# SUMMON: A Sensitivity And Uncertainty Methodology For MONte Carlo Codes

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**Abstract** - Accurate and reliable tools for sensitivity analysis and uncertainty quantification are needed to estimate the uncertainties in reactor key parameters ( $k_{eff}$ ,  $\beta_{eff}$ , safety coefficients, ...) and to identify possible nuclear data weaknesses. Sensitivity and Uncertainty Methodology for MONte carlo codes has been developed to address the limitations of existing tools and to calculate the sensitivities of the criticality safety parameters and their uncertainties due to uncertainties in nuclear data. The methodology is currently based on the use of the KSEN card of MCNP6 code to perform the eigenvalue sensitivity calculations, although any Monte Carlo code that can produce sensitivity coefficients can be used. The sensitivity coefficients of a reactivity response are derived using the eigenvalue definition of reactivity, which is equivalent to applying Equivalent Generalized Perturbation Theory. Uncertainties are propagated using the "Sandwich Rule" of the "Propagation of Moments" method employing state-of-the-art covariance libraries. The methodology has been applied to the Advanced Lead-cooled Fast Reactor European Demonstrator and a sensitivity and uncertainty analysis has also been performed with the SCALE6.1 system. Good agreement between SUMMON and SCALE has been obtained in the sensitivity analysis and uncertainty quantification. Preliminary results for a change of coolant density scenario are also presented.

# I. INTRODUCTION

Numerous efforts are being made nowadays to address the increasing demand from nuclear research, industry and regulators to support calculations of best-estimate predictions with their confidence bounds (1) and to address the nuclear data needs for the design and safety analysis of Advanced Fast Reactors and Advanced Fuel Cycle Scenarios. The uncertainty in nuclear data is one of the most important sources of uncertainty in reactor physics simulations (2). Hence, accurate and reliable tools for sensitivity analysis and uncertainty quantification are needed to estimate the uncertainties in reactor key parameters ( $k_{eff}$ ,  $\beta_{eff}$ , safety coefficients, ...) and to identify possible nuclear data weaknesses. Sensitivity and Uncertainty Methodology for MONte carlo codes (SUMMON) has been developed to calculate the sensitivities of the criticality safety parameters and their uncertainties due to uncertainties in nuclear data. Tools that can calculate the uncertainty of a response due to uncertainties in nuclear data are available, however they possess several limitations such as no parallel processing capabilities, user selection of isotope and reaction channels to be included in the analysis, use of multi-group (MG) nuclear data, use of specific nuclear data library and/or specific covariance matrix. SUMMON addresses all these previously mentioned limitations.

In this paper, the methodology is presented and applied to an innovative Generation IV design, the Advanced Leadcooled Fast Reactor European Demonstrator. Additionally, sensitivity analysis and uncertainty quantification results are compared against reference calculations and SCALE6.1 system results.

# **II. SUMMON**

The SUMMON methodology is based on the use of the KSEN card of the MCNP6 code (3), taking advantage of MCNP parallel computing implementation and allowing the use of any neutron induced nuclear data library, though any Monte Carlo code that can calculate the energy dependent sensitivity of  $k_{eff}$  with respect to cross section changes can be used. Moreover, any covariance matrix can be used to propagate the uncertainties.

First, the evaluated data files are processed with the NJOY code (4) to obtain MCNP-compatible ACE files. Any evaluated nuclear library in ENDF-6 format (5) can be used. Then, if the parameter to be studied is the effective neutron multiplication factor, one single MCNP calculation with the KSEN card is run to obtain the sensitivity coefficients. On the other hand, if the sensitivity analysis and uncertainty quantification are going to be performed for a reactivity response corresponding to the difference in critical eigenvalues for two distinct states of a system (temperature effects, void worth, control rod worth, expansion effects, ...), two different MCNP/KSEN calculations are run, one for the nominal state and other for the perturbed state. Once the energy dependent sensitivity coefficients have been obtained the KSEN2sdf code translates the MCNP/KSEN output to sensitivity data file (.sdf) format (6). Finally, the SUMMONR code is called to perform the sensitivity analysis of the reactivity response (if required) and the uncertainty quantification. The KSEN2sdf and SUMMONR codes have been developed in the framework of this work.

