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Calculation of the higher α-modes of the VENUS-F fast subcritical system

Máté Szieberth,* Péter German,* Lajos Nagy,* József Kóphází,† Danny Lathouwers‡

*Institute of Nuclear Techniques, Budapest University of Technology and Economics, H-1111 Budapest, Műegyetem rkp. 3., Hungary

†Nuclear Engineering Group, Department of Mechanical Engineering, City and Guilds Building, Imperial College London, Exhibition Road, South Kensington, London, SW7 2AZ, United Kingdom

‡Department of Radiation, Radionuclides and Reactors, Delft University of Technology, Mekelweg 15, 2629 JB Delft, The Netherlands

szieberth@reak.bme.hu

Abstract - *The paper presents α-modes calculations in a state-of-the art discrete ordinates transport code with 10 energy groups and finite element mesh for the VENUS-F fast subcritical assembly. The first 10 prompt α-modes have been calculated both in the forward and adjoint cases. Besides the analysis of the structure of the modes their detector contributions in a beam trip or pulsed neutron source experiment are also presented for different detector positions and detector deposits. Among the calculated modes the mode #9 has a dominant effect but the calculation of many more α-modes are needed for the description of the system.*

I. INTRODUCTION

A basic theoretical approach for the description of the temporal behavior of a source driven subcritical system is the α-modes expansion, i.e. the determination of the kinetic eigenvalues and the corresponding eigenfunctions. Dynamics of reactors close to criticality can be well approximated by the eigenvalue of the largest magnitude and the corresponding eigenfunction (the fundamental mode) but the description of a subcritical core calls for the higher order terms. The presence of the higher α-modes explains the spatial effects in different subcritical measurement methods, e.g. pulsed neutron source and neutron noise measurements. The theoretical description of these effects can help to improve the accuracy and reliability of the reactivity monitoring techniques, which have crucial importance in the operation of an Accelerator Driven System (ADS).

In recent years an extensive experimental campaign has been performed at the VENUS-F subcritical reactor core within the framework of the GUINEVERE [1] and FREYA [2] projects. Evaluation of these measurements can help the understanding of the neutronics behavior of subcritical systems and provide a validation basis for computer codes.

The development of transport codes capable of the computation of the higher kinetic of static eigenmodes started only in the last decade due to the high computational costs. Calculations were performed for PWR and BWR reactor cores in two-group diffusion approach [3, 4]. The applicability of the Implicitly Restarted Arnoldi Method (IRAM) to calculate either the λ- (static) [5] or the α-eigenfunctions (kinetic) [6, 7] in transport approximations was also demonstrated.

Efforts have already been taken for the modal analysis of the VENUS-F core including λ-modes calculation in a 4 group diffusion approach [8] and α-modes calculation in a state-of-the art discrete ordinates transport code with 10 energy groups and finite element mesh [9]. This paper presents the first results of the continuation of the latter work aiming at the improvement of the modeling based on the findings of the experimental campaign and by the application of an unstructured finite element mesh. Due to the computationally more demanding transport approximation only the first 10 modes have been determined, yet. Besides the analysis of the structure of the modes their detector contributions in a beam trip or pulsed neutron source experiment are also presented for different detector positions and detector deposits. Results can be applied to interpret measurement results affected by the higher modes.

II. THEORY

The Boltzmann transport equation for the neutron flux $\Phi(\boldsymbol{r}, \boldsymbol{v}, t)$ in a case of a system with an external source $S(\boldsymbol{r}, \boldsymbol{v}, t)$ can be written in the following form:

$$\left(\frac{1}{v}\frac{\partial}{\partial t} - \mathbf{L}\right)\Phi(\boldsymbol{r}, \boldsymbol{v}, t) = S(\boldsymbol{r}, \boldsymbol{v}, t) \tag{1}$$

where $\mathbf{L}$ is the transport operator defined as e.g. in [10].

