GT} + 0.01(\Sigma_x^R - \Sigma_x^{GT})t, \quad 0 \le t < 1s \\ \Sigma_x(t) &= \Sigma_x^{GT} + 0.01(\Sigma_x^R - \Sigma_x^{GT})(2-t), \quad 1s \le t < 2s \\ \Sigma_x(t) &= \Sigma_x^{GT}, \quad t > 2s \end{aligned} \tag{1}$$

where Σ refers to the seven-group macroscopic cross sections, while subscription x being denoted as the reaction type, which includes absorption and scattering. The superscription R and GT stand for the domain of control rod and guide tube, respectively.

The third exercise (TD3) is intended as a simulation of a transient event of the core moderator density change. The moderator density in all assemblies is at its nominal value as the starting point, and decreases linearly before reaching its minima after 1 s into the transient, then returns to its initial value linearly within the next 1 s. This minimum value is represented as a fraction, denoted as ω ($0 \le \omega \le 1$), of its initial value to account for the rate of change of moderator density.

There are four test problems in TD3, as listed below, each with its own value of ω varying from 0.80 to 0.95, demonstrating different levels of intensity of the transient. The rate of change of moderator density for each of these test problems is illustrated in Fig. 5. Take TD3-1 for example, the moderator density decreases with time linearly and reaches 95% of its initial value 1 s into the transient and then returns to the nominal value at the end of the next 1 s. The simulation of TD3 transient can be realized by the linear perturbation of the moderator cross sections in all energy groups in Zone 2 (see Fig. 3) across the core, i.e. cross sections decrease linearly to ω times their initial values at the end of 1 s, and made equal to the nominal level at the end of the next 1 s. It should be noted that this change mechanism affects all cells in the core uniformly except for that of the water in the reflector.

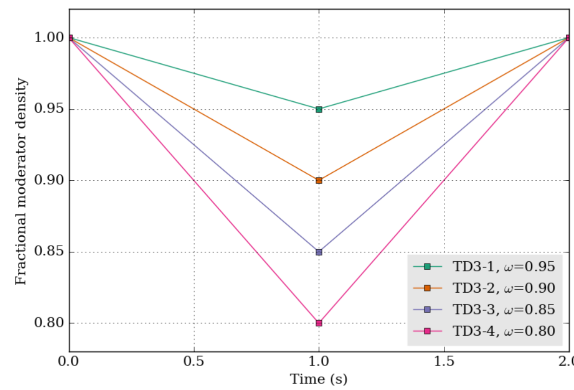

Fig. 5. Core average moderator density change in TD3 exercises.

4. 3-D Transient Exercises

The initial core condition of the 3-D transient problems is referred to as the Unrodded case, where the control rod banks are positioned inside the upper axial water reflector as indicated by the shading in Fig. 1. The explicit modelling of the fission chambers and control rods present in the axial reflector region is required during the simulation.

Two exercises have been specified: TD4 is driven by the control rods movement and TD5 is concerned with the moderator density change.

In TD4, it is assumed that the rod bank moves at a constant speed, which allows it to be completely inserted to the core from the Unrodded position within 6 s. Note that this is a hypothetic value proposed only for the purpose of shortening the transient event thus reducing the computational efforts in the time-space calculation. Five test problems are defined with different scenarios in the way similar to the 2-D transient exercises and the corresponding visualizations are given in Figs. 6-10. An example of understanding these figures is given as the following. In the TD4-3 case (Fig. 8) both rod banks 1 and 3 are involved: 2 s after the rod bank of Assembly 3 is inserted, the rods of Assembly 1 start to insert at the same speed. At the end of 4 s both rod banks 1 and 3 begin to be withdrawn simultaneously until the core condition returns to the Unrodded configuration at the end of 8 s.

The assembly or combination of assemblies affected in the 5 proposed control rods transient scenarios in TD4 exercise are given in the second column of Table II.