State-of-the-art covariance libraries are employed by the SUMMONR code to propagate the uncertainties in  $k_{eff}$  and in

the criticality safety parameters due to uncertainties in nuclear data. COMMARA-2.0 (7), SCALE6.1 (8), SCALE6.2 (6) and covariance matrices from the evaluated nuclear data libraries ENDF/B-VII.1 (9) and JENDL-4.0 (10) can be used in the uncertainty calculations. These matrices have been processed to BOXER format (4) which allows large data compression factors significantly reducing the size of the covariance files, except COMMARA-2.0 which is directly used by SUMMONR without any previous processing. Additionally, any new nuclear data library which contains covariance data can be easily processed with the NJOY code to BOXER format, thus giving SUMMONR the capability of always using state-of-the-art covariance libraries.

Although the SUMMON methodology is based on the use of MCNP6 to calculate the sensitivities, other Monte Carlo codes can be used to perform this task such as SCALE, SERPENT (11), etc. The only requirement is that the group structure of the energy dependent sensitivity coefficients contains at least the 308 standard groups used by SUMMONR, which have been defined to be compatible with the energy binning of the existing covariance matrices.

### **III. THEORY**

The methods implemented in the KSEN card of MCNP6 to calculate the sensitivity coefficients are based upon linearperturbation theory using adjoint weighting. The adjoint weighting is performed in a forward calculation by means of the Iterated Fission Probability (IFP) methodology (12, 13, 14), which is based on the interpretation of the adjoint flux as the neutron importance function, also called iterated fission probability, originally formulated by Hurwitz (15). The theory for the derivation of the sensitivity coefficients is presented below.

# 1. Eigenvalue sensitivity coefficients

Let the Boltzmann transport equation be written in the form:

$$[A - \lambda B]\phi = 0 \tag{1}$$

where  $\phi$  is the neutron flux,  $\lambda$  represents the eigenvalues where the largest eigenvalue is  $1/k_{eff}$ , A is the operator that represents the scattering and total interactions terms and B is the operator that represents the fission term.

A perturbation in the transport operators and the eigenvalues can be defined as:

$$A' = A + \partial A$$
  

$$B' = B + \partial B$$
  

$$\lambda' = \lambda + \partial \lambda$$
(2)

where  $\partial A$  and  $\partial B$  represent small linear perturbations in the transport operators and  $\partial \lambda$  represents the resulting change in the eigenvalues. The perturbed neutron transport equation can be written as:

$$[A' - \lambda'B']\phi' = 0 \tag{3}$$

The equation adjoint to Eq. 1 is:

$$[A^* - \lambda B^*]\phi^* = 0 \tag{4}$$

where  $\phi^*$  is the adjoint flux and A<sup>\*</sup> and B<sup>\*</sup> are the adjoint operators.

Multiplying Eq. 3 by  $\phi^*$ , integrating over all phase space and expanding in terms of Eq. 2 yields:

$$\langle \phi^*(A - \lambda B + \partial A - \lambda \partial B - B \partial \lambda - \partial \lambda \partial B) \phi' \rangle = 0 \quad (5)$$

Using the property of adjointness and Eq. 4:

$$\langle \phi^* (\partial A - \lambda \partial B - B \partial \lambda - \partial \lambda \partial B) \phi' \rangle = 0 \tag{6}$$

Ignoring the second order perturbation term (i.e.,  $\partial\lambda\partial B$ ) and assuming that perturbations in the transport operator do not cause significant perturbations in the flux ( $\phi' = \phi$ ), the eigenvalue perturbation becomes:

$$\frac{\partial \lambda}{\lambda} = \frac{\langle \phi^*(\partial A - \lambda \partial B) \phi \rangle}{\langle \phi^*(\lambda B) \phi \rangle} \tag{7}$$