The main difference between critical and subcritical systems is that in a critical case a time independent non-trivial solution can be found for the $S(\boldsymbol{r}, \boldsymbol{v}, t) \equiv 0$ case, i.e. the homogeneous form of (1). It is known (e.g. [10]) that this solution can be found by separating the time variable and seeking it in the following form:

$$\Phi(\boldsymbol{r}, \boldsymbol{v}, t) = \sum_{i=0}^{\infty} A_i e^{\alpha_i t}\phi_i(\boldsymbol{r}, \boldsymbol{v}) \tag{2}$$

where α_i and $\phi_i(\boldsymbol{r}, \boldsymbol{v})$ are the corresponding eigenvalues and eigenfunctions of the following equation obtained from the homogeneous form of (1) by the substitution of (2):

$$\frac{\alpha_i}{v}\phi_i(\boldsymbol{r}, \boldsymbol{v}) = \mathbf{L}\phi_i(\boldsymbol{r}, \boldsymbol{v}). \tag{3}$$

α_i are called the *kinetic eigenvalues*. This approach is called the α-modes expansion. Due to orders of magnitudes difference in the characteristic time constants prompt and delayed modes are often treated separately in practice. In the case of a fast system this separation is especially justified therefore in this paper we addresses only prompt modes, meaning that operator $\mathbf{L}$ in (1) contains only the prompt neutron production.

The solution presented in (2)-(3) also holds for the adjoint flux $\Phi^+(\boldsymbol{r},\boldsymbol{v},t)$ applying the adjoint source $S^+(\boldsymbol{r},\boldsymbol{v})$, the adjoint transport operator $\mathbf{L}^+$ and its $\phi_i^+(\boldsymbol{r},\boldsymbol{v})$ eigenfunctions. However, it is important to note that the α_i eigenvalues are the same for both the direct and the adjoint case. Based on the above, having obtained the eigenvalues and eigenfunctions, one can calculate the detector response for various measurements and interpret its spatial variations.

In a subcritical system a source must be present to obtain a constant solution and the amplitudes A_i can be derived from the initial flux distribution $\Phi(\boldsymbol{r},\boldsymbol{v},0)$ and the source $S(\boldsymbol{r},\boldsymbol{v},t)$. Assuming a constant source $S(\boldsymbol{r},\boldsymbol{v})$ present in a subcritical system since the remote past, one has to expand it also according to the $\phi_i(\boldsymbol{r},\boldsymbol{v})$ eigenfunctions and the time independent solution $\Phi(\boldsymbol{r},\boldsymbol{v})$ can be found in the following form:

$$\Phi(\boldsymbol{r},\boldsymbol{v}) = \sum_{i=0}^{\infty} \frac{1}{(-\alpha_i)} \frac{(\phi_i^+, S)}{(\frac{1}{v}\phi_i^+, \phi_i)} \phi_i(\boldsymbol{r},\boldsymbol{v}). \quad (4)$$

In case of beam trip measurements the continuous neutron source is interrupted for a short time an the time evolution of the neutron flux is observed in order to determine the kinetic parameters of the core. Assuming (4) as initial conditions at $t = 0$, based on (2) one can obtain the following solution for this case:

$$\Phi(\boldsymbol{r},\boldsymbol{v},t) = \sum_{i=0}^{\infty} \frac{1}{(-\alpha_i)} \frac{(\phi_i^+, S)}{(\frac{1}{v}\phi_i^+, \phi_i)} e^{\alpha_i t} \phi_i(\boldsymbol{r},\boldsymbol{v}). \quad (5)$$

In a measurement with a pulsed neutron source (PNS) in a fast system one may assume that the contribution of the prompt modes completely decays between two pulses, while the delayed neutron term produces a constant background. With a square shaped pulse of t_p width the time behaviour of the flux from the prompt modes after the pulse can be described as:

$$\Phi(\boldsymbol{r},\boldsymbol{v},t) = \sum_{i=0}^{\infty} \frac{1}{(-\alpha_i)} \frac{(\phi_i^+, S)}{(\frac{1}{v}\phi_i^+, \phi_i)} \left(1 - e^{\alpha_i t_p}\right) e^{\alpha_i t} \phi_i(\boldsymbol{r},\boldsymbol{v}). \quad (6)$$

As in practice t_p is expected to be not long enough to saturate the fundamental mode, one may conclude that in the PNS measurement a higher amplitude of the higher modes is expected compared to the beam trip measurements.