The exercise 5 (TD5) models a series of transient events of moderator density change. It is assumed that all control rods are positioned in the fully withdrawn position (Unrodded configuration) throughout the transient and the moderator density is at the nominal level as the starting point. Totally 4 test problems have been defined to demonstrate different tran-

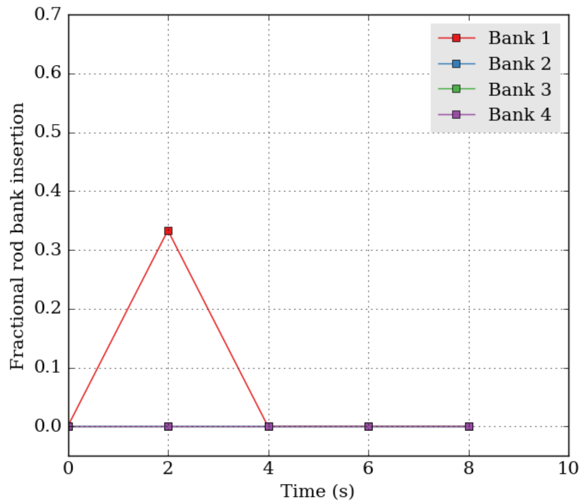

Fig. 6. Relative control rod position in exercise TD4-1.

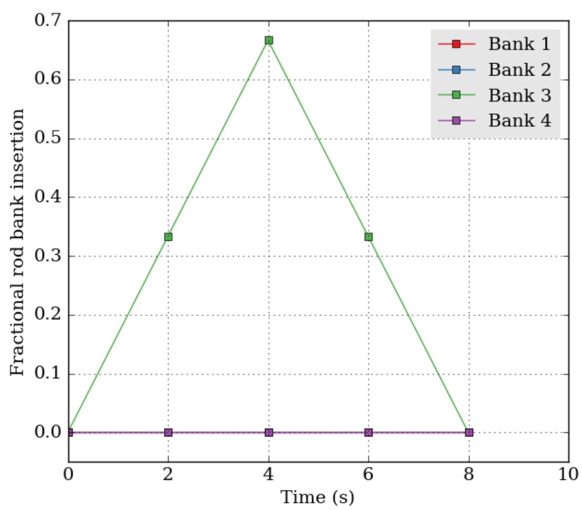

Fig. 7. Relative control rod position in exercise TD4-2.

sient mechanisms with variation in the rate and location of the density change, as shown in Figs. 11-13.

For example, TD5-1 transient (Fig. 11) is initiated by the moderator density decrease in Assembly No 1 at the constant rate of 5% per second, and after 1 s into the transient the moderator density in Assembly No 3 starts to drop at the same rate. The moderator density starts to increase right after 2 s into the transient in both assemblies at the rate of 5% per second, and thus returns to the nominal value within another 2 s and 1 s for Assembly No 1 and 3, respectively. The moderator density in Assembly No 2 and 4 is not affected in this transient. Note that all the density change is expected to take place uniformly with the assembly, i.e., no spatial dependence is assumed. In addition, the water density in both radial and axial reflector is maintained throughout the transient.

The rightmost column of Table II shows the fuel assem-

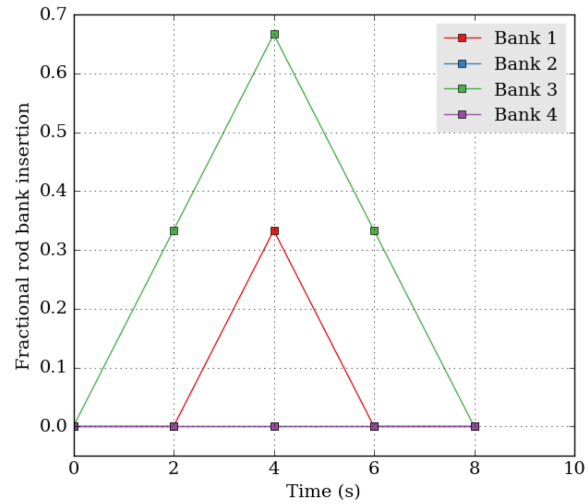

Fig. 8. Relative control rod position in exercise TD4-3.