The sensitivity of k to a change in some nuclear data  $\alpha$ , which can be a cross section, the average fission neutron multiplicity v or the average fission spectra,  $\chi$ , is defined as:

$$S_{k,\alpha} = \frac{\alpha}{k} \frac{\partial k}{\partial \alpha} \tag{8}$$

Therefore, considering that  $\lambda = 1/k$ , where  $k = k_{eff}$ , and considering Eq. 7 and Eq.8, the sensitivity coefficient can be expressed as:

$$S_{k_{eff},\alpha} = -\frac{\alpha}{\partial\alpha} \frac{\langle \phi^* \left( \partial A - \frac{1}{k_{eff}} \partial B \right) \phi \rangle}{\langle \phi^* \left( \frac{1}{k_{eff}} B \right) \phi \rangle} \tag{9}$$

# 2. Eigenvalue-difference response sensitivity coefficients

The methodology implemented in the SUMMONR code to derive the sensitivity coefficients of a reactivity response is based on the eigenvalue definition of reactivity, which is equivalent to applying Equivalent Generalized Perturbation Theory (16) in the case of reactivity responses. This methodology is also employed by the TSAR module of the SCALE system (6, 17). As discussed in Ref. 16, an alternative formulation based on Generalized Perturbation Theory (GPT) exist (18), however, GPT requires generalized solutions that are more difficult to compute and not available in all Monte Carlo codes. Therefore, the eigenvalue-difference approach is more convenient. This approach is derived in the lines below.

The change in reactivity (reactivity response  $\rho_{1\rightarrow 2}$ ) associated with the change in the state of the system can be defined as:

$$\rho_{1\to 2} = \rho_2 - \rho_1 = \lambda_1 - \lambda_2 \tag{10}$$

The relative sensitivity coefficient for the reactivity response is:

$$S_{\rho_{1\to 2},\alpha} = \frac{\alpha}{\rho_{1\to 2}} \frac{\partial \rho_{1\to 2}}{\partial \alpha}$$
(11)

Therefore, introducing Eq. 10 in Eq. 11, the relative sensitivity coefficient of the reactivity response  $\rho_{1\rightarrow 2}$  is:

$$S_{\rho_{1\to2},\alpha} = \frac{\alpha}{\rho_{1\to2}} \left( \frac{\partial \lambda_1}{\partial \alpha} - \frac{\partial \lambda_2}{\partial \alpha} \right) = \frac{\lambda_1 S_{1,\alpha} - \lambda_2 S_{2,\alpha}}{\rho_{1\to2}}$$
(12)

And the standard deviation, *std*, can be derived from Eq. 12:

$$S_{\rho_{1\to 2},\alpha} = f(\lambda_1, \lambda_2, S_1, S_2) = f(k_1, k_2, S_1, S_2)$$
(13)

$$std_{S_{\rho_{1}\rightarrow_{2},\alpha}}^{2} = \left(\frac{\partial S_{\rho_{1}\rightarrow_{2},\alpha}}{\partial k_{1}}\right)^{2} \sigma_{k_{1}}^{2} + \left(\frac{\partial S_{\rho_{1}\rightarrow_{2},\alpha}}{\partial k_{2}}\right)^{2} \sigma_{k_{2}}^{2} + \left(\frac{\partial S_{\rho_{1}\rightarrow_{2},\alpha}}{\partial S_{1}}\right)^{2} \sigma_{S_{1}}^{2} + \left(\frac{\partial S_{\rho_{1}\rightarrow_{2},\alpha}}{\partial S_{2}}\right)^{2} \sigma_{S_{2}}^{2} + 2 \left(\frac{\partial S_{\rho_{1}\rightarrow_{2},\alpha}}{\partial k_{1}}\right) \left(\frac{\partial S_{\rho_{1}\rightarrow_{2},\alpha}}{\partial k_{2}}\right) \sigma_{k_{1}}\sigma_{k_{2}} + 2 \left(\frac{\partial S_{\rho_{1}\rightarrow_{2},\alpha}}{\partial S_{1}}\right) \left(\frac{\partial S_{\rho_{1}\rightarrow_{2},\alpha}}{\partial S_{2}}\right) \sigma_{S_{1}}\sigma_{S_{2}}$$
(14)

