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An Extended Theory of Multiplicity Counting from Fission Chamber Signals in the Current Mode

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Abstract - *This paper concerns the derivation of the individual and joint statistics of the signals of up to three fission chambers operating in the current mode, detecting neutrons emitted from a sample containing fissioning material. The purpose is to develop an alternative method to the traditional pulse detection based multiplicity counting for the determination of the sample parameters. The underlying theory and corresponding method of unfolding the parameters of the sample from such continuous signals was recently developed by the authors for the case when multiple neutrons emitted simultaneously were assumed to be also detected simultaneously in the same or different detectors. In the present paper the method is generalized by extending it to the case when the detection of the multiply emitted neutrons occurs with a random time delay individually for each neutron, such that the delays are independent, identically distributed random variables. It is seen that in the arising formulas, in addition to the detector pulse shape and amplitude distribution, the properties (parameters) of the time delay distribution appear as well. At the same time it is also seen that, although at the expense of a somewhat more involved calibration procedure, the unfolding of the sample parameters from the three lowest order auto- and cross-cumulants of the detector signals is still possible, in a procedure similar to using the singles, doubles and triples count rates of traditional multiplicity counting. In contrast to this latter method, the procedure proposed here is free from the dead time problem, and requires a somewhat simpler data processing. In particular, being free from the dead-time problem and by the relative insensitivity of the fission chambers to gamma contributions, especially for the higher order cumulants, makes the method particularly suitable for the multiplicity analysis of spent fuel.*

I. INTRODUCTION

In nuclear safeguards, one of the frequently used non-destructive assay methods for estimating sample parameters is the multiplicity counting, based on determining the singles (S), doubles (D) and triples (T) detection rates from the measured signals of several neutron detectors. Traditionally, these measurements are performed with thermal neutron detectors operating in pulse mode and require the use of multi-channel analyzers as well as various dead time correction techniques [1, 2]. An alternative method has been proposed recently, based on fast neutron measurements with fission chambers operating in current mode [3]. Although the new approach needs a more involved calibration, it does not require dead time corrections and has a much simpler data processing procedure. The fact that no dead time problems are present also makes the method suitable for measurements on spent fuel, where high count rates are encountered. The relative insensitivity of fission chambers to gamma contributions and, in particular, the ability of the higher order moments of the detector signal for suppressing minority components (cf. the higher order Campbelling theorems, e.g. [4, 5]) makes the proposed method a very promising alternative for multiplicity measurements, including measurement on spent fuel. Because of their small size, fission chambers can be inserted even inside the fuel assembly in BWRs [6].

A significant limitation of both the traditional and the newly proposed method is that their underlying theory is based on a spatial and energy independent mathematical model of the emission and detection process of neutrons. As a consequence, they are unable to describe inherently the temporal separation of neutrons: due to the source energy spectrum, the stochastic nature of the neutron transport, and the difference in the distances of various detectors from the source, particles originating from the same emission event (hence being emitted simultaneously) will reach the detectors after a (random) detection time, different for each neutron. It is worth noting that, at least in principle, the detection times of the individual particles may not be totally independent. If the effect is a result of the difference in the energies (hence velocities) of the neutrons, the correlations between the energies would lead to non-zero covariances between the detection times. Such energy correlations are expected to be small, in particular after the internal multiplication of the source neutrons in the sample. However, there is an increasing awareness of the significance of the energy correlations of neutrons emitted in a single fission event, hence in measurements on small sources with negligible internal multiplication, such energy correlations might play a role. This effect will be investigated in a future work.

Clearly, the above mentioned time delay between the emission and detection of neutrons should be taken into account in the relevant theoretical expressions. For the traditional case of pulse counting with thermalised neutrons, the delay is actually dominated by the slowing down and thermal diffusion, and is represented by a "detector die-away time". Quantitatively, it is accounted for with the doubles and triples gate factors in the multiplicity formulas [1, 2]. On the other hand, since the proposed alternative method is based on fast neutron detection without neutron thermalisation being involved, the difference in the detection times is several orders of magnitudes smaller than in thermal detector systems and mainly arise from geometrical factors (different flight lengths to different detectors or different parts of the same detector), as well as from the different velocities of the emitted neutrons. The corresponding fluctuations in the detection times are therefore several orders of magnitude smaller than in thermal detection systems, so much so that in pulse counting systems no correction factors (gate factors) would be necessary. Nevertheless, it can be expected from simple considerations, that the suggested method of using fission chambers in the current mode is still sensitive even to such small time delay effects. This is because the pulse width is rather small (in the range of tens of nanoseconds), and an arrival time difference equal to or larger than the pulse width can make two completely overlapping pulses to totally non-overlapping. In the previous work, no such differences in the detections times have been taken into account so far: neutrons emitted simultaneously were assumed to be detected instantly, hence also simultaneously. Reflecting these random detection times, however, might result in a substantially different relationship between the signal cumulants and the multiplicity rates compared with the model in which simultaneous detection of simultaneously born neutrons is assumed.

The objective of the present paper is thus to account for the fluctuations in the detection times of jointly born neutrons by extending the theory to account for the different arrival times of neutrons from the same source event. This will be achieved by using a random time delay of the detection of each neutron after their birth, specified by a probability distribution. It is shown that the S, D and T rates are still uniquely related to the first three cumulants of the detector signal. In this case though, as expected, the scaling factors will depend not only on the detector pulse shape and its amplitude distribution, as in the previous model, but also on the properties of the random time delay distribution of the detection event. Nonetheless, the results also suggest that the non-simultaneous detection of neutrons from the same source event does not decrease the potentials of the method for multiplicity counting in a crucial manner.

II. THEORY

The two substantial elements in a multiplicity counting measurement are the emission and subsequent detection of neutrons. Due to the probabilistic nature of the underlying physical phenomena (including the production and transport of neutrons as well as the formation of the detector signal), they need to be described as stochastic processes. The topic of this section is the mathematical formulation of these processes.

1. Emission Statistics

The emission of neutrons from the sample is completely independent from the detection process, hence it is the common starting point for both the pulse-based traditional coincidence counting and the fission chamber signal based method proposed in [3] and generalized in this paper. A suitable probabilistic model of the emission of neutrons in a heavy-nuclide sample is provided by the theory of *superfission* [7, 1, 2], which describes the emission as a compound Poisson process [8].

