A Stability Analysis Of The IBR-2M Pulsed Reactor Of Periodic Operation At Self-Regulating Regime

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Abstract - The power feedback (PFB) is used in the dynamics model of the IBR-2M reactor instead of the temperature feedback. The PFB is significantly influenced to operation and stability of the reactor. This replacement allowed the PFB structure and parameters to be experimentally estimated through the mathematical processing of the recorded power transition processes caused by deliberate change in the reactivity. Experimental and model studies of the parameters of the PFB on power as a function of the average power of the IBR-2M have been performed in 2015-2016. The model of the IBR-2M with the power feedback parameters which correspond to one series of experiments is investigated for stability by the frequency method. In this work we present results of the stability analysis of the IBR-2M reactor at different level of the average power in the self-regulating regime (i.e., without an automatic regulator).

I. INTRODUCTION

The IBR-2M pulsed reactor of periodic operation (upgraded version of IBR-2 reactor) was commissioned with an average power of 2 MW at the Frank Laboratory of Neutron Physics in 2012. The reactor generates short neutron pulses (200 µs at half width) with period 0.2 s and amplitude of 1830 MW. In the IBR-2M reactivity pulses are produced by the reactivity modulators rotating near the core. When the two reflectors pass the core simultaneously, a reactivity pulse develops and for a short time (450 µs) the reactor stops being in a super-critical state with prompt neutrons. As the reflectors move away from the reactor core, the reactor becomes deeply sub-critical. The controlled parameter of the reactor is the deviation of the power pulse amplitude $P_m$ from its basic value $P^0_m$ in relative units ($\Delta P_m = (P_m - P^0_m)/P^0_m$) [1]. The basic value is the average amplitude corresponding to the stationary regime, i.e., the regime in which the reactor average power is generally constant [2]-[4].

Shown that a power feedback (PFB) is significantly influenced to operation and stability of reactor, on the experience of the IBR-2 reactor. The PFB parameters are changed in the process of reactor operation. Therefore, it is necessary periodically to record transient processes caused by square wave reactivity. The automatic regulator generates the waves when taken out of the control loop. And at time the reactor is operated in a self-regulating regime. The PFB parameters of the IBR-2M reactor were estimated at average power ranging 0.5, 1.0, 1.5 and 2.0 MW [5]. In this work we present results of the stability analysis of the IBR-2M reactor in the self-regulating regime (i.e., without an automatic regulator).

II. BRIEF DESCRIPTION OF THE IBR-2M REACTOR

In contrast to stationary reactors characterized by a constant neutron flux, the IBR-2M reactor generates periodic neutron pulses whose duration at half maximum is shorter than the interval between pulses by three orders of magnitude. The reactor core is shaped as an irregular hexahedral prism positioned vertically. Two rotating movable reflectors, representing a reactivity modulator (MR), pass beside one of the prism faces. Stationary reflectors, used for emergency protection (EP) and control of the reactor, adjoin five other faces. The schematic representation of the IBR-2M reactor is shown in Fig. 1.
Fig. 2. Reactivity ($\epsilon$), power ($P$) and intensity ($S$) of delayed neutron sources of the IBR-2M reactor as function of time ($t$) under operating conditions with a power pulse frequency of 5 Hz: $T_p$ is the power pulse repetition interval, $\epsilon_m$ is the maximum reactivity achieved over the time of the reactivity pulse, $E_p$ is the energy of power pulse and half orders of magnitude lower than the power pulse amplitude. In the intervals between pulses, i.e., when there are neither MMR nor AMR in front of the core, the reactor power (background) is approximately four orders of magnitude lower than the power pulse amplitude. Due to such a principle of the pulsing reactivity formation, almost the total energy released in the reactor is released during power pulse (92%), and the fraction of pauses is as low as 8% of the total energy.

III. MODEL DYNAMICS OF THE IBR-2M PULSED REACTOR OF PERIODIC OPERATION

This model is designed to simulate the transient processes, calculate the frequency characteristics of the reactor for the system as a whole and the constituent components, and assess the stability of the reactor. The block diagram of the IBR-2M dynamics model is shown in fig.3. The units of the block diagram correspond to the dynamic equations.