$$std_{\rho_{1\to2},\alpha}^{2} = \left(\frac{s_{1}-s_{2}}{k_{2}\left(1-\frac{k_{1}}{k_{2}}\right)^{2}}\right)^{2}\sigma_{k_{1}}^{2} + \left(\frac{k_{1}(s_{2}-s_{1})}{(k_{2}-k_{1})^{2}}\right)^{2}\sigma_{k_{2}}^{2} + \left(\frac{1}{1-\frac{k_{1}}{k_{2}}}\right)^{2}\sigma_{S_{1}}^{2} + \left(\frac{1}{1-\frac{k_{2}}{k_{2}}}\right)^{2}\sigma_{S_{2}}^{2} - 2\frac{k_{1}k_{2}(s_{2}-s_{1})^{2}}{(k_{2}-k_{1})^{4}}\sigma_{k_{1}}\sigma_{k_{2}} + 2\frac{1}{2-\frac{k_{2}}{k_{1}}-\frac{k_{1}}{k_{2}}}\sigma_{S_{1}}\sigma_{S_{2}} \quad (15)$$

Eq. 15 represents the standard deviation of the reactivity response sensitivity coefficient due to the propagation of the statistical uncertainties in the parameters  $k_1$ ,  $k_2$ ,  $S_1$  and  $S_2$ . Correlations between the two states of the system are taken into account. Generally, the sensitivity coefficients are nearly independent of the nuclear data library used although  $k_{eff}$  is not, as can be seen in Ref. 19. Therefore, significant variations in  $k_{eff}$  have very small impact in the sensitivity coefficients of a defined state of a system and the correlation between k and S can be considered negligible.

#### 3. Uncertainty quantification

Once the sensitivity coefficients have been obtained, the relative variance in  $k_{eff}$  can be calculated using the "Sandwich Rule" of the "Propagation of Moments" method formulated in Ref. 20:

$$\sigma_{k_{eff}}^2 = S_{k_{eff},\alpha} C_{\alpha\alpha} S_{k_{eff},\alpha}^T \tag{16}$$

where  $C\alpha\alpha$  denotes the covariance matrix for the nuclear data  $(\alpha_1, ..., \alpha_k)$  and  $S_{keff,\alpha}$  are the sensitivity coefficients collapsed to the same energy group structure as the covariance matrix.

The relative variance in the reactivity due to nuclear data uncertainties can be obtained combining Eqs. 12 and 16:

$$\sigma_{\rho_{1\to2}}^{2} = \left(\frac{\lambda_{1}}{\rho_{1\to2}}\right)^{2} S_{\lambda_{1}} C_{\alpha\alpha} S_{\lambda_{1}}^{T} + \left(\frac{\lambda_{2}}{\rho_{1\to2}}\right)^{2} S_{\lambda_{2}} C_{\alpha\alpha} S_{\lambda_{2}}^{T} - \frac{\lambda_{1} \lambda_{2}}{\rho_{1\to2}^{2}} \left(S_{\lambda_{1}} C_{\alpha\alpha} S_{\lambda_{2}}^{T} + S_{\lambda_{2}} C_{\alpha\alpha} S_{\lambda_{1}}^{T}\right) \quad (17)$$

## **IV. VALIDATION**

The above-presented SUMMON methodology has been applied to an innovative reactor concept, the Advanced Leadcooled Fast Reactor European Demonstrator (ALFRED) (21), which is currently in the conceptual design phase. The reactor core at Beginning of Cycle and nominal conditions has been modelled with MCNP6 code and SCALE6.1 system and a sensitivity analysis has been carried out using the ENDF/B-VII.0 nuclear data library (22) with the KSEN card of MCNP6 and the consolidated and well-validated TSUNAMI-3D MG module (8) of SCALE, which is also based on linear-perturbation theory. To ensure the consistency of the calculations, the results have been compared against sensitivity coefficients obtained by the direct perturbation method (23), which employs variations of nuclide density that result in small keff variations consistent with the linear perturbation approximation and allows determining the total sensitivity coefficients. Furthermore, nuclear data uncertainties have been propagated with SUMMONR and TSUNAMI-3D MG using SCALE6.1 (ENDF/B-VII.0 based) and SCALE6.2-56g (ENDF/B-VII.1 based) covariance matrices.



