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Estimation of the External Source Contribution to a Driven Subcritical Reactor

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Abstract - *Three cases of the driven subcritical reactors are analyzed for their neutron amplification and TRU depletion characteristics. Neutrons coming from ADSR, a fusion source, and a fast reactor (FR) are used to represent the corresponding driving sources. A modified MCNPX 2.6.0 in fixed source configuration is used to analyze neutron multiplication and to compare the contribution of the driving sources in depletion of the selected TRU isotopes. For a given subcritical core (keff=0.97731) neutron multiplication from these driving sources is about 53, 32 and 31, respectively. So, corresponding source strength required from ADSR is much less than its counterparts. Burning rate (expressed in grams per ton of initial TRU per MWD/kg of Burnup) of most of the isotopes in a driven system weakly depends on the driving source. Fissile isotopes (like Pu-239) burn more rapidly in a FR driven system (733 in FR driven system and about 720 in each of the other two). Production of some elements (like Curium) also depends on the driving source. It is concluded that the difference of driving source strengths, required to produce same power from a subcritical core is not proportionate to burning rate of the TRU isotopes.*

I. INTRODUCTION

Modeling of the driving source in a subcritical reactor is analyzed, and the source effect is approximately incorporated into the Monte Carlo code MCNPX 2.6.0. The external source (i.e. driving source) is modeled by analyzing equivalence of the source neutrons with fission neutrons by introducing an embedded fission source (EFS) approximation. The performance of the two most promising candidate driving sources for subcritical reactors is compared with a critical fast reactor.

Currently, literature gives scattered and partial insight into driving sources of the driven systems, because different parameters have been used to define the sub-criticality level and also different definitions of the multiplication factor have been proposed. Gandini et al recommended utilization of the perturbation theory to address inhomogeneous flux distribution and importance of neutrons due to their differing source of origin [1], [2]. Salvatores extended the g* factor concept introduced by Spriggs et al [3] and quantitatively analyzed the transmutation of fission products (FP) by using D parameter (net number of neutrons emitted per fission), and transmutation of the transuranic (TRU) isotopes by using α parameter (ratio of average fission to capture cross section) and termed the technique as neutron economy concept [4]. Kobayashi et al even separated effect of the fission neutrons and the driving source neutrons [5]. They defined two multiplication factors and estimated combined effect in the core using Green's Function method. Similarly, other attempts are also made to quantify sub-criticality level of the core and its transmutation capability [6], [7]. However, up to now, there is no rigorous treatment to clearly show the perils and merits of using different driving sources.

This study concerns with three candidate reactors; (1) a typical fast reactor (FR) (2) accelerator-driven subcritical reactor (ADSR) and a fusion-fission hybrid reactor (FF-Hyb). We make use of the importance function concept introduced by Gandini et al [2] to calculate core multiplication level of a driven system. Then we introduce embedded fission source (EFS) approximation for the driving source and compare amounts of different isotopes burnt or produced in equal time duration. The study clearly shows the benefit of using softer neutron spectrum (FR case) to burn fissile isotopes. FF-Hyb is a better candidate to burn more important TRU isotopes like Am-243. ADSR is proven to be capable of producing an equal amount of power as the other two, with least source strength required.

MCNPX 2.6.0 is used to calculate core multiplication with an external source, M, and hence multiplication factor. The only difference among the selected sources is their ability to change neutron flux level and also they provide neutrons of energy spectrum different from Watt fission spectrum. Both these effects are approximated by replacing fission neutrons from a thin layer of the core with the driving source neutrons. This embedded fission source approximation is the key to use MCNPX 2.6.0 with its existing CINDER90 and MCNP coupling scheme [8].

Calculated isotope masses are compared with each other instead of some experimental data because, to date, no such reference driven system exists. Many other important and decisive factors like feasibility of manufacturing, economics etc. are not discussed.

Most of the notations and parameters used in this manuscript have standard meanings. For instance, k_{eff} represents eigenvalue of the neutron balance equation without any driving source, ν the mean number of neutrons produced in a fission reaction etc. The less common notations are explained below.