Numerical solution of the eigenvalue equation (3) can be obtained with Krylov subspace methods as the Implicitly Restarted Arnoldi Method (IRAM)[11]. This method has been implemented in the freely available ARPACK package[12]. For the solution with the ARPACK routines the eigenvalue problem in (3) has to rewritten into a standard form:

$$\frac{1}{v}\mathbf{L}^{-1}\phi_i(\boldsymbol{r},\boldsymbol{v}) = \frac{1}{\alpha_i}\phi_i(\boldsymbol{r},\boldsymbol{v}), \quad (7)$$

where the numerical representation of the operator $\frac{1}{v}\mathbf{L}^{-1}$ is the matrix the eigenvalues of which will provide the reciprocals of α_i. In order to solve a standard eigenvalue problem using ARPACK, the user is required to supply a routine to apply the given matrix to a vector $\boldsymbol{x}$. This operation can be interpreted as the solution of the following fixed source problem for the flux ϕ:

$$\mathbf{L}\phi(\boldsymbol{r},\boldsymbol{v}) - \frac{1}{v}x(\boldsymbol{r},\boldsymbol{v}) = 0. \quad (8)$$

The consequence of having to solve a fixed source problem is that the method is applicable only to subcritical systems. The number of fixed source problems to be solved depends on the number of requested eigenvalues n and the dimension of the subspace d (it is recommended that $d \approx 3n$). Besides the large computation time the number of calculated eigenvalues is also limited by the memory required for the basis vectors of the subspace.

III. THE VENUS-F SUBCRITICAL SYSTEM

Originally, the VENUS reactor was a zero power thermal critical assembly at SCK•CEN and was mainly used for neutronics code validation and experimental neutronics studies. In 2006 the GUINEVERE project [13, 14] was initiated aiming to construct a zero power experimental facility to investigate the feasibility of accelerator driven systems. In the framework of the project, the thermal reactor was modified to a fast spectrum lead containing system and the core was coupled to the GENEPI-3C neutron generator [15] resulting in the VENUS-F accelerator driven system [16]. In 2011 the FREYA project [17] has started in order to provide experimental support for the development of ADSs and lead-cooled fast reactors.

The reactor takes place in a cylindrical vessel of approximately 80 cm in radius and 140 cm in height. The central part of the system is a 12×12 grid of square assemblies with 8 cm side length, constituting the reactor core of 60.96 cm in height, the 40 cm thick top lead reflector as well as the safety and control elements. The grid is located in an approximately 3 cm thick stainless steel core container box which is surrounded by the cylindrical radial reflector of 77.5 cm in radius and by the 40 cm thick bottom reflector. The whole set-up is placed onto a steel supporting structure. Several types of assemblies were used during the project, among which the most relevant are: normal and experimental fuel assembly, normal and experimental lead assembly, safety rod, control rod and pellet absorber rod.

The accelerator can be coupled to the core by removing the four central fuel assemblies and inserting a vertical beam tube containing the target. The neutron source is provided by ${}^3_1T(d,n){}^4_2He$ fusion reactions: the device accelerates deuteron ions to energy of 220 keV and guides them on a tritiated target, located at the core mid-plane, providing a quasi-isotropic field of neutrons with approximately 14 MeV energy.

IV. METHODS AND MODELLING

The eigenmode-calculations were performed using the PHANTOM transport code [9] developed at the Delft University of Technology. The implementation of the code is based on the three-dimensional multigroup S_N-equations spatially discretized with the discontinuous finite element method [18]. Among its capabilities, the code can solve the (forward or

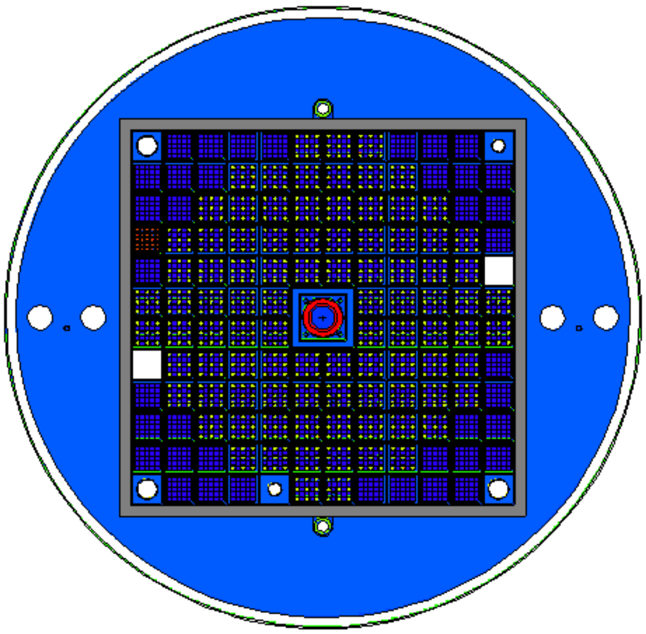

Fig. 1. Horizontal cross-section of the VENUS-F subcritical core SC1 at the core midplane.

adjoint) multi-mode λ and α-eigenvalue problems using the implicitly restarted Arnoldi method.