Fig. 1. <sup>239</sup>Pu v sensitivity profile using different statistics.

Statistical uncertainties are the most severe limitation of the method used by SUMMONR code to derive the sensitivity coefficients of the reactivity response due to the formulation presented in Section III.2 (Eq. 15), consequently several calculations with MCNP6/KSEN were performed in order to study the impact of statistics in the sensitivity coefficient calculations. In Figs. 1 and 2, the sensitivity profiles of  $^{239}$ Pu v and  $^{238}$ U(n,n) using different statistics are shown. The standard deviation of the neutron multiplication factor of the initial calculation, given in pcm (1 pcm =  $10^{-5}$ ), has been reduced from 8 pcm to 1 pcm using higher amount of neutron histories (Table II), therefore, improving the statistics of the successive calculations. Small differences can be observed in the calculated sensitivity profiles of  $^{239}$ Pu v (Fig. 1), however, the impact of statistics in the <sup>238</sup>U elastic scattering cross section is very high (Fig. 2). This is reflected in the <sup>238</sup>U(n,n) energy-integrated sensitivity coefficients presented in Table I, where differences between the TSUNAMI-3D MG value, taken as a reference, and the MCNP6/KSEN sensitivity coefficient values reduce considerably when the statistics are improved. Additionally, the statistical uncertainty of the calculated sensitivity coefficients reduces up to an 86% when the statistical uncertainty in  $k_{eff}$  is reduced to 1 pcm.

Table I.  $^{238}U(n,n)$  energy-integrated sensitivity coefficients obtained with MCNP6/KSEN.  $S_{TSUMANI-3D}=1.3515\ x\ 10^{-2}\pm8.1\ x\ 10^{-5}$ 

k <sub>eff</sub> std.	S	<b>Diff.</b> (%)
8 pcm	$1.6741 \text{ x } 10^{-2} \pm 7.8 \text{ x } 10^{-5}$	23.87
4 pcm	$1.5564 \ge 10^{-2} \pm 4.5 \ge 10^{-5}$	15.16
2 pcm	$1.1200 \ge 10^{-2} \pm 2.3 \ge 10^{-5}$	17.13
1 pcm	$1.2370 \ge 10^{-2} \pm 1.1 \ge 10^{-5}$	8.48



Fig. 2. <sup>238</sup>U(n,n) sensitivity profile using different statistics.

The same behaviour can be observed in the elastic and inelastic cross section sensitivity profiles and statistical uncertainties of other key elements. Hence, when using the KSEN card of MCNP6 is highly recommended to perform calculations with a sufficient number of neutron histories so that the statistical uncertainty in  $k_{eff}$  is not greater than 2 pcm, in order to have a low statistical uncertainty in the sensitivity coefficients. On the other hand, achieving such statistical uncertainties might be too computationally costly depending on the application, so the user must evaluate the accuracy of

the calculations when less statistics are employed. For ALFRED sensitivity calculations, CPU times are presented in Table II.

Table II. CPU times for SCALE6.1 and MCNP6 sensitivity calculations

	Number of processors	Number of neutrons	<i>k<sub>eff</sub></i> std.	t (minutes)
SCALE6.1	1*	$1.20 \ge 10^8$	9 pcm	6120
MCNP6	64	$6.00 \ge 10^7$	8 pcm	330
		$1.60 \ge 10^8$	4 pcm	878
		$5.60 \ge 10^8$	2 pcm	2793
		2.28 x 10 <sup>9</sup>	1 pcm	13514

\*Note: SCALE6.1 doesn't have parallel capabilities

The results of MCNP6/KSEN calculations with a statistical uncertainty in  $k_{eff}$  of 1 pcm are shown from this point forward. In Table III, the sensitivity coefficients obtained with MCNP6 and SCALE are compared with direct perturbation calculations. It can be seen that the KSEN and TSUNAMI-3D MG total sensitivity coefficients for the fuel materials are consistent with the direct perturbation calculations, with differences less than 4%, except for <sup>242</sup>Pu calculated with MCNP6/KSEN. For <sup>56</sup>Fe and <sup>208</sup>Pb, KSEN sensitivity coefficients are closer to the reference direct perturbation values than TSUNAMI-3D MG.