A. Neutron Multiplication M

M is defined as the ratio of the total neutrons observed in the system per source neutron. Obviously, for a subcritical core with k_{eff} close to unity the core multiplication parameter M is very high and conversely, if there was no fissile material in the core then M value approaches unity. If F and S denoted total number of neutrons available from fission reactions and from the driving source respectively, then M is calculated from equation (1).

$$M = \frac{F+S}{S} \tag{1}$$

B. Subcritical Multiplication factor k_s

The subcritical multiplication factor k_s is defined as a ratio of the fission neutrons (F) to the total neutrons (F+S), where S is a contribution from the driving neutron source in that generation. It is related to the core multiplication M by equation (2).

$$k_s = \frac{F}{F+S} = 1 - \frac{1}{M} \tag{2}$$

C. Equivalent Source Parameter

The equivalent fundamental mode source proposed by Spriggs et al [3] is a fictitious source that is identical to the fundamental mode fission source. True core multiplication with a fixed source is obtained for any arbitrary source from the parameter g*, define by:

$$g^* = \frac{1-k_{eff}}{k_s} \tag{3}$$

D. Source Neutron Importance φ^*

The source neutron importance was initially introduced by Salvatores et al as a ratio of the expectation value of the perturbed (i.e. driven) subcritical reactor operator to its non-driven counterpart. They concluded that it is related to the multiplication factors by equation (4).

$$\varphi^* = \frac{\dfrac{1}{k_{eff}} - 1}{\dfrac{1}{k_s} - 1} \tag{4}$$

II. REACTOR MODEL

The ideal reactor model used in this study is shown in Fig. 1. The coaxial cylinders making region no. 1 and region no. 2 house fuel and HT-9 reflector respectively. The extent of the region no. 1 ~~extends~~ ranges from 10 cm to 27 cm radially while that of region no. 2 reaches up to 47 cm (Fig. 1). This simple reactor model is similar to Vologymyr's model [9], though with different dimensions and TRU composition.

Reflecting boundary conditions are used for the top and bottom surfaces to effectively make the core infinite axially, eliminating any neutron reflections from these boundaries. Purpose of using such an idealistic reactor is not to design some specific reactor but to compare the external source effect under identical conditions. This is also the reason for using 10 cm radius cylinder (region no. 3) for fusion neutron source. In real situations it will obviously be much larger than this figure. In real world, size of one driving source is never identical to the other. Same is true for fuel layout and core arrangements. For instance fuel need to be placed very close to the ADSR beam while it is more reasonable to put fissile or fertile materials containing 'fission fuel' in the blanket shape i.e. around the periphery of the Fusion system. In other words, we are assuming that there is some way to guide source neutrons into the subcritical core region.

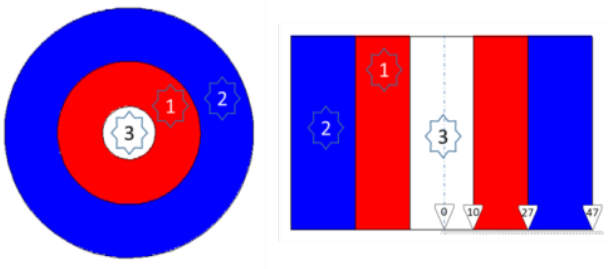

Fig. 1. Two cross sectional views of the ideal reactor model

Fuel content is chosen following the work of Tariq and Kim [10]. This chosen TRU content of 6 % (by volume) gives initial k_{eff} ~ 0.97731. The remaining 94 % of the core region is constituted by everything besides fuel i.e. coolant, spacers, cladding materials etc. This fuel region is enclosed by a cylindrical shell of HT-9 steel to serve as a reflector material.