The geometrical model of the reactor was created based on the validated VENUS-F MCNP model [17] (see in Fig. 1) provided by SCK•CEN, using the GMSH program [19], which is a general finite element mesh generator tool able to produce a MSH-ASCII-formatted geometry description. A two-dimensional finite element mesh has been created in the horizontal plane (see in Fig. 2) and extruded along the vertical axis. In the reactor core region and in the steel container enclosing it a structured hexahedral mesh has been applied, while in the reflector region outside the steel container an unstructured mesh has been created, which reduces the required number of elements.

Starting from the detailed material description of the MCNP model, a 10 group cross-section library (see group structure in Table I) representing the PHANTOM material model has been created with the modules of the SCALE 6.1 code system [20] using the ENDF/B-VII (point-wise continuous and corresponding 238-group master) library [21]. Homogenized regions can be observed in Fig. 3. S_4 quadrature was used for the angular discretization.

V. RESULTS AND ANALYSIS

Applying the above described tools and methodology, calculations have been performed for two VENUS-F core configurations: the critical CR0, which served as a validation basis, and the subcritical SC1, for which the α-modes have also been calculated.

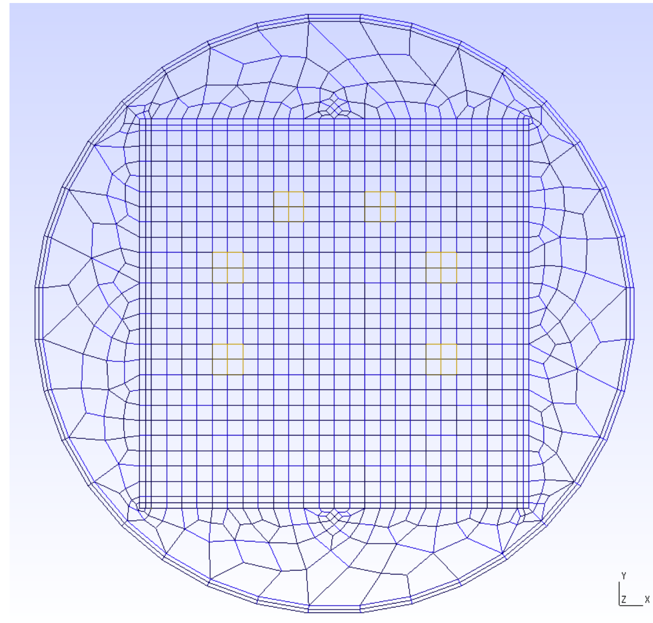

Fig. 2. Top view of the finite element mesh of the VENUS-F core.

group	upper boundary	
	energy (eV)	lethargy (-)
1	2.0000×10^{7}	-6.9314×10^{-1}
2	6.4340×10^{6}	4.4098×10^{-1}
3	1.3560×10^{6}	1.9980×10^{0}
4	4.9952×10^{5}	2.9966×10^{0}
5	7.3000×10^{4}	4.9198×10^{0}
6	9.5000×10^{3}	6.9590×10^{0}
7	2.2000×10^{3}	8.4218×10^{0}
8	3.0500×10^{2}	1.0397×10^{1}
9	6.7500×10^{1}	1.1906×10^{1}
10	4.0000×10^{0}	1.4731×10^{1}
11	1.0000×10^{-5}	2.7631×10^{1}

TABLE I. The group structure of the 10-group library applied in the material model of VENUS-F reactor.