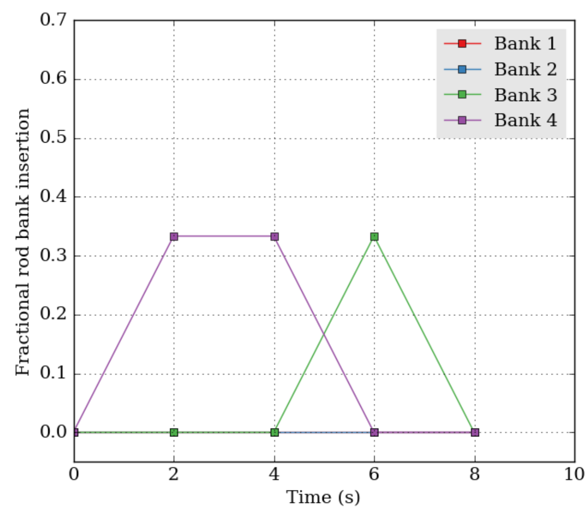

Fig. 9. Relative control rod position in exercise TD4-4.

blies where the moderator density will change according to the specification.

5. Output Requirements

In order to fully capture the temporal behavior of the core during the postulated transients, simulations should be performed with sufficiently small time step size, especially at the beginning of the events. In principle, the time step should be no longer than 25 ms during the transient and can be increased gradually towards the end of simulation; however, the participants may choose the time step size based on the method employed in the calculation without sacrificing the resolution of the results. Parameters to be collected from the participants are the following:

- Core dynamic reactivity.

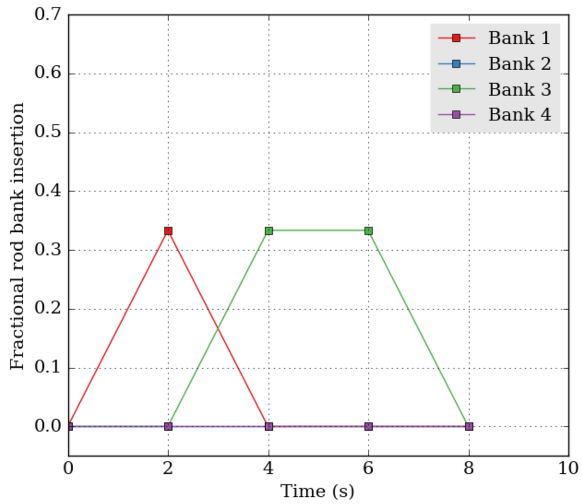

Fig. 10. Relative control rod position in exercise TD4-5.

TABLE II. Assemblies affected in the transient scenarios of TD4 and TD5 excercises.

Test cases	TD4 (control rod bank)	TD5 (moderator density)
1	Bank 1	Assembly No 1 and 3
2	Bank 3	Assembly No 1 and 3
3	Bank 1 and 3	Assembly No 1, 3 and 4
4	Bank 3 and 4	Assembly No 2, 3 and 4
5	Bank 1 and 3	–

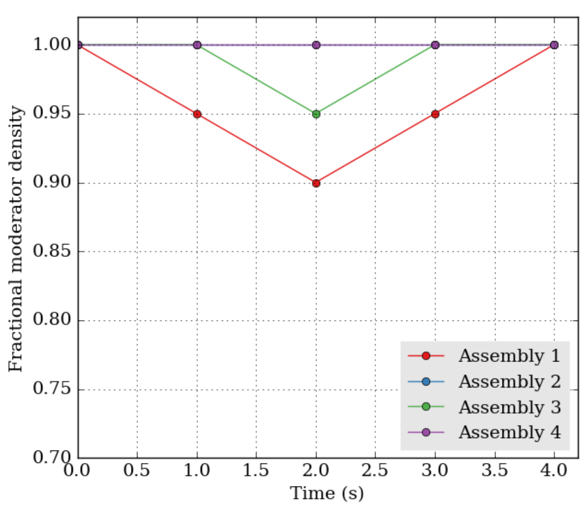

Fig. 11. Relative control rod position in exercise TD5-1.

- Fractional total core fission rate: the fraction of total core fission rate to its initial value at $t = 0$. The fission rate in the fission chamber should be neglected.