The source emission events are characterized by an intensity Q_s which is given in terms of the fission source intensity F as

$$Q_\mathrm{s} = F\,(1 + \alpha\nu_{\mathrm{sf},1}), \tag{1}$$

where α stands for the so-called α-ratio. The emission of neutrons in a source event (spontaneous fission or (α, n) reaction) leads to a number distribution $P(n)$ of the neutrons leaving the sample (per source event), which also takes into account the internal multiplication in the sample. For later use we define the generating function of this number distribution as

$$G(z) = \sum_{n=0}^{\infty} P(n)\, z^n \tag{2}$$

The first three factorial moments of $P(n)$, which are obtained from the derivatives of $G(z)$ with respect to z, are given as [7]

$$\nu_1 = \frac{M}{(1 + \alpha\nu_{\mathrm{sf},1})}\,\nu_{\mathrm{sf},1}(1+\alpha), \tag{3a}$$

$$\nu_2 = \frac{M^2}{(1 + \alpha\nu_{\mathrm{sf},1})}\left[\nu_{\mathrm{sf},2} + \left(\frac{M-1}{\nu_{\mathrm{i},1}-1}\right)\nu_{\mathrm{sf},1}(1+\alpha)\,\nu_{\mathrm{i},2}\right], \tag{3b}$$

$$\nu_3 = \frac{M^3}{(1 + \alpha\nu_{\mathrm{sf},1})}\left\{\nu_{\mathrm{sf},3} + \left(\frac{M-1}{\nu_{\mathrm{i},1}-1}\right)\times\left[3\nu_{\mathrm{sf},2}\nu_{\mathrm{i},2} + \nu_{\mathrm{sf},1}(1+\alpha)\,\nu_{\mathrm{i},3}\right] + 3\left(\frac{M-1}{\nu_{\mathrm{i},1}-1}\right)^2\nu_{\mathrm{sf},1}(1+\alpha)\,\nu_{\mathrm{i},2}^2\right\}, \tag{3c}$$

where M is the so-called net leakage multiplication, whereas $\nu_{\mathrm{sf},k}$ and $\nu_{\mathrm{i},k}$ are the factorial moments of the number of neutrons emitted in one spontaneous or induced fission event, respectively.

In order to simplify the upcoming formulas, it is convenient to introduce the modified factorial moments[1]

$$\tilde{\nu}_k = \nu_k\,(1 + \alpha\nu_{\mathrm{sf},1}), \tag{4}$$

hence one has

$$Q_\mathrm{s}\,\nu_k = F\,\tilde{\nu}_k. \tag{5}$$

2. Detection Statistics

The probabilistic model of the detection of neutrons should provide equations for a proper set of quantities that *a*) can be expressed as functions of the (known and unknown)

[1] It has to be mentioned that the word "modified" here is different from its meaning in the expression "modified factorial moments", where it means the nth factorial moments divided by $n!$.

sample parameters defined in Section 1., and *b*) can be determined from the registered responses of the detectors.

Such a model is based on a simplified experimental setup in which a neutron-emitting sample is surrounded by three detectors characterized by efficiencies ε_i ($i = 1, \ldots, 3$), such that $\sum_{i=1}^{3} \varepsilon_i \leq 1$, and by some other parameters specified later. It is further assumed, that each emitted neutron might be detected *independently* by one of the detectors with a corresponding probability ε_i, and triggers a response. The actual form of the response depends on the operation mode of the detectors:

- in pulse mode, the individual detection events are registered e.g. by counting them within a suitable time-gate (pulse counting); this forms the basis of the traditional method of multiplicity counting and is summarized briefly in Section A.
- in current mode, the time-dependent current (or voltage) signal is registered with a suitable time-resolution; this forms the basis of the new method of multiplicity counting and is described in detail in Section B.

With both modes of operation, the observed statistics of detection is highly influenced by the fluctuations in the detection times of the individual neutrons, discussed already in Section I.. Therefore, for any practical applications, this effect needs to be taken into account in the derivations of the formulas.

A. Detection in Pulse Mode

The traditional method of multiplicity counting utilizes the counting statistics obtained from the detectors operating in pulse mode and characterized by identical efficiencies $\varepsilon_i = \varepsilon$ ($i = 1, \ldots, 3$). Specifically, the k-multiplet detection rates, that is, the expected number of events per unit time, when a response is triggered in k detectors ($k = 1, 2, 3$) within a suitable time-gate is determined [1]. It has to be mentioned that it is necessary to turn to these detection intensities, since the factorial moments of the neutrons emitted in one source event cannot be measured.

In particular, the first three multiplet detection rates, called the singles (S), doubles (D) and triples (T) rates can be written as

$$S = F\,\varepsilon\,\tilde{\nu}_1, \tag{6a}$$

$$D = F\,\frac{\varepsilon^2\tilde{\nu}_2}{2}\,f_d, \tag{6b}$$

$$T = F\,\frac{\varepsilon^3\tilde{\nu}_3}{6}\,f_t, \tag{6c}$$

which, besides the sample parameters and the detector efficiency, contain two empirical quantities, the f_{d} doubles and f_{t} triples gate fractions. These latter represent a "detector die-away time" accounting for the detection time fluctuations by compensating the underestimation of the corresponding multiplet detection rate: since the detection events are counted within a time-gate initiated by the first (triggering) count and this time-gate is finite, only a fraction of the coincident detections will be registered.

By substituting the modified factorial moments (4) into the expressions (6) of the detection rates, a system of algebraic equations is obtained. Using these equations, the three unknown sample parameters (F, M and α) can be obtained from the measured values of the S, D and T rates by algebraic inversion [1, 2]. The corresponding inversion formulas were shown in the previous paper, hence they will not be given here.

B. Detection in Current Mode

The new method of multiplicity counting, based on a recently developed formalism [9], utilizes the signal statistics obtained from the detectors operating in current mode. Specifically, the auto and cross cumulants of the stationary signals of different groups of detectors are determined. In the previous work, the cumulants were derived assuming the coincident detection of simultaneously emitted neutrons. The aim of this section is to provide a similar derivation of another set of formulas, which now will take into account the fluctuating detection time of neutrons.