1. Validation of the model

In order to validate the models, criticality calculations have been performed for the critical CR0 and the subcritical SC1 cores. The obtained k_{eff} values can be found in Table II and show good agreement with the measurements especially in the case of the subcritical core, which has major importance for the accurate calculation of the α-modes. A mesh sensitivity study has also been performed for the SC1 core. As it can be seen in Fig. 4 the doubling of the number of mesh elements can produce a further 100 pcm improvement. However, considering the computational time and the required memory the mesh with 23240 elements were chosen. The calculated spectral indexes show slightly softer spectrum compared to the measurements[22]. Radial and axial flux profiles have also

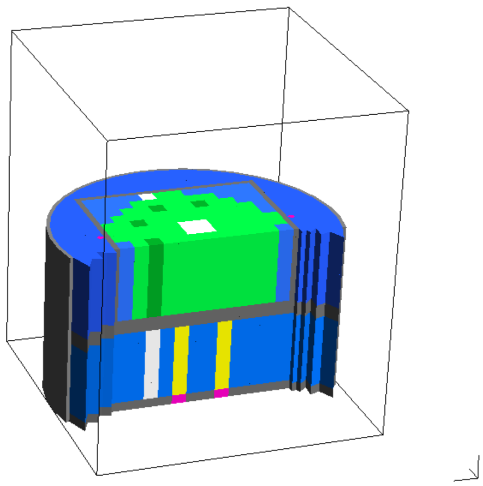

Fig. 3. Homogenized regions of the VENUS-F model with horizontal and vertical cut.

been calculated and compared with the core characterization measurements with satisfactory agreements (see in Figs. 5-6). The asymmetry in the axial ^{235}U distribution, which is due to the moderating effect of the polyethylene parts of the start-up chambers under the core (represented by the yellow region in Fig. 3) is well reflected by the calculations. Discrepancies can be found in the reflector peak, where the ^{235}U fission rates suggests higher thermal neutron flux in the measurements. It has to be noted that considerable moderating effect of the concrete wall of the bunker surrounding the VENUS-F facility was discovered during the measurement campaign. For this reason the provided MCNP model contained also a simplified model of the bunker, but it was omitted from the PHANTOM model, since the inclusion of the large air and concrete volume around the facility would have extensively increased the number of the mesh elements.

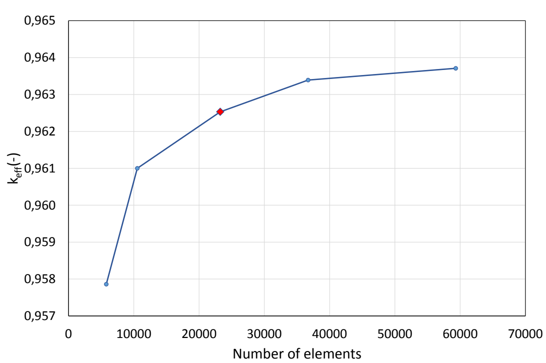

Fig. 4. Mesh sensitivity study for the SC1 core. The mesh denoted with the red marker was applied for the α-mode calculations.

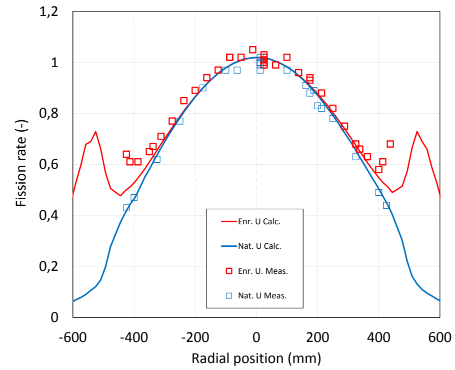

Fig. 5. Comparison of measured and calculated radial fission reaction rate distributions.

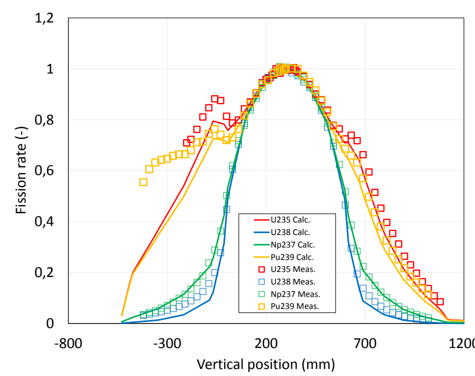

Fig. 6. Comparison of measured and calculated axial fission reaction rate distributions in the E1 position.

The comparison with the measurement results confirmed that the models produce reasonable agreement considering also the fact that further refinement of the mesh and the group structure was not feasible due to the large computational effort necessary for the eigenvalue calculations.