External source surface (boundary of the region no. 1 and 3) is a regular surface with regular passage of neutrons allowed in all directions. Source region, region no. 3, is modeled as vacuum and every neutron entering this region re-enters the TRU at some other point, so the presence of this region is meaningless to neutron interactions. The outer most surface of the reflector is also a regular surface with leakage permitted normally. Outside of the reflector is vacuum region, and history of every neutron entering this region is terminated.

The composition of the HT-9 steel and the TRU region was studied previously [10], [11]. The choice of Plutonium content in the TRU core region depends on many factors. If the purpose of the reactor was to burn Pu then Artioli et al [12] suggested a cap on the minor actinide (MA) content i.e. (MA/Pu ratio to be less than 1.2) and from the external source point of view, they suggested using Pu enrichment close to 42%. Resultantly there was negligible variation in the k_{eff} over entire cycle length and external source would be running at constant current. Such a "non-natural" Pu content needs extraction of Pu from the spent nuclear fuel which is not an option for many countries. Currently, we are using candidate fuel coming from DUPIC cycle and consisting of all the non-gaseous elements in the used nuclear fuel besides Uranium. The assumption that Uranium has been extracted is desired because most of the subcritical reactors are intended to serve as waste burning facilities, which is best served by a uranium-free fuel.

III. EXTERNAL SOURCE MODELING

The external source contribution was concerned both in its location and fraction. Core multiplication level M and the subcritical multiplication factor (k_s) are assessed by running original MCNPX 2.6.0 in fixed source mode with fission source only i.e. without a driving source. Contribution from the external source is then directly proportional to the difference from critical state i.e. ($1\text{-}k_s$), as given in Fig 2 below.

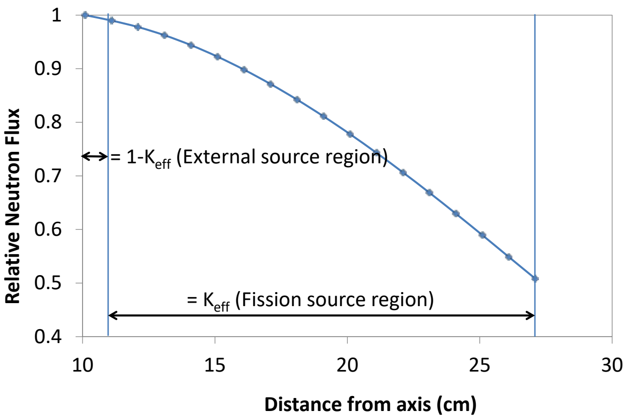

Fig. 2. Assessment of thickness of the TRU region serving as external source

The Ratio of neutrons from an external source to the fission source is determined from their respective neutron importance. That is a source providing S neutrons per second is effectively contributing $S \times M$ fission neutrons to the system. If ν and ϵ_f are the average numbers of fission neutrons and energy released per fission reaction then the total fission power being produced is given by

$$P = \frac{S \times M}{\upsilon} \times \epsilon_f = \frac{S \times k_s}{\upsilon(1-k_s)} \times \epsilon_f \qquad (5)$$

Physically, neutron importance, φ^* gives the equivalence of a source neutron to the fission neutrons. That is one source neutron effectively contributes as much power to the system as φ^* fission neutrons. So, the prevalent neutron spectrum in the system is given by equation (6) and plotted in Fig. 3.

$$\chi(\mathrm{E}) = \{S\}\chi_{ext}(\mathrm{E}) + \{\mathrm{S} \times \mathrm{M} - \varphi^* \times S\}\chi_{fission}(\mathrm{E}) \qquad (6a)$$

$$\chi(\mathrm{E}) = \{S\}\chi_{ext}(\mathrm{E}) + \left\{\frac{S \times k_s}{(1-k_s)} - \varphi^* \times S\right\}\chi_{fission}(\mathrm{E}) \quad (6b)$$

Equation (6) also gives the relative neutron population from the two sources. That is the probability of each source to contribute a neutron in MCNPX is given by:

$$\frac{External\ Source}{Fission\ Source} = \frac{1}{\dfrac{k_s}{(1-k_s)} - \varphi^*} \qquad (7)$$