Table III. Direct perturbation calculations for the isotopes with the highest total sensitivity

Isotope	Direct	MCNP6/KSEN		SCALE/TSUNAMI	
	Perturbation	Total Sensitivity	Diff. (%)	Total Sensitivity	Diff. (%)
<sup>239</sup> Pu	4.4988 x 10 <sup>-1</sup>	4.5724 x 10 <sup>-1</sup>	1.64	4.4520 x 10 <sup>-1</sup>	1.05
<sup>238</sup> U	-1.2187 x 10 <sup>-1</sup>	-1.2073 x 10 <sup>-1</sup>	0.94	-1.2134 x 10 <sup>-1</sup>	0.44
<sup>241</sup> Pu	6.3706 x 10 <sup>-2</sup>	6.4335 x 10 <sup>-2</sup>	0.99	6.3264 x 10 <sup>-2</sup>	0.70
<sup>240</sup> Pu	3.0313 x 10 <sup>-2</sup>	3.0941 x 10 <sup>-2</sup>	2.07	2.9943 x 10 <sup>-2</sup>	1.24
<sup>16</sup> O	-3.1652 x 10 <sup>-2</sup>	-3.0412 x 10 <sup>-2</sup>	3.92	-3.1648 x 10 <sup>-2</sup>	0.01
<sup>56</sup> Fe	-2.0278 x 10 <sup>-2</sup>	-2.1564 x 10 <sup>-2</sup>	6.34	-1.7899 x 10 <sup>-2</sup>	13.29
<sup>242</sup> Pu	4.2390 x 10 <sup>-3</sup>	4.5176 x 10 <sup>-3</sup>	6.57	4.2049 x 10 <sup>-3</sup>	0.81
<sup>208</sup> Pb	7.9280 x 10 <sup>-3</sup>	8.8876 x 10 <sup>-3</sup>	12.10	6.8858 x 10 <sup>-3</sup>	15.14

The uncertainty quantification analysis has been performed with SCALE/TSUNAMI-3D MG and with the SUMMONR code using the sensitivities from TSUNAMI-3D MG and KSEN, and SCALE6.1 (ENDF/B-VII.0 based) and SCALE6.2-56g (ENDF/B-VII.1 based) covariance matrices. The results of the analysis for the top 10 contributors to the uncertainty in  $k_{eff}$  are presented in Table IV. The uncertainties obtained by SCALE and SUMMON using TSUNAMI-3D MG sensitivities and SCALE6.1 covariance matrix are identical. Differences arise in the uncertainty contribution of <sup>239</sup>Pu v, <sup>238</sup>Pu(n,f) and <sup>238</sup>U v

calculated with SUMMON using KSEN sensitivities and SCALE6.1 covariance matrix, due to differences in the sensitivity coefficients. Additionally,  $\nu$  and  $^{238}$ Pu(n,f) uncertainty contributions are significantly reduced when using SCALE6.2-56g ENDF/B-VII.1 based covariance matrix.

Table IV. Uncertainty quantification for the 10 top contributors in  $\Delta k_{eff}/k_{eff}(\%)$ 