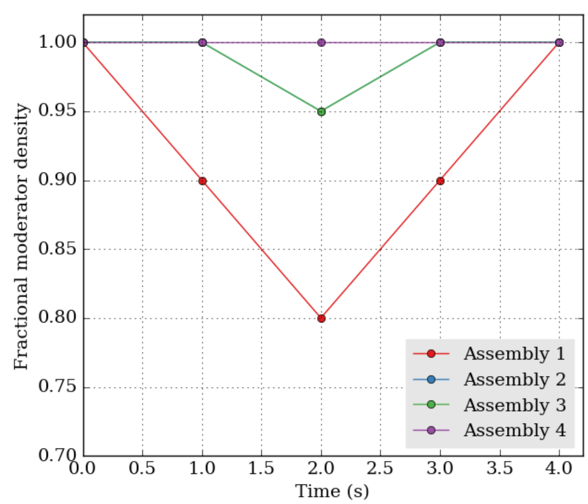

Fig. 12. Relative control rod position in exercise TD5-2.

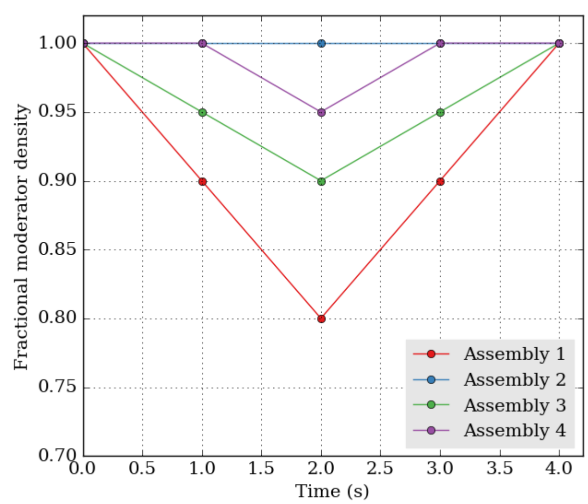

Fig. 13. Relative control rod position in exercise TD5-3.

- Effective delayed neutron fraction.
- Prompt neutron life time.
- Radial distribution of axially integrated fission rate on the fuel assembly basis.
- Radial distribution of axially integrated fission rate on the pin-by-pin basis.

The fission rate distribution should be normalized in such a way that the values of all assemblies (or pin cells) across the core is summed up to the ratio of total core fission rate at a given time point to that of the beginning of the transient. This is achieved by the use of a single normalization factor for the entire process ensuring that the summation of the distribution is 1.0 at time $t = 0$. In 3-D transient problems similar core integral parameters are requested as in the 2-D problems with the addition of the 3-D distributions of normalized fission rate on both assembly and pin level according to the pre-defined axial discretization of the active core. Templates for result submission have been developed for the convenience of the participants.

III. PRELIMINARY RESULTS

The eigenvalue of the initial steady-state of C5G7-TD benchmark is calculated to be $k_{\mathbf{eff}} = 1.18646 \pm 0.07\%$ using the Monte Carlo code MCNP5 [15]. The corresponding initial core fission rate distribution on the pin cell level is depicted in Fig. 14 for the south-east quadrant as a reference, where the total fission rate is equal to unity.

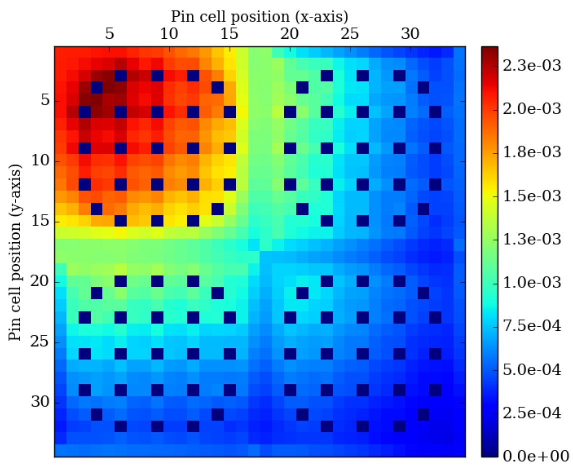

Fig. 14. Initial core fission rate distribution of C5G7-TD.