A very similar variation of the two parameters, M and φ^* is expected because both are related by the equation (8). M, being much larger than unity, translates into the direct proportionality between the neutron importance ϕ^* and the multiplication M.

$$\phi^* = \frac{\dfrac{1}{k_{eff}} - 1}{\dfrac{1}{k_s} - 1} = \left(\frac{1}{k_{eff}} - 1\right)(M-1) \qquad (8)$$

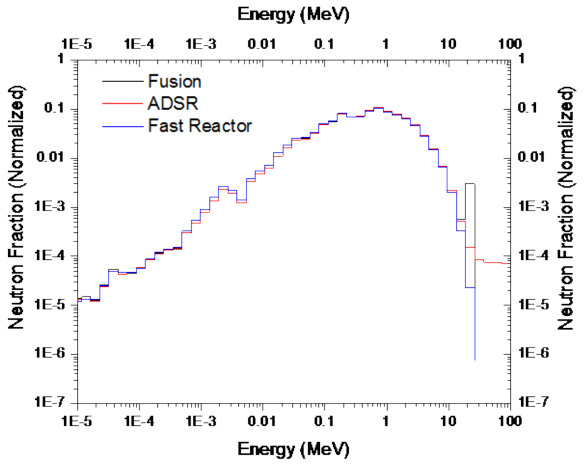

Fig. 3. Core averaged neutron spectra with external sources

The tabulated values of the subcritical multiplication factor k_s (Table I) are directly proportional to the source strength required to run the three driven reactors at equal power. However, if the driving sources have equal strengths (neutrons/second) then ADSR driven system would just be producing more power and burning TRU at a faster rate.

Table I. Comparison of the source dependent parameters for a subcritical driven reactor with k_{eff} =0.97731.

Parameter	**ADSR**	**FUSION**	**FR**
Multiplication (M)	52.827	32.232	30.695
Subcritical multiplication factor (k_s)	0.98753	0.97908	0.97803
Equivalent Source Parameter (g^*)	0.026025	0.026249	0.026277
Source Neutron Importance (φ^*)	1.393466	0.823835	0.783292

IV. DEPLETION OF SELECTED ISOTOPES

To get maximum benefit from the driving source while remaining within safe limits and, producing maximum power as well as burning TRU, the more widely proposed case of k_{eff} ~ 0.98 (=0.97731, to be more precise) is used. This choice rendered to a limited contribution (~2%) from the external source. The burning rate, defined by equation (9), is chosen as the figure of merit for comparison.

$$Burning\ Rate = \frac{Mass\ at\ BOC - Mass\ at\ EOC}{TRU\ at\ BOC \times Burnup} \qquad (9)$$

1. External Source Effect on Pu Burning

Fig. 4 shows that a fast reactor is more effective in burning Pu-239 than the other two, namely ADSR and fusion driven systems. Thanks to high fission cross section of Pu-239 at lower energies.

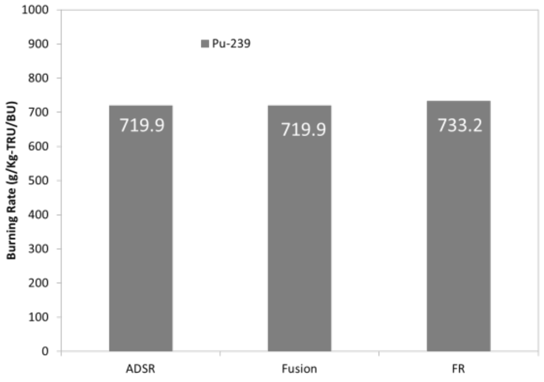

Fig. 4. Burning rate of Pu-239 (grams of Pu-239 per ton of the total TRU per burnup of 1.0 MWD/kg-TRU)

Fig. 5 shows that besides the difference of the source spectra, total Plutonium burning rate (Pu-239 included) is not very different. The increased burning rate of Pu-239 in FR spectrum is counter-balanced by the production of other Plutonium isotopes. This is important because all the isotopes of Pu are extracted together in reprocessing and other spent nuclear fuel separation techniques.