The complete signal of a detector is the sum of the signals induced by all the detections after a single emission which, in turn, is a sum of the pulses induced by the individual detections. In order to structure the following lengthy discussion, the statistics of the detector signal will be described in three steps, each utilizing the results of the previous ones: 1. characterization of the response to a single detection; 2. characterization of the response to detections from a single emission; 3. characterization of the response to detections from a series of emissions.

Let $\xi(t)$ denote the stochastic process representing the signal of a detector after detecting *one neutron* originating from an emission at time $t = 0$. We shall assume that the detection occurs after a random time τ, which is independent and identically distributed for each neutron, and is characterized by a density function $u(\tau)$. Each detection induces a stochastic pulse with a deterministic shape $f(t)$, such that $f(t) = 0$ for $t < 0$, and with a random amplitude a characterized by a probability density function $w(a)$. Under these assumptions, the one-point distribution function $H(y, t)$ of the process can be written as

$$H(y,t) = \mathcal{P}\{\xi(t) \leq y\} = \tag{7}$$

$$\int_0^\infty \int_0^\infty \Delta\left[y - af(t-\tau)\right] w(a)\, u(\tau)\, \mathrm{d}a\mathrm{d}\tau, \tag{8}$$

where Δ denotes the unit step function. The corresponding one-point density function $h(y, t)$ and its characteristic function $\chi(\omega, t)$ reads as

$$h(y,t) = \int_0^\infty \int_0^\infty \delta\left[y - af(t-\tau)\right]\, w(a)\, u(\tau)\, \mathrm{d}a\mathrm{d}\tau \tag{9}$$

and

$$\chi(\omega,t) = \int_0^\infty \int_0^\infty \mathrm{e}^{\imath\omega a f(t-\tau)}\, w(a)\, u(\tau)\, \mathrm{d}a\mathrm{d}\tau, \tag{10}$$

respectively, with δ denoting the Dirac delta function. When no delay between the emission and the detection is considered, i.e., when $u(\tau) = \delta(\tau)$, the above forms of $h(y, t)$ and $\chi(\omega, t)$

reduce to the corresponding expressions in [3]. Actually, since in the stationary state of the process, all quantities are invariant to an arbitrary time shift, it is not the zero value of the time delay which counts, rather whether it is deterministic (the same for all neutron arrivals), or has a random distribution. If all neutrons are delayed with the same time lapse, the particles emitted simultaneously will still arrive simultaneously. That is, if one has $u(\tau) = \delta(\tau - \tau_0)$, then one still should have formulas identical with the case of instantaneous detection. Indeed, using such a delay distribution in (10) leads to

$$\chi(\omega, t) = \int_0^\infty e^{\imath\omega a f(t-\tau_0)}\, w(a)\, da, \tag{11}$$

and since in the stationary formulas $\chi(\omega, t)$ is always integrated from $t = 0$ to $t = \infty$, it is easy to see that even (11) reverts to the corresponding formula in [3]. On the other hand, as will be seen more clearly on the expressions of the cumulants, if $u(\tau)$ is a random distribution, the arising expressions and results are different from the previous one. Hence, the theoretical model presented in this paper can be considered as a generalization of that proposed in [3].

Let $\xi_k(t)$ denote the stochastic process representing the signal of a detector after the (not necessarily simultaneous) detection of *k neutrons originating from the same emission event* at time $t = 0$. Since each detection generates a corresponding pulse $\xi(t)$, the one-point distribution function $U_k(y, t)$ of $\xi_k(t)$ can be expressed with that of $\xi(t)$ as

$$\begin{aligned} U_k(y, t) = \mathcal{P}\{\xi(t) \le y_1, \ldots, \xi(t) \le y_k\} = \\ H(y_1, t) \cdots H(y_k, t). \end{aligned} \tag{12}$$

with the condition $y_1 + \cdots + y_k = y$; this expresses the fact that although the pulses are generated independently, their contribution to the signal is not independent. As a consequence, the density function u_k of the process ξ_k can be written as a k-fold convolution of h:

$$u_k(y, t) = \int\limits_{y_1+\cdots+y_k=y} \cdots \int h(y_1, t) \cdots h(y_k, t)\, dy_1 \cdots dy_k,$$

whereas, after utilizing the *convolution theorem* [10], its characteristic function ζ_k can be expressed as the k-th power of χ:

$$\zeta_k(\omega, t) = \chi^k(\omega, t). \tag{13}$$

Now, let the stochastic process $\eta_i(t)$ represent the fluctuating signal of the i-th detector. Then, the joint distribution of the signals of all m detectors is defined as

$$P(y_1, \ldots, y_m, t) = \mathcal{P}\{\eta_1(t) \le y_1, \ldots, \eta_m(t) \le y_m\}. \tag{14}$$

Specifically, we are interested in the statistics of at most three detectors. Therefore, using a backward type master equation formalism, integral equations will be formulated for the single and joint density functions of one, two and three detectors. By taking their Fourier-transforms, another set of equations will be obtained for the corresponding characteristic functions, which will have very simple solutions. Finally, the desired cumulants will be obtained by differentiating the natural logarithm of the characteristic functions of the stationary detector signals.

The details of this derivation will be considered separately for one, two and three detectors. In order to maintain generality, we shall assume that all detectors are distinct and have different parameters. As will be seen below, the primary quantities that enter into the expressions of the cumulants are the modified factorial moments (4) and the spontaneous fission rate F. These expressions are sufficient to perform the inversion procedure, i.e. expressing the parameters F, M and α in terms of the cumulants of the detector current and the (known) factorial moments of spontaneous and induced fission.

The formulas corresponding to identical detectors will also be presented. As was done in the previous work, it will be shown that in this special case, the cumulants can also be expressed in terms of the S, D and T rates of the traditional method (at least after disregarding the empirical gate factors, relevant only in an actual pulse counting measurement). Although this step is not necessary, there is a trivially simple relationship between the multiplicity rates on one hand and the fission rate and modified factorial moments on the other. Expressing the S, D and T rates in terms of the cumulants is thus simple, and it has the practical advantage that the relationship to the traditional pulse counting methods becomes transparent, and the known, traditional inversion formulas can be used also in the proposed method.