Fig. 3. Block-scheme of the dynamics model of the IBR-2M reactor in the self-regulating regime. $\epsilon_m^0$ is the basic value of the maximum reactivity achieved over the time of the reactivity pulse; $\epsilon_{mn}$, $\epsilon_{Fn}$ and $\epsilon_{Tn}$ are the reactor reactivity, external (excitation) and power feedback reactivity, respectively; $\Delta \epsilon_n$ and $\Delta \epsilon_{pm}$ are the deviation of the total and energy of power pulse from their basic values, respectively

To begin with, known equations of the kinetics of a one-group point model of the reactor are taken[6]:

$$\frac{dn}{dt} = \frac{\rho - \beta}{\tau} n + \sum_{i=1}^{6} \lambda_i C_i$$  \hspace{1cm} (1)

$$\frac{dC_i}{dt} = \frac{\beta_i}{\tau} n - \lambda_i C_i$$  \hspace{1cm} (2)

where $n$ is the neutron density; $t$ is the time; $\rho$ is the reactivity; $\epsilon = \frac{\beta}{\sum_i \beta_i}$ is the total fraction of delayed neutrons ($\beta_i$ is the fraction of delayed neutrons in the $i$th group, $\tau$ is the effective lifetime of prompt neutrons; $i = 1, \ldots, 6$ is the index of the delayed neutron group); $C_i, \lambda_i$ are the concentration and the decay constant of the sources of delayed neutrons of the group $i$, respectively.

After we go over from the neutron density to the power and from the intensity of the neutron to the normalized intensity in units of power, kinetic equations (1) and (2) take the form:

$$\frac{\tau dP}{\beta dt} = \frac{\epsilon}{\beta} + S$$  \hspace{1cm} (3)

$$S = \sum_{i=1}^{6} S_i$$  \hspace{1cm} (4)

$$\frac{1}{\lambda_i} \frac{dS_i}{dt} + S_i = \mu_i P$$  \hspace{1cm} (5)

where $P$ is the power, $\epsilon$ is the reactivity on prompt neutrons; $S$ and $S_i$ are, respectively, the total number of delayed neutrons and the neutrons in the $i$th group (expressed in units of power); $\mu_i = \beta_i/\beta$ is the relative fraction of delayed neutrons in the $i$th group.

The following values of parameters have been used in equations: $\mu_i = 0.038, 0.028, 0.216, 0.328, 0.103, 0.035$; $\lambda_i = 0.0129, 0.0311, 0.134, 0.331, 1.26, 3.21$ s$^{-1}$ [6]; $\beta = 2.16 \times 10^{-3}$; $\tau = 6.5 \times 10^{-8}$ s [7].

The relative amplitude of the power pulse was taken as a controllable parameter of IBR-2M. This parameter is actually equal to the energy of power pulse $E_p$, scaled to the prescribed (base) value $E_p^0$. The energy of power pulse is expressed by the relation [8]

$$E_p = MS,$$  \hspace{1cm} (6)

where $S$ is the total intensity of delayed neutrons prior to the growth of the reactivity pulse; $M$ is the pulse transfer coefficient, which is a nonlinear function of the maximum reactivity in a pulse $\epsilon_m$ [9]. In the working range, the dependence of $M$ on $\epsilon_m$ can be approximated by an exponential function to a good of accuracy.

As a result, from equations (3)–(5) we obtain equation (7) corresponding to the $n$th power pulses:

$$\frac{E_n}{E^0} = \frac{E_{pm}}{E^0} + \frac{E_{hn}}{E^0}, \quad \frac{E_{pn}}{E^0} = \frac{E_{pm}}{E^0} \quad \frac{E_{hn}}{E^0} = k_{hn} S_n \frac{S}{S^0},$$

$$k_{hn} = \frac{E_{mn} S_0}{S_n E^0} = \frac{\beta T_p}{\Delta k_{MR} - \epsilon_{mn} E^0};$$

$$\frac{E_{pm}}{E^0} = \frac{S_n M_n}{S^0 M^0} = \frac{S_n}{S^0} \exp[\ln M_n - \ln M^0]$$
\[
\frac{S_n}{S^0} = \sum_{i=1}^{6} \frac{S_{in}}{S^0}, \quad \frac{S_m}{S^0} = \left( \frac{S_m}{S^0} + \mu_i A_i \frac{E_{n-1}}{E^0} \right) \exp(-\lambda T_p) \quad (7)
\]

where, \(E_p, E_s, E = E_p + E_0\) are the energy of the power pulse, the energy of interval between power pulses (background energy), and the total energy for period of pulses \(T_p\), respectively; \(k_b\) is the fraction of the background energy over a period in the total energy over the same period; \(\Delta k_{MR} = 0.027\) is the movable reflector efficiency. Here and below the base values of the parameters are labelled by the index 0.