Configuration	Measured	Calculated
CR0	1.000	1.0085
SC1	0.964	0.9625

TABLE II. Measured and calculated k_{eff} values for VENUS-F core configurations.

2. Calculated α-modes

The eigenvalue calculations were performed for the subcritical SC1 core, which contains a deuteron beam guide and a tritium target as neutron source approximately in the middle of the core (see in Fig. 1). Prompt α-modes have been calculated both in the forward and adjoint cases neglecting the delayed neutron production. Available computing resources limited the calculations to the determination of the 10 largest magnitude eigenvalue (see in Table III) since both the CPU time and the memory usage increase with the number of requested eigenvalues. The eigenvalues from the forward and adjoint calculations agrees within the set tolerances, which is the theoretical expectation. The largest eigenvalue (the prompt decay constant) is in the expected range, while the value of -6651.03 s^{-1} for the 10th eigenvalue suggests that even more modes needs to be calculated since in the measurements much more quickly decaying mode can also be observed.

#	Eigenvalue [s^{-1}]
1	-3233.41
2	-4008.28
3	-4026.65
4	-5006.59
5	-5101.97
6	-5628.16
7	-6271.80
8	-6283.40
9	-6520.64
10	-6651.03

TABLE III. Calculated first 10 α eigenvalues of the SC1 core.

In the analysis of the eigenfunctions the most important question is the expected contribution of a given mode to the detector response, which can be estimated based on (4) as the product of the adjoint mode at the source position and energy group and the forward mode at the detector position. In order to illustrate this, some examples of the forward and adjoint eigenfunctions at the reactor mid-plane can be seen in Fig. 8 and Fig. 7. Adjoint eigenfunctions are shown in the first energy group (6.43 - 20 MeV), which is the source group, while forward eigenfunctions in the 4th group (73 keV - 499.5 keV) and the 10th group (thermal) which are expected to contribute the most to the reaction rates. As expected, most higher modes (mode #2-5 and #7-8) are antisymmetric and have a zero crossing around the symmetry axis passing through the source position, which cancels out their contribution. Adjoint modes #6 and #9, however, are symmetric and have a local maximum at the source position in the 1st group, which suggest that they are amplified by the source and a significant contribution can be expected. #10 is highly asymmetric and has also a significant value at the source position, however it is expected to have a mirrored pair in mode #11 with a similar contribution of the opposite sign.

In order to qualitatively evaluate the eigenmodes, the coefficients according to (4) have been calculated applying an isotropic point source in the middle of the core in the 1st energy group. Detector responses were calculated assuming four different representative positions as shown in Fig. 9 and three kinds of deposits (^{235}U,^{238}U and ^{237}Np). In line with the expectations the contribution of the antisymmetric modes described above were all negligible, therefore only the coefficients for modes #1, #6, #9 and #10 are listed in Tab. IV in a way that the sum of them are normalized to 1 for each detector. It can be observed that surprisingly mode #9 dominates all detector responses with at least one order of magnitude in all cases, which means that the actual fundamental mode would be practically hidden in a measurement. This suggests that the calculated 10 modes are not enough to explain the spatial variation observed in the measurements as it was also found in [8].

Deposit	A_1	A_6	A_9	A_{10}
D-1				
^{235}U	2.79×10^{-3}	3.04×10^{-4}	0.938	5.87×10^{-2}
^{238}U	2.13×10^{-2}	3.83×10^{-3}	0.944	3.05×10^{-2}
^{237}Np	1.93×10^{-2}	3.50×10^{-3}	0.944	3.33×10^{-2}
D-2				
^{235}U	6.73×10^{-3}	4.06×10^{-4}	1.04	-5.21×10^{-2}
^{238}U	2.14×10^{-2}	3.83×10^{-3}	0.973	1.94×10^{-3}
^{237}Np	2.10×10^{-2}	3.78×10^{-3}	0.974	8.09×10^{-4}
D-3				
^{235}U	7.97×10^{-3}	1.51×10^{-3}	1.07	-8.03×10^{-2}
^{238}U	1.94×10^{-2}	3.67×10^{-3}	0.996	-1.89×10^{-2}
^{237}Np	2.03×10^{-2}	3.85×10^{-3}	0.990	-1.41×10^{-2}
D-4				
^{235}U	2.42×10^{-2}	4.80×10^{-3}	0.960	1.14×10^{-2}
^{238}U	2.41×10^{-2}	4.79×10^{-3}	0.960	1.10×10^{-2}
^{237}Np	2.41×10^{-2}	4.79×10^{-3}	0.960	1.13×10^{-2}

TABLE IV. A_i amplitudes of the modes for different detector positions and fissile deposits. Sums of the amplitudes are normalized to 1.