As a notational convenience, we introduce a function of the detection efficiencies of k detectors as

$$c_k(z_1, \ldots, z_k) = \sum_{i=1}^{k} \varepsilon_i\ (z_i - 1) + 1. \tag{15}$$

One detector To obtain the master equation for the density function $p(y, t)$ of the signal of one detector, the following two mutually exclusive events are accounted for:

1. there will be no source emission on the interval $[0, t)$;
2. there will be a source emission (following other emissions) on the interval $[0, t)$ producing n neutrons, whereafter the detector detects k neutrons and $n - k$ neutrons escape without being detected.

With the above considerations, the master equation reads as

$$\begin{aligned} p(y, t) &= e^{-Q_s t}\, \delta(y) \\ &+ Q_s \int_0^t e^{-Q_s(t-t')} \sum_{n=0}^{\infty} P(n) \sum_{k=0}^{n} \binom{n}{k} \varepsilon^k (1-\varepsilon)^{n-k} \\ &\times \int_0^y u_k(y', t')\, p(y - y', t')\, dy' dt'. \end{aligned}$$

By utilizing (13), for the characteristic function one obtains

$$\begin{aligned} \pi(\omega, t) &= \int_{-\infty}^{\infty} e^{\imath\omega y} p(y, t)\, dy \\ &= e^{-Q_s t} + Q_s \int_0^t e^{-Q_s(t-t')} \sum_{n=0}^{\infty} P(n) \sum_{k=0}^{n} \binom{n}{k} \\ &\times [\chi(\omega, t')]^k\ \varepsilon^k (1-\varepsilon)^{n-k}\, \pi(\omega, t')\, dt' \end{aligned}$$

which, using (2) and (15), can be further simplified to

$$\pi(\omega,t) = \mathrm{e}^{-Q_s t} + Q_s \int_0^t \mathrm{e}^{-Q_s(t-t')}\, G\left[c_1\left(\chi(\omega,t')\right)\right]\,\pi(\omega,t')\,\mathrm{d}t'.$$

The well-known solution of this integral equation is

$$\pi(\omega,t) = \exp\left\{Q_s \int_0^t \left[G\left[c_1\left(\chi(\omega,t')\right)\right] - 1\right]\mathrm{d}t'\right\},$$

from which the logarithm of the characteristic function of the stationary signal can be expressed as

$$\gamma(\omega) = \lim_{t\to\infty} \ln\left[\pi(\omega,t)\right] = Q_s \int_0^\infty \left\{G\left[c_1\left(\chi(\omega,t')\right)\right] - 1\right\}\mathrm{d}t'. \tag{16}$$

As mentioned earlier, since in the above stationary formula, the characteristic function (and other functions of it) are integrated from zero to infinity, a constant, deterministic time delay would lead to the same results as the zero time delay.

The k-th order cumulant of the stationary signal is obtained from the derivatives of (16):

$$\kappa_k = \frac{1}{\iota^k}\left.\frac{\mathrm{d}^{(k)}\gamma(\omega)}{\mathrm{d}\omega^k}\right|_{\omega=0}.$$

Using the explicit form (10) of $\gamma(\omega)$, for the cumulants up to order three one obtains

$$\kappa_1 = F\tilde{\nu}_1\varepsilon\langle a\rangle \int_0^\infty f(t)\,\mathrm{d}t, \tag{17}$$

$$\kappa_2 = F\tilde{\nu}_1\varepsilon\left\langle a^2\right\rangle \int_0^\infty f^2(t)\,\mathrm{d}t + F\tilde{\nu}_2\varepsilon^2\langle a\rangle^2 \int_0^\infty \left[\int_0^\infty f(t-\tau)\,u(\tau)\,\mathrm{d}\tau\right]^2 \mathrm{d}t, \tag{18}$$

and

$$\begin{aligned}\kappa_3 = {} & F\tilde{\nu}_1\varepsilon\left\langle a^3\right\rangle \int_0^\infty f^3(t)\,\mathrm{d}t \\ & + 3F\tilde{\nu}_2\varepsilon^2\langle a\rangle\left\langle a^2\right\rangle \int_0^\infty \left[\int_0^\infty f(t-\tau)\,u(\tau)\,\mathrm{d}\tau \right. \\ & \left. \times \int_0^\infty f^2(t-\tau)\,u(\tau)\,\mathrm{d}\tau\right]\mathrm{d}t \\ & + F\tilde{\nu}_3\varepsilon^3\langle a\rangle^3 \int_0^\infty \left[\int_0^\infty f(t-\tau)\,u(\tau)\,\mathrm{d}\tau\right]^3 \mathrm{d}t.\end{aligned} \tag{19}$$

It is interesting to compare these results with those obtained for the case of no time delay. It is seen from (17) that the first cumulant is insensitive to the presence of any time delay, whether random or deterministic. This is intuitively clear: since the first cumulant is only related to the total number of detections, which is determined by the source intensity and source multiplicity, it does not matter when the individual detections take place. The second cumulant (18) consists of two parts. The first one is the same as in the case of the zero delay case. This is because this term corresponds to the "auto-correlation" part of the signal, when a pulse is correlated with itself. This is seen if the integral expression is rewritten as

$$\int_0^\infty \int_0^\infty f^2(t-\tau)\,u(\tau)\,\mathrm{d}\tau\,\mathrm{d}t \tag{20}$$

which, by noting that $f(t) = 0$ for $t < 0$, using the convolution theorem and further noticing that $u(\tau)$ is a probability density function whose integral is unity, one arrives to the expression in the first term of (18).

The second term corresponds to the "cross-correlation" between two different pulses, which is seen if the integral expression is rewritten as

$$\int_0^\infty \int_0^\infty \int_0^\infty f(t-\tau_1)\,f(t-\tau_2)\,u(\tau_1)\,u(\tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2\,\mathrm{d}t \tag{21}$$

One can show that the dependence of the integrand on τ_1 and τ_2 is reduced to that on $\tau_1 - \tau_2$, hence the expression is invariant to a time shift of the time delay distributions, as is expected on physical grounds. One can also see that, in general, the value of this integral is smaller than that in the first term, due to the fact that pulses arriving in different time do not fully overlap. This also means that if the time delay distribution is not taken into account in the formulas, i.e. the measurement is evaluated with the formulas corresponding to no time delay, the fissile mass will be underestimated.