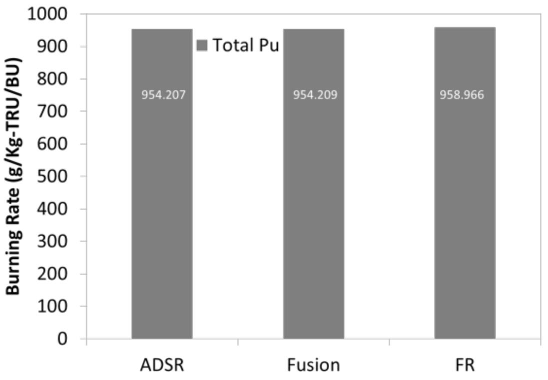

Fig. 5. Burning rate of all isotopes of Pu (grams of Pu per ton of the total TRU per burnup of 1.0 MWD/kg-TRU)

2. External Source Effect on Curium Isotopes

Although Curium is one of the most feared actinide products of the thermal reactors, but its production is not limited to the legacy reactors only. In the current study, it is observed that irrespective of the driving source Curium isotopes are always produced. However, their production rate (or their net amount at EOC) depends on the driving source. Fast reactor shows the highest Curium production rate (54.7 g/MWD)

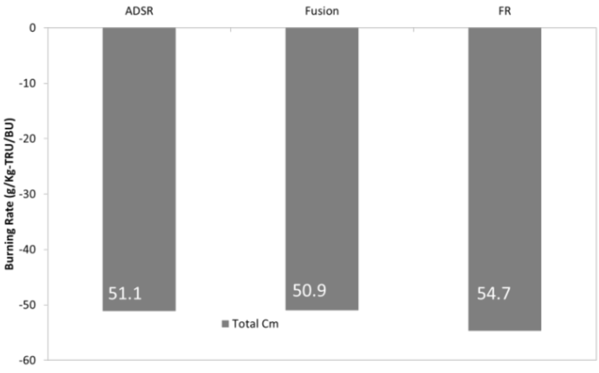

Fig. 6. Burning rate of all isotopes of Curium (grams of Cm per ton of the total TRU up to burnup of 1 MWD/kg-TRU)

Fusion and Spallation sources, both being source of harder neutrons produce bit less Curium than the Fast reactor source under identical conditions. The difference between these two options (FF-Hyb and AFSR) is also very small (0.2

g/MWD). The difference between the best (FF-Hyb) and the worst case (FR) is less than 10%. So, production of the Curium only could not be taken as a decisive metric to prefer one driving source over the other.

V. CONCLUSIONS

An approximate technique to include the driving source into the analysis of subcritical driven systems is proposed and then utilized to compare the two most widely researched subcritical reactor options (ADSR and FF-Hyb). Their TRU burning characteristics are compared with a lead-cooled fast reactor.

In a driven system, change of neutron flux and energy spectrum caused by the driving source has a direct effect on the core multiplication M, which in turn depends on Eta value, absorption/fission/etc. cross sections and similar other parameters. Net neutron multiplication in a subcritical core, caused by a certain driving source is found to be proportional to the driving source importance and hence, the true representation of its value. This corollary is found useful to analyze the driven system.

Using ability of a driving source to change the prevalent neutron characteristics in the core by changing net multiplication, M, and modeling this change via embedded fission source approximation, we compared the amount of the most important TRU isotopes burned or produced in the core. It is found that all the parameters studied are inclined in the favor of the harder spectrum source i.e. fusion source. That is, overall larger quantities of the TRU isotopes are burned in a core driven by Fusion neutrons. Similarly, for the isotopes being produced, smaller quantities are produced with the same driving source. The difference, however, is not considerable enough to dump the option of TRU burning in a fast reactor altogether.

This analysis of the driving sources is limited to use of these sources under identical conditions. A driving source could be selected due to mechanical differences or due to existing infrastructures.

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