Concerning the spectral distribution of the detector response one can observe that for ^{238}U and the ^{237}Np fission rates the most important contribution is in the 2nd and 3rd group, respectively, practically in all positions and modes. For the fissile ^{235}U the fundamental mode and mode #6 have the highest contribution in the 4th group, as their thermal group is maximal in the reflector region under the core due to the polyethylene parts below and the withdrawn safety rods above the core. On the other hand modes #9 and #10 have their maxima in the thermal flux around the core, where the detectors are located. Therefore in positions D-1-3 the ^{235}U fission rate is determined by the 10th group in modes #9 and #10. Position D-4 very close to the center of the core produces very similar response for all the deposits as even the ^{235}U fission is determined by the 4th group.

VI. CONCLUSIONS

The paper presents calculations to determine the α-modes in the VENUS-F fast subcritical facility by a finite element transport code. Criticality calculations have been performed first for the created models, which showed satisfactory agreement with the results of the core characterization measurements. The first 10 prompt α-modes have been calculated for

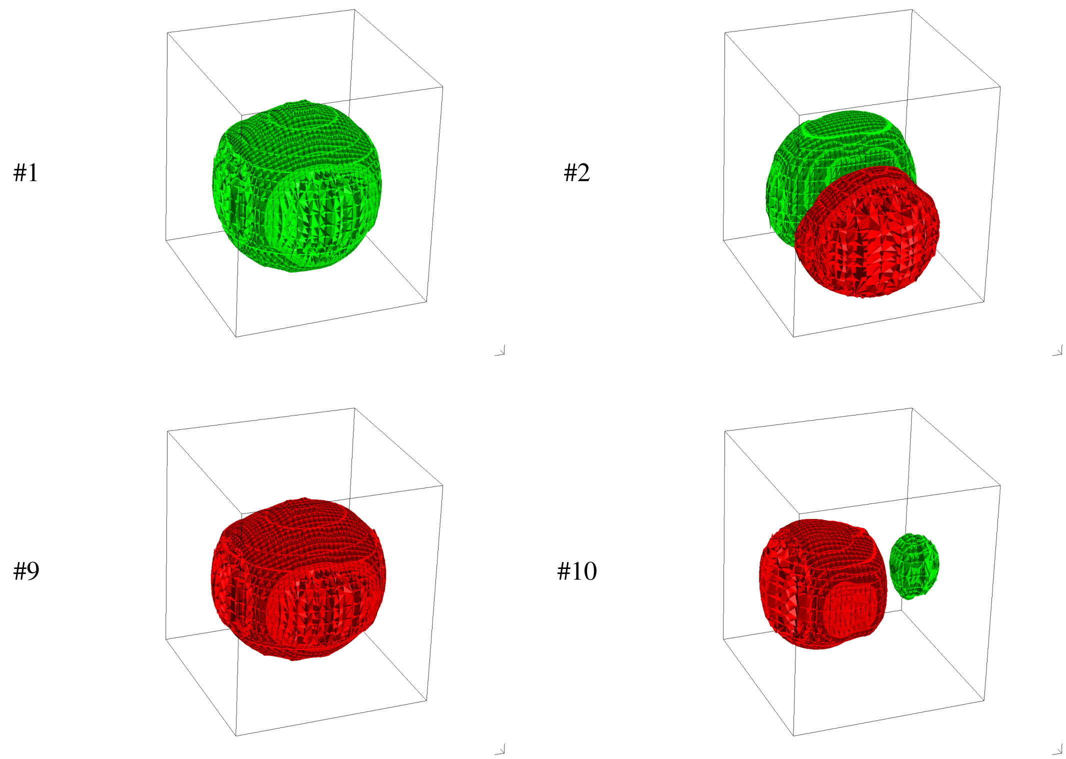

Fig. 7. Adjoint modes in the 1st group. Green surface denotes the 15% of the maximum, while red surface is at -15%.

the subcritical core SC1 and detector responses for different positions and deposits. It has been observed that modes #1, #6, #9 and #10 contribute significantly to the detector responses and #9 dominates them in all positions and for all deposits. Future work aims at the extension of the calculated set of eigenfunctions in order to obtain a more complete description of the system and make possible the comparison with measurements.