	SCALE	SUMMON		
	TSUNAMI	TSUNAMI	KSEN	KSEN
	SCALE6.1	SCALE6.1	SCALE6.1	SCALE6.2
<sup>239</sup> Ρu ν	7.03 x 10 <sup>-1</sup>	7.03 x 10 <sup>-1</sup>	7.13 x 10 <sup>-1</sup>	5.63 x 10 <sup>-2</sup>
<sup>238</sup> U(n,n')	5.33 x 10 <sup>-1</sup>	5.33 x 10 <sup>-1</sup>	5.34 x 10 <sup>-1</sup>	5.44 x 10 <sup>-1</sup>
<sup>239</sup> Pu(n,γ)	2.66 x 10 <sup>-1</sup>	2.66 x 10 <sup>-1</sup>	2.62 x 10 <sup>-1</sup>	2.52 x 10 <sup>-1</sup>
<sup>240</sup> Ρu ν	2.43 x 10 <sup>-1</sup>	2.43 x 10 <sup>-1</sup>	2.45 x 10 <sup>-1</sup>	6.14 x 10 <sup>-2</sup>
<sup>239</sup> Ρu χ	2.13 x 10 <sup>-1</sup>	2.13 x 10 <sup>-1</sup>	1.66 x 10 <sup>-1</sup>	1.27 x 10 <sup>-1</sup>
<sup>238</sup> U(n,γ)	2.00 x 10 <sup>-1</sup>	2.00 x 10 <sup>-1</sup>	1.98 x 10 <sup>-1</sup>	1.74 x 10 <sup>-1</sup>
<sup>239</sup> Pu(n,f)	1.98 x 10 <sup>-1</sup>	1.98 x 10 <sup>-1</sup>	2.02 x 10 <sup>-1</sup>	1.96 x 10 <sup>-1</sup>
<sup>238</sup> Pu(n,f)	1.49 x 10 <sup>-1</sup>	1.49 x 10 <sup>-1</sup>	2.90 x 10 <sup>-2</sup>	1.46 x 10 <sup>-3</sup>
<sup>238</sup> U(n,n) - <sup>238</sup> U(n,n')	1.31 x 10 <sup>-1</sup>	1.30 x 10 <sup>-1</sup>	1.56 x 10 <sup>-1</sup>	1.66 x 10 <sup>-1</sup>
<sup>238</sup> U v	1.01 x 10 <sup>-1</sup>	1.01 x 10 <sup>-1</sup>	1.98 x 10 <sup>-1</sup>	1.03 x 10 <sup>-1</sup>

The SUMMON methodology has also been applied to a 20% increase of the coolant density scenario and preliminary results have been obtained. In Fig. 3, the sensitivity profile of the simulated reactivity response for <sup>239</sup>Pu v is shown. There is a good agreement between SUMMON and the TSAR module of SCALE and the statistical uncertainties in the average prompt neutron fission multiplicity of <sup>239</sup>Pu obtained with the SUMMONR code are found to be low. Nevertheless, the results need to be validated for scattering cross sections and additional scenarios are being studied.



Fig. 3.  $^{239}$ Pu v sensitivity profile of the reactivity response corresponding to a change in coolant density of +20% in ALFRED.

# V. CONCLUSIONS

In this paper, SUMMON, a general methodology to calculate the sensitivities and uncertainties of the criticality safety parameters has been presented.

This methodology can be used by any Monte Carlo transport code that can calculate the sensitivity of  $k_{eff}$  with respect to cross section changes. Currently, it has been implemented with MCNP6, which removes the single library usage restriction that other transport codes have. Furthermore, several state-of-the-art covariance matrices have been processed and can be used to propagate the uncertainties.

The SUMMON methodology has been applied to the GEN-IV Lead-cooled Fast Reactor ALFRED. A sensitivity and uncertainty analysis has been performed and the effect of the Monte Carlo statistical uncertainties has been studied. Good agreement between KSEN and TSUNAMI-3D MG has been found in the sensitivity analysis when using a sufficient number of neutrons in the calculations so that the statistical uncertainty in  $k_{eff}$  does not exceed 2 pcm. Good agreement has also been found between TSUNAMI-3D MG and the SUMMONR code in the uncertainty quantification.

SUMMON's capability of calculating sensitivity coefficients of reactivity responses has also been presented and preliminary results have been obtained for a change of coolant density scenario in the ALFRED reactor. Additional scenarios are being studied. The propagation of reactivity response uncertainties due to uncertainties in nuclear data will be validated in future work. Additionally, SUMMON will be able to provide energy-dependent uncertainties.

SUMMON capabilities will allow calculating the uncertainties in the criticality safety coefficients of reactor designs. The propagation of these uncertainties to transient scenarios will provide an exhaustive picture of the influence of nuclear data on core performance, identifying key parameters and proposing specific actions to achieve an improved safety level.

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