The expression for the third cumulant, (18), can be interpreted in a similar way. The first term corresponds to the triple "auto-correlation" when a pulse is correlated with itself twice, and hence the integral is independent of the time delay distribution; the second term corresponds to the mixed case when a pulse is correlated with itself and with another (different) pulse; and finally the last term to the case of three different pulses, each with its own time delay.

It is also seen in Equations (18) and (19) that, due to the appearing higher moments, or higher powers of the lower moments of the detector pulse amplitudes, the second and third cumulants suppress the minority components, notably the gamma contributions, similarly to the application of the higher order Campbelling methods. This decreases the possible bias of the method in measurements in a high gamma background.

Equations (17) - (19) above can also be expressed with the multiplet detection rates (6) of the traditional method as

$$\kappa_1 = S\,\langle a\rangle \int_0^\infty f(t)\,\mathrm{d}t, \tag{22}$$

$$\kappa_2 = S\left\langle a^2\right\rangle \int_0^\infty f^2(t)\,\mathrm{d}t + 2D\langle a\rangle^2 \int_0^\infty \left[\int_0^\infty f(t-\tau)\,u(\tau)\,\mathrm{d}\tau\right]^2 \mathrm{d}t, \tag{23}$$

and

$$\begin{aligned}\kappa_3 &= S\left\langle a^3\right\rangle \int_0^\infty f^3(t)\,\mathrm{d}t \\ &+ 6D\langle a\rangle\left\langle a^2\right\rangle \int_0^\infty \left[\int_0^\infty f(t-\tau)\,u(\tau)\,\mathrm{d}\tau\right. \\ &\times \left.\int_0^\infty f^2(t-\tau)\,u(\tau)\,\mathrm{d}\tau\right]\mathrm{d}t \\ &+ 6T\langle a\rangle^3 \int_0^\infty \left[\int_0^\infty f(t-\tau)\,u(\tau)\,\mathrm{d}\tau\right]^3 \mathrm{d}t.\end{aligned} \tag{24}$$

These expressions make it possible to determine the S, D and T rates from the cumulants. It is important to emphasize that the expressions used for calculating the D and T rates also contain the S rate as well as the S and D rates, respectively. As a consequence, the accuracy of the estimation of D and T is burdened by the accuracy of the estimation of the lower order rates.

Two detectors To obtain the master equation for the joint density function $p(y_1, y_2, t)$ of the signals of two detectors, the following two mutually exclusive events are accounted for:

1. there will be no source emission on the interval $[0, t)$;
2. there will be a source emission (following further emissions) on the interval $[0, t)$ producing n neutrons, whereafter the two detectors detect k_1 and k_2 neutrons, respectively, whereas $k_0 = n - k_1 - k_2$ neutrons escape without being detected.

With the above considerations, the master equation reads as

$$\begin{aligned}p(y_1, y_2, t) &= \mathrm{e}^{-Q_s t}\,\delta(y_1)\,\delta(y_2) \\ &+ Q_\mathrm{s}\int_0^t \mathrm{e}^{-Q_\mathrm{s}(t-t')}\sum_{n=0}^\infty P(n)\sum_{k_0+k_1+k_2=n} \\ &\times \frac{n!}{k_0!\,k_1!\,k_2!}\,\varepsilon_1^{k_1}\varepsilon_2^{k_2}(1-\varepsilon_1-\varepsilon_2)^{k_0} \\ &\times \int_0^{y_1}\int_0^{y_2} u_{k_1}(y_1', t-t')\,u_{k_2}(y_2', t-t') \\ &\times p(y_1-y_1', y_2-y_2', t')\,\mathrm{d}y_1'\mathrm{d}y_2'\mathrm{d}t',\end{aligned}$$

By utilizing (13), for the characteristic function one obtains

$$\begin{aligned}\pi(\omega_1,\omega_2, t) &= \int_{-\infty}^\infty\int_{-\infty}^\infty \mathrm{e}^{\imath(\omega_1 y_1+\omega_2 y_2)}\,p(y_1, y_2, t)\,\mathrm{d}y_1\mathrm{d}y_2 \\ &= \mathrm{e}^{-Q_s t} + Q_\mathrm{s}\int_0^t \mathrm{e}^{-Q_\mathrm{s}(t-t')}\sum_{n=0}^\infty P(n)\sum_{k_0+k_1+k_2=n} \\ &\times \frac{n!}{k_0!\,k_1!\,k_2!}\,(1-\varepsilon_1-\varepsilon_2)^{k_0}\,\varepsilon_1^{k_1}\varepsilon_2^{k_2} \\ &\times [\chi_1(\omega_1, t')]^{k_1}\,[\chi_2(\omega_2, t')]^{k_2}\;\pi(\omega_1,\omega_2, t')\,\mathrm{d}t',\end{aligned}$$

which, using (15), can be further simplified to

$$\begin{aligned}\pi(\omega_1,\omega_2, t) &= \mathrm{e}^{-Q_s t} + Q_\mathrm{s}\int_0^t \mathrm{e}^{-Q_s(t-t')} \\ &\times G_2[c_2(\omega_1,\omega_2, t')]\,\pi(\omega_1,\omega_2, t')\,\mathrm{d}t',\end{aligned} \tag{25}$$

The solution of this integral equation is

$$\begin{aligned}&\pi(\omega_1,\omega_2, t) = \\ &\exp\left\{Q_\mathrm{s}\int_0^t \left[G\left[c_2\left\{(\chi_1(\omega_1, t'), \chi_2(\omega_2, t')\right\}\right] - 1\right]\,\mathrm{d}t'\right\},\end{aligned}$$

from which the logarithm of the characteristic function of the stationary signals can be expressed as

$$\begin{aligned}\gamma(\omega_1,\omega_2) &= \lim_{t\to\infty}\ln[\pi(\omega_1,\omega_2, t)] \\ &= Q_\mathrm{s}\int_0^\infty \left\{G\left[c_2\left\{(\chi_1(\omega_1, t'), \chi_2(\omega_2, t')\right\}\right] - 1\right\}\,\mathrm{d}t'.\end{aligned} \tag{26}$$