VII. ACKNOWLEDGMENTS

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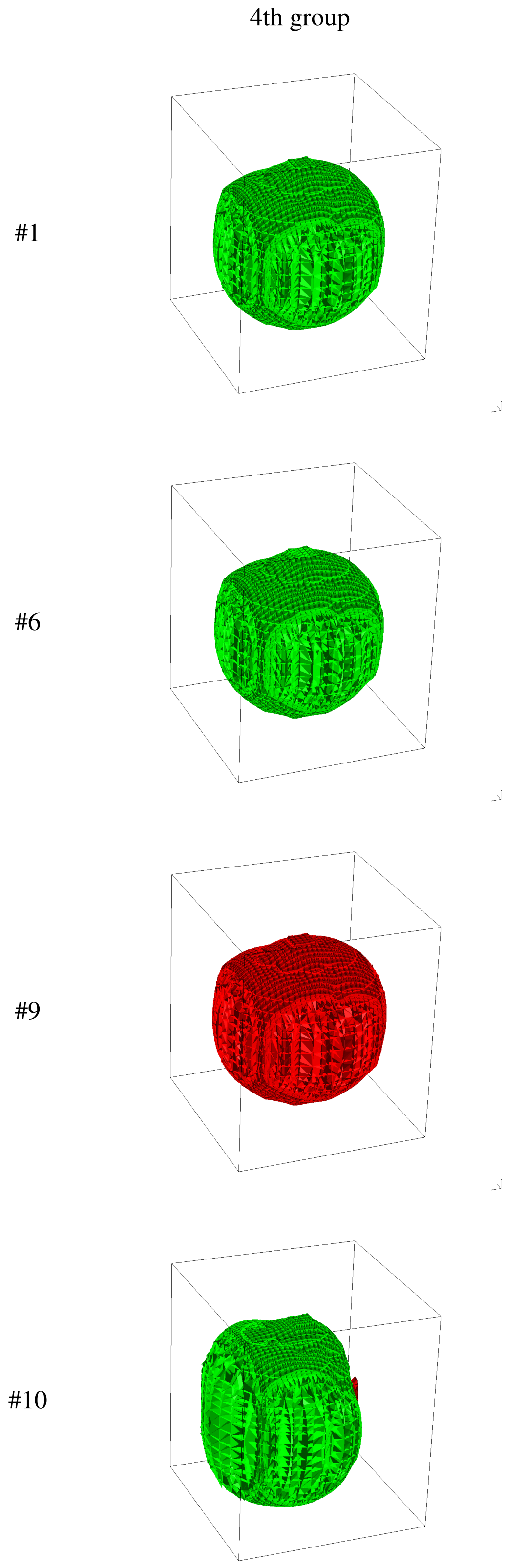

Fig. 8. Forward modes. Green surface denotes the 15% of the maximum, while red surface is at -15%. (Except the 4th group of mode #10, where it is ± 6.5%.)

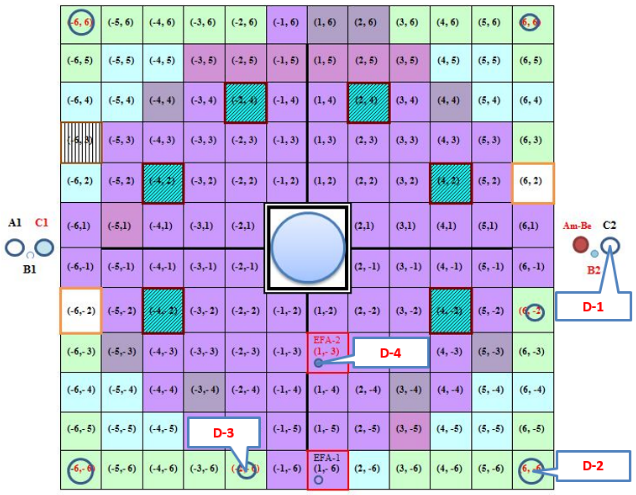

Fig. 9. Detector positions where contributions of the eigenmodes were calculated.

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