The constants \(S^0/E^0\) and \(E^0/E_0\) in equation (7) are calculated by the following formulas:

\[
\frac{S^0}{E^0} = \frac{6}{\sum_{i=1}^{6} \frac{\mu_i A_i}{\exp(\lambda T_p)} - 1};
\]

\[
\frac{E^0}{E_0} = \frac{\beta T_p}{\Delta k_{MR} - \epsilon_{m0}} \sum_{i=1}^{6} \frac{\mu_i A_i}{\exp(\lambda T_p)} - 1;
\]

\[
\frac{E^0}{E_0} = 1 - k_b^0
\]

where, \(\epsilon_{m0} = 0.001\) is the basic value of the maximum value of reactivity achieved over time of reactivity pulse.

The model intended for investigating the dynamics of IBR-2M uses the deviations of the variables from their base values:

\[
\Delta E_p = E_p - E_0; \quad \Delta E = E - E_0;
\]

\[
\Delta S_i = S_i - S_0; \quad \Delta S = S - S^0
\]

and relative deviations of the variables:

\[
\frac{\Delta e_p}{\Delta E_p} = \frac{\Delta e}{\Delta E}; \quad \frac{\Delta s_i}{\Delta S_i}; \quad \frac{\Delta s}{S}
\]

(9)

For the sake of convenience, reactivity in the model is expressed both in absolute units (\(e\)) and in fractions \(\beta_p\), \(\beta = e/\beta_p\) [10]. The expression of the pulsed reactor reactivity is analogous to the expression of the stationary reactor reactivity in fractions of \(\beta\). For the IBR-2M reactor, \(\beta_p = 1.54 \times 10^{-4}\).

By analogy with the 2R-2 reactor, the IBR-2M PFB reactivity corresponding to the \(n\)-th power pulse is a sum of three components [2]-[5]:

\[
r_{Tj} = \sum_{j=1}^{3} r_{Tj};
\]

\[
r_{Tj} = \left( r_{Tj} + \Delta E_{n-1} \frac{k_{Tj}}{T_{Tj}} \right) \exp\left( -\frac{T_p}{T_{Tj}} \right)
\]

(11)

where, \(k_{Tj}\) and \(T_{Tj}\) are the transfer coefficient of the PFB and the time constant of the PFB \(j\)th component \((j = 1, 2, 3)\), respectively.

Linear equations relate discrete values of the variables associated with the neighboring power pulses. Therefore, in the model used following discrete transfer functions:

1. For the equation of the kinetics (7),

\[
W_s(z) = \frac{\Delta s(z)}{\Delta e(z)} = \sum_{i=1}^{6} W_{ai}(z),
\]

\[
W_{ai}(z) = \frac{\Delta s(z)}{\Delta e(z)} = \frac{b_{ai}}{z - z^{-1}}
\]

(12)

where,

\[
\begin{align*}
\epsilon_{ai} &= \exp(\lambda T_p) \quad \text{where}, \\
\beta &= \frac{\mu_i A_i}{\sum_{i=1}^{6} \mu_i A_i \exp(-\lambda T_p)}
\end{align*}
\]

2. For the equation of the PBF (11),

\[
W_{Tj}(z) = \frac{r_{Tj}(z)}{\Delta E(z)} = \frac{b_{Tj}}{z - z^{-1}}
\]

(13)

where,

\[
\begin{align*}
\epsilon_{Tj} &= \exp(T_p/T_{Tj}) \quad \text{where}, \\
\beta &= \frac{k_{Tj}}{T_{Tj}}
\end{align*}
\]