The (k_1, k_2)-th order cumulant of the stationary signals is then obtained from the derivatives of (26):

$$\kappa_{k_1,k_2} = \frac{1}{\imath^{(k_1+k_2)}}\left.\frac{\partial^{(k_1+k_2)}\gamma(\omega)}{\partial\omega_1^{k_1}\,\partial\omega_2^{k_2}}\right|_{\omega_1=\omega_2=0}.$$

In particular, using (10), for the cumulants up to order three one obtains

$$\begin{aligned}\kappa_{1,1} &= F\tilde{\nu}_2\varepsilon_1\varepsilon_2\langle a_1\rangle\langle a_2\rangle \int_0^\infty\int_0^\infty f_1(t-\tau)\,u_1(\tau)\,\mathrm{d}\tau \\ &\times \int_0^\infty f_2(t-\tau)\,u_2(\tau)\,\mathrm{d}\tau\,\mathrm{d}t\end{aligned} \tag{27}$$

and

$$\begin{aligned}\kappa_{2,1} &= F\tilde{\nu}_2\varepsilon_1\varepsilon_2\left\langle a_1^2\right\rangle\langle a_2\rangle \int_0^\infty\int_0^\infty f_1^2(t-\tau)\,u_1(\tau)\,\mathrm{d}\tau \\ &\times \int_0^\infty f_2(t-\tau)\,u_2(\tau)\,\mathrm{d}\tau\,\mathrm{d}t + F\tilde{\nu}_3\varepsilon_1^2\varepsilon_2\langle a_1\rangle^2\langle a_2\rangle \\ &+ \int_0^\infty\left[\int_0^\infty f_1(t-\tau)\,u_1(\tau)\,\mathrm{d}\tau\right]^2 \\ &\times \int_0^\infty f_2(t-\tau)\,u_2(\tau)\,\mathrm{d}\tau\,\mathrm{d}t.\end{aligned} \tag{28}$$

Considering the case, when both detectors are characterized by the same parameters, Equations (27)–(28) can also be expressed with the multiplet detection rates (6) of the traditional method as

$$\kappa_{1,1} = 2D\langle a\rangle^2 \int_0^\infty\left[\int_0^\infty f(t-\tau)\,u(\tau)\,\mathrm{d}\tau\right]^2 \mathrm{d}t \tag{29}$$

and

$$\begin{aligned}\kappa_{2,1} &= 2D\left\langle a^2\right\rangle\langle a\rangle \int_0^\infty\int_0^\infty f^2(t-\tau)\,u(\tau)\,\mathrm{d}\tau \\ &\times \int_0^\infty f(t-\tau)\,u(\tau)\,\mathrm{d}\tau\,\mathrm{d}t \\ &+ 6T\langle a\rangle^3 \int_0^\infty\left[\int_0^\infty f(t-\tau)\,u(\tau)\,\mathrm{d}\tau\right]^3 \mathrm{d}t.\end{aligned} \tag{30}$$

These expressions make it possible to determine the D and T rates from the cumulants. Unlike the corresponding formulas (23)–(24) for one detector, they do not contain the S rate. Hence, the accuracy of the calculated values of D and T will no longer be burdened by the accuracy of S.

Three detectors To obtain the master equation for the joint density function $p(y_1, y_2, y_3, t)$ of the signals of three detectors, the following two mutually exclusive events are accounted for:

1. there will be no source emission on the interval $[0, t)$;
2. there will be a source emission (following further emissions) on the interval $[0, t)$ producing n neutrons, whereafter the three detectors detect k_1, k_2 and k_3 neutrons, respectively, whereas $k_0 = n - k_1 - k_2 - k_3$ neutrons escape without being detected.

With the above considerations, the master equation reads as

$$\begin{aligned}
p(y_1, y_2, y_3, t) &= \mathrm{e}^{-Q_s t} \prod_{i=1}^{3} \delta(y_i) + Q_s \int_0^t \mathrm{e}^{-Q_s(t-t')} \\
&+ \sum_{n=0}^{\infty} P(n) \sum_{k_0+\cdots+k_3=n} \frac{n!}{k_0!\ \prod_{i=1}^{3} k_i!} \left(1 - \sum_{i=1}^{3} \varepsilon_i\right)^{k_0} \prod_{i=1}^{3} \varepsilon_i^{k_i} \\
&\times \int_0^{y_1}\int_0^{y_2}\int_0^{y_3} \prod_{i=1}^{3} u_{k_i}(y_i', t-t') \\
&\times p(y_1 - y_1', y_2 - y_2', y_3 - y_3', t')\, \mathrm{d}y_1'\mathrm{d}y_2'\mathrm{d}y_3'\mathrm{d}t',
\end{aligned}$$

By utilizing (13), for the characteristic function one obtains

$$\begin{aligned}
\pi(\omega_1, \omega_2, \omega_3, t) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{\imath(\omega_1 y_1 + \omega_2 y_2 + \omega_3 y_3)}\, \mathrm{d}y_1\mathrm{d}y_2\mathrm{d}y_3 \\
&= \mathrm{e}^{-Q_s t} + Q_s \int_0^t \mathrm{e}^{-Q_s(t-t')} \sum_{n=0}^{\infty} P(n) \sum_{k_0+\cdots+k_3=n} \\
&\times \frac{n!}{k_0!\ \prod_{i=1}^{3} k_i!} \left(1 - \sum_{i=1}^{3} \varepsilon_i\right)^{k_0} \prod_{i=1}^{3} \left[\varepsilon_i \chi_i(\omega_i, t')\right]^{k_i} \\
&\times\ \pi(\omega_1, \omega_2, \omega_3, t')\, \mathrm{d}t',
\end{aligned}$$

which, using (15), can be further simplified to

$$\begin{aligned}
\pi(\omega_1,\omega_2, \omega_3, t) &= \mathrm{e}^{-Q_s t} + Q_s \int_0^t \mathrm{e}^{-Q_s(t-t')} \\
&\times G[c_3 \{(\chi_1(\omega_1, t'), \chi_2(\omega_2, t'), \chi_3(\omega_3, t')\}] \\
&\times \pi(\omega_1, \omega_2, \omega_3, t')\, \mathrm{d}t',
\end{aligned} \tag{31}$$