The study of selected benchmark exercises has been carried out and the transient calculations were performed using the SUHAM-TD code, which is being developed at Kurchatov Institute for solving the time-dependent neutron transport group equations for a full-scale reactor core. The Surface Harmonic Method (SHM) was selected as the transport solver because it demonstrates reasonably sufficient calculation accuracy of major reactor physics parameters without sacrificing the computational cost at the same time [16].

In the current study temporal behaviors of the solution were obtained for two 2-D exercises driven by the control rod movement, among which TD0 involves abrupt transient scenario while TD2 featuring the ramp scenario at different speed. The time-dependent calculations were performed over a period of 10 s with the fixed time step of 1 ms. Comparison of normalized core fission rate of various test cases are shown in Figs. 15 and 16, respectively, for TD0 and TD2.

IV. CONCLUSIONS

The space-time neutron kinetics benchmark C5G7-TD has been developed and proposed for the verification of numerical methods which aim to solve the time-dependent neutron transport equation without employing the spatial homogenization. It is based on the well-studied C5G7 benchmark of which the 2-D geometry configuration and few-group cross

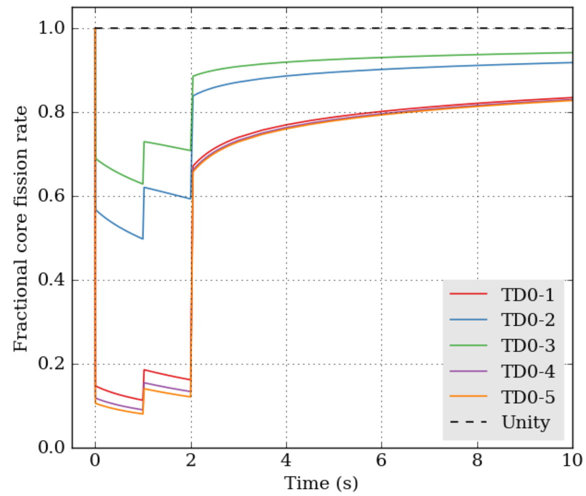

Fig. 15. Fractional core fission rate as a function of time for TD0 cases.

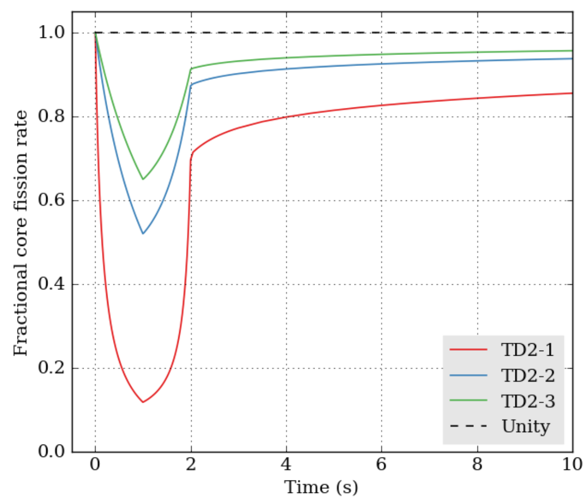

Fig. 16. Fractional core fission rate as a function of time for TD2 cases.

sections were inherited, while extensive effort being made to complete the 3-D geometry configuration, to prepare kinetics related data in support of the modelling and simulation, as well as to define various perturbation scenarios based on the control rod movement and moderator density change. A list of critical time-dependent physics parameters has also been identified based on which the comparative analysis of the obtained results will be carried out for the purpose of the cross-verification of numerical methods. In addition, transient calculations were performed for selected 2-D benchmark exercises using the Surface Harmonic Method (SHM) and preliminary calculation results were presented.

Authors welcome researchers to participate in the benchmark and utilize these problems in the development and veri-

fication of numerical methods for solving the time-dependent transport equations. The participants are also encouraged to propose the expansion of list of compared parameters as well as the perturbation scenarios. International cooperation in the organization of cross-verification of the C5G7-TD benchmark and its modifications would allow obtaining the reference solutions for these test problems and would provide a quantitative assessment of computational uncertainties of modern codes solving the time-dependent neutron transport equation.

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