To study the feedbacks of a reactor in fast processes, a sequence of the energy of the power pulses under the action of periodic modulation of external reactivity was recorded. To estimate the dependence of the feedbacks on the average power of the reactor, a series of experiments was conducted at power ranging 0.5, 1, 1.5 and 2 MW. In addition, the noise of the energy in the power pulses was investigated to analyse the stability of the reactor. In the course of the experiments, the reactivity in the reactor operating in a self-regulation regime, i.e., without an automatic regulator tour, was modulated. The reactor was taken out of the regulation loop and used as a generator of periodic square oscillations of reactivity. To eliminate any effect of energy production on the parameters, the power feedbacks were estimated in one cycle of measurements. The experiments were performed under identical conditions: the coolant flow rate through the core 100 m\(^3\)/h, period of power pulses 0.2 sec, the number of power pulses over a period of square reactivity fluctuations 80-160, driving reactivity amplitude from ±0.0325 to ±0.0565. One transient process of power variation over a modulation period of the reactivity is displayed in Fig. 4. The parameters of power feedback, which are introduced into the model of the reactor dynamics, were determined by analyzing these processes using a special search program (Table 1) [5].

After the mathematical processing of the relevant data from the series of experiments performed at the IBR-2M reactor the pulsed transition characteristic of the PBF (a change in the PBF reactivity caused by a power pulse of unit area) was calculated. For illustration and convenience of further analysis we pass onto feedback pulse characteristic which is a change with time \(t = T_p \cdot n\) of the feedback reactivity \(r_T\) caused by a single power pulse with energy of 1 MJ:

\[
r_T(t) = \sum_{j} \frac{k_{Tj}}{T_{Tj}} \exp\left( -\frac{t}{T_{Tj}} \right)
\]

(14)
TABLE I. Power feedback parameters of the IBR-2M reactor of periodic operation [5].

<table>
<thead>
<tr>
<th>Power</th>
<th>Parameter</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 MW</td>
<td>(k_{T1}, \beta_p/MW)</td>
<td>-6.54</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(T_{T1}, s)</td>
<td>5.98</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.0 MW</td>
<td>(k_{T1}, \beta_p/MW)</td>
<td>-5.61</td>
<td>1.16</td>
<td>-1.09</td>
</tr>
<tr>
<td></td>
<td>(T_{T1}, s)</td>
<td>6.90</td>
<td>0.40</td>
<td>0.37</td>
</tr>
<tr>
<td>1.5 MW</td>
<td>(k_{T1}, \beta_p/MW)</td>
<td>-5.87</td>
<td>1.31</td>
<td>-0.96</td>
</tr>
<tr>
<td></td>
<td>(T_{T1}, s)</td>
<td>8.20</td>
<td>1.33</td>
<td>1.13</td>
</tr>
<tr>
<td>2.0 MW</td>
<td>(k_{T1}, \beta_p/MW)</td>
<td>-5.91</td>
<td>1.59</td>
<td>-0.82</td>
</tr>
<tr>
<td></td>
<td>(T_{T1}, s)</td>
<td>7.60</td>
<td>1.02</td>
<td>0.46</td>
</tr>
</tbody>
</table>

where \(T_T\) is the time constant and \(k_T\) is the corresponding feedback transfer coefficient, \(j = 1, 2, 3\).

The sum \(k_T = \sum_j k_{Tj}\) is an asymptotic value of the fast power coefficient of reactivity. It is obvious that the necessary condition for stability is \(k_T < 0\). A most probable scenario of feedback weakening as well as a sufficient condition for stability can be determined by direct modelling of pulse characteristics. The sufficient condition for stability can be a negative pulse characteristic, \(r_p(t) < 0\). The characteristic behaviour of \(r_p(t)\) calculated from power transients measured in different level of average power of the IBR-2M reactor are shown in Fig.5.

IV. FREQUENCY CHARACTERISTICS

An IBR-2M reactor without automatic regulator is represented as a linearised closed single-loop system with negative feedback (Fig.6). The reactivity deviation \(\Delta r_n = (\varepsilon_m - \varepsilon_m^0)/\beta_p\) is taken as the input signal of the closed system, and the relative deviation \(\Delta e_p\) of the energy of the power pulses from the basic value is taken as the output signal. The main dynamic properties of a pulsed reactor can be determined if its amplitude-phase-frequency characteristics (APFCs), or Nyquist plots, are known. The APFC is a plot of the pulse frequency transfer function of a reactor \(W(j\omega) = U(\omega) + jV(\omega)\) on the complex plane. The transfer function \(W(j\omega)\) is given in the next equation:

\[
W(j\omega) = W_E(j\omega) (E^0 W_T^* (j\omega) - W_S^*)
\]  
(15)

where \(\omega\) is the dimensionless frequency; \(W_E(j\omega), W_T^* (j\omega)\) and \(W_S^*(j\omega)\) are given the following equations.

\[
W_E(j\omega) = \frac{1 - k_0}{1 - k_0 W_S^*(j\omega)},
\]

\[
W_S^* = \sum_{i=1}^{6} W_{si}(j\omega),
\]

\[
W_T^* = \sum_{j=1}^{3} W_{Tj}(j\omega).
\]

where \(W_{si}\) and \(W_{Tj}\) are defined in the equations (12) and (13), respectively.