The solution of this integral equation is

$$\begin{aligned}
\pi(\omega_1, \omega_2, \omega_3, t) &= \exp\Bigg\{ Q_s \\
&\times \int_0^t [G_3 [c_3 \{(\chi_1(\omega_1, t'), \chi_2(\omega_2, t'), \chi_3(\omega_3, t')\}] - 1]\, \mathrm{d}t' \Bigg\},
\end{aligned}$$

from which the logarithm of the characteristic function of the stationary signals can be expressed as

$$\begin{aligned}
\gamma(\omega_1, \omega_2, \omega_3) &= \lim_{t\to\infty} \ln[\pi(\omega_1, \omega_2, \omega_3, t)] = Q_s \int_0^{\infty} \\
&\times \{G_3 [c_3 \{(\chi_1(\omega_1, t'), \chi_2(\omega_2, t'), \chi_3(\omega_3, t')\}] - 1\}\, \mathrm{d}t'.
\end{aligned} \tag{32}$$

The (k_1, k_2, k_3)-th order cumulant of the stationary signals is then obtained from the derivatives of (32):

$$\kappa_{k_1,k_2,k_3} = \frac{1}{\imath^{(k_1+k_2+k_3)}} \left.\frac{\partial^{(k_1+k_2+k_3)}\gamma(\omega)}{\partial\omega_1^{k_1}\,\partial\omega_2^{k_2}\,\partial\omega_3^{k_3}}\right|_{\omega_1=\omega_2=\omega_3=0}.$$

In particular, using (10), for the only cumulant of order three one obtains

$$\begin{aligned}
\kappa_{1,1,1} &= F\tilde{\nu}_3\, \varepsilon_1\varepsilon_2\varepsilon_3\, \langle a_1\rangle\langle a_2\rangle\langle a_3\rangle \int_0^{\infty} \\
&\times \int_0^{\infty} f_1(t-\tau)\, u_1(\tau)\, \mathrm{d}\tau \int_0^{\infty} f_2(t-\tau)\, u_2(\tau)\, \mathrm{d}\tau \\
&\times \int_0^{\infty} f_3(t-\tau)\, u_3(\tau)\, \mathrm{d}\tau\, \mathrm{d}t
\end{aligned} \tag{33}$$

Considering the case when all three detectors are characterized by the same parameters, Equation (33) can also be expressed with the third multiplet detection rate (6) of the traditional method as

$$\kappa_{1,1,1} = 6T\, \langle a\rangle^3 \int_0^{\infty} \left[\int_0^{\infty} f(t-\tau)\, u(\tau)\, \mathrm{d}\tau\right]^3 \mathrm{d}t. \tag{34}$$

This expression makes it possible to determine the T rate from the cumulant. Unlike the corresponding formula (30) for two detectors, it does not contain the D rate. Hence, the accuracy of the calculated value of T will not be burdened by the accuracy of any lower order detection rate.

III. VERIFICATION

In order to verify the new detection model presented in Section B., specific values were selected for the parameters characterizing the emission and detection processes (discussed in Section II.). In the possession of these values the corresponding statistics of the emission and detection can be obtained in two ways:

- With theoretical calculations, using the proper formulas of Section II. The singles, doubles and triples detection rates can be obtained from Equations (6), after disregarding the gate fractions which are relevant only in the pulse counting based multiplicity measurements. The low order cumulants of the detector current might be calculated from Equations (17)–(19), (27)–(28) and (33).
- With the simulation of measurements in an experimental setup based on the theoretical model of Section II. Analyzing the simulated detector signals directly, estimated values of their cumulants can be obtained. When the detectors have identical properties, also the values of the singles, doubles and triples rates can be estimated using Equations (22)–(24), (29)–(30) and (34).

By comparing the calculated and simulated values, the correctness of the derivations presented in Section II. can be checked.

The following parameters of the emission and detection processes were selected. The source was characterized by an intensity $Q_\mathbf{s} = 10^8\ \mathrm{s}^{-1}$ and by an emission number distribution $P(n)$ given in Table I. All the detectors had the same detection efficiency $\varepsilon = 0.3$, and each neutron was detected independently by one of the detectors after a Gamma$(2, 5\cdot10^7)$ distributed time delay τ with a corresponding probability density function

$$u(\tau) = \beta^2\,\tau\,\mathrm{e}^{-\beta\tau} \qquad \text{for} \quad \beta = 5\cdot10^7\ \mathrm{s}^{-1}; \tag{35}$$

this resulted in an expected time delay $\langle\tau\rangle = 4\cdot10^{-8}$ s. The response of each detector was characterized by an exponentially decaying random pulse

$$f(t) = a\,\mathrm{e}^{-\alpha t} \qquad \text{for} \quad t \geq 0 \tag{36}$$

with a time constant $\alpha = 10^8\ \mathrm{s}^{-1}$ and a Gamma$(2, 1)$ distributed random amplitude a with a corresponding probability density function

$$w(a) = a\,\mathrm{e}^{-a}. \tag{37}$$

TABLE I. Emission number distribution of the simulated neutron source.

n	0	1	2	3	4	5
$P(n)$	0.03	0.20	0.35	0.35	0.05	0.02

1. Performing Simulations

For the simulation of measurements, a simple Monte Carlo tool was prepared with the Python programming language [11]. The tool consists of three programs responsible for the following subtasks:

1. generating a sequence of detection times, by simulating the source emission as well as the detection process, including the random time delay of the individual neutrons arising from one source event;
2. calculating a time-resolved detector signal, by producing the detector response for each detection event;
3. estimating the first few cumulants of the time-resolved signal, using their unbiased estimators.

In order to characterize the uncertainty of the cumulants, the entire process was repeated 100 times using the same parameters, from which the average value of the cumulants as well as their standard deviations were obtained.

In the following, the above listed three steps are discussed in more detail.

A. Generation of Detection Times

Given a neutron source and an arbitrary number of detectors, a program was created to simulate the emission and detection of neutrons, in order to produce a sequence of detection times for each detector in the system.