To this characteristic there corresponds the pulse transfer function \(W(z)\), where \(z = \exp(j\omega)\). The oscillation frequency varies from 0 to 0.5 \(f_p\), where \(f_p = 1/T_p\) is the power

Fig. 4. Transient process caused by square wave reactivity \(r_p\) (1) at average power 0.5 MW(a) and 2 MW(b). \(\Delta e_p\) and \(\Delta e_p^0\) are recorded (2) and approximated (3) relative deviation of the pulse energy, respectively. \(n\) is number of pulses.

Fig. 5. Reactivity versus time at the output of linearized power feedback for a unit power pulse at the input with different values of average power. 1 – 0.5 MW, 2 – 1.0 MW, 3 – 1.5 MW, 4 – 2.0 MW.

Fig. 6. A linearised model of IBR-2M in the self-regulation regime, represented as a single-loop closed system.
pulse frequency, and the dimensionless frequency $\omega = 2\pi f_p T_p$ varies from 0 to $\pi$. The length of the vector from the origin of coordinates to the point with frequency $\omega (|W(j\omega)|)$ is the amplitude–frequency characteristic (gain), and the angle of rotation of the vector from the real positive semi axis $\varphi(\omega) = \arctan|V(\omega)|/U(\omega)$ is the phase–frequency characteristic of the reactor. For example, for the pulsed reactor APFC (this characteristic will be considered in more detail below, Fig. 7) the length of the vector drawn from the origin to any point on the curve corresponding to frequency $\omega$ is equal to the ratio between the amplitude of the sine power pulse energy oscillation in relative units of $\Delta E_p = (E_p - E_p^0)/E_p^0$ (output value) and the amplitude of the sine reactivity oscillation (input value) expressed in terms of parts of $\beta_p$. Here $E_p$ and $E_p^0$ are the energy of the current power pulse and its basic value. The angle of rotation of the vector with respect to the real positive semi axis is a phase shift between the power and reactivity oscillations.

Assessment of the reactor stability using the Nyquist stability criterion is done as follows. The reactor is represented as a closed one-loop system. While the open system is stable or neutral, the closed system is stable on condition that the APFC of the open system does not include the point with the coordinates $(-1, 0)$. The amplitude (gain) stability margin shows how many times this coefficient must be increased to bring the system from the stable state to the stability boundary $(-1, 0)$. The phase margin is defined as an angle of rotation of the unit-length vector at which its end turns out to be at the point with the coordinates $(-1, 0)$ and the APFC of the open system goes through this point, i.e., the closed system reaches the stability boundary. In the Fig. 7a using unit circle (dashed) to define phase margin at the different level of the average power. The phase margin reflects possible influence of the additional neglected inertia of the system.

V. CONCLUSIONS

The dynamics of the reactor power feedback is significantly influenced in the self-regulatory regime. Estimation of power feedback parameters is obtained by the mathematical treatment of recorded transients of power caused by square wave reactivity. Oscillations reactivity performs automatic regulator (AR) which output from the control loop and operating as the reactivity set point. Power feedback parameters are established for different levels of average power and calculated amplitude and phase frequency characteristics of the open-loop system, which corresponds to the reactor operation in the self-regulation regime.

![Fig. 7. Amplitude-phase frequency characteristic of the open part of the system $W(j\omega)$ in the complex plane (a). $V(\omega)$ is shown on the expanded $U(\omega)$-scale during 0.5 (b). Average power level is: 0.5 MW (1), 1.0 MW (2), 1.5 MW (3) and 2 MW (4)](image)

![Fig. 8. The depends of stability margin in amplitude (a) and phase (b) on the average power value of the IBR-2M reactor](image)
2M reactor. Pulse characteristics corresponding to the nominal power of 2 MW, demonstrates that the power feedback model is necessary to describe three exponents, i.e. presented in the form of three components.

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