The actual simulation was performed using three detectors, as illustrated in Figure 1. The detection process was simulated for a measurement time $T = 10^{-3}$ s hence, with the applied source strength Q_s, the simulation comprised an average of $T\cdot Q_\mathrm{s} = 10^5$ source emission events. In practical applications, however, much longer measurement times are advisable to be used.

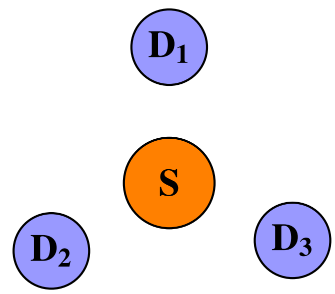

Fig. 1. A conceptual scheme of the simulated experiment. A neutron-emitting source (S) is surrounded by three identical detectors (from D_1 to D_3).

B. Production of Detector Signals

Given a sequence of detection times corresponding to a single detector, a second program was created to produce a time-resolved detector signal by calculating the detector responses to the individual detection events.

For each detection at time t_i a pulse $f(t-t_i)$ was generated for $t \geq t_i$ using a time resolution $\Delta t = 10^{-11}$ s, and the complete signal of the detector was calculated as a sum of these individual pulses. Considering the selected values of the time resolution Δt and the measurement time T, the final signals consisted of approximately $T/\Delta T = 10^8$ points. A short sample of such a signal is shown in Figure 2.

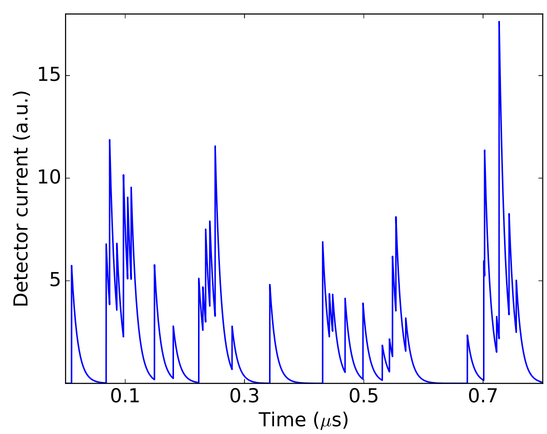

Fig. 2. An example of a generated detector signal comprising of the sum of exponentially decaying pulses.

C. Estimation of Signal Cumulants

Given the time-resolved signals of several detectors, a third program was created to estimate the auto as well as cross-cumulants of the signals.

Only the orders of the cumulants discussed in Section B. were considered, and their actual values were calculated using their unbiased estimators, the k-statistics [12].

2. Comparing the Results

Table II shows a comparison of the cumulants obtained from analyzing the simulated signals as well as from calculations using the theoretical formulas proposed in Section B.. In order to better illustrate the temporal separation effect for the neutrons, the theoretical calculations were performed by both considering and neglecting the specified random time delay between the emission and the detection; these two cases will be referred to as delayed detection and instant detection. The first two columns identify the actual cumulants displayed in a particular row of the table. The third and fourth columns contain the theoretically calculated values for the instant and delayed detection cases, respectively. In the fifth column, the simulated values are displayed with their relative uncertainties at a 95% confidence level. Regarding the presented data, the following observations can be made:

- Whereas the theoretical value of the first order cumulant is insensitive to the presence of any time delay, the higher order cumulants, on the other hand, show a lower value in the delayed case. This behavior is expected based on the discussion of the cumulant expressions in Section B..
- The values of the simulated cumulants (obtained by taking into account the random time delay between the neutron emission and detection) can only be predicted with the formulas corresponding to the delayed case, which is a rather obvious conclusion.
- Higher order simulated cumulants tend to have higher relative uncertainties in general; within the same order, the cumulants corresponding to more detectors also have higher relative uncertainties. This is a general property of the *k*-statistics, used for estimating the cumulants [12].

TABLE II. Comparison of the cumulants obtained from analyzing simulated detector signals as well as from theoretical calculations. The theoretical values are presented for the cases when the the random time delay is neglected (instant) or considered (delayed). The relative uncertainties of the simulated values are given at a 95% confidence level.

order	type	theory		simulation
		instant	delayed	
1	1	1.350	1.350	$1.350 \pm 0.06\%$
2	2	2.709	2.177	$2.180 \pm 0.10\%$
	1,1	0.684	0.152	$0.152 \pm 0.58\%$
3	3	9.828	6.116	$6.125 \pm 0.20\%$
	2,1	1.692	0.248	$0.249 \pm 1.23\%$
	1,1,1	0.324	0.014	$0.014 \pm 13.48\%$

Table III shows a comparison of the singles, doubles and triples detection rates obtained from "pure" theoretical calculations and from the simulated cumulants presented in Table II. The third column contains the type of the rate displayed in a particular row. In the fourth and fifth column, the theoretical and simulated values of the rates are presented in the units of $10^6\,\mathrm{s}^{-1}$, with the relative uncertainties of these latter at a 95% confidence level. The first two columns identify the cumulants from which the simulated values were obtained. It is important to note that in some cases, as suggested by the formulas of Section B., in order to calculate the value of a rate, the calculated values of the lower order rates needs to be used (e.g. to calculate the doubles rate D from κ_2, the value of the singles rate S determined from κ_1 was used). Regarding the presented data, the following observations can be made:

- The simulated values of the detection rates shows a good agreement with the theoretical predictions. This is expected from the good agreement between the simulated and theoretical cumulants.
- Higher order simulated detection rates tend to have higher relative uncertainties due to the higher uncertainty of the cumulants involved in their calculation. Within the same order, the detection rates calculated from cumulants corresponding to less detectors have higher uncertainties, since the uncertainties from the lower order rates are carried over in their calculation.

TABLE III. Comparison of the simulated and theoretically calculated values of the singles (S), doubles (D) and triples (T) detection rates obtained from cumulants of different orders. The values are presented in units of $10^6\,\mathrm{s}^{-1}$. The uncertainties of the simulated values are given at a 95% confidence level.

order	type	rate	theory	simulation
1	1	S	67.500	$67.503 \pm 0.06\,\%$
2	2	D	17.100	$17.410 \pm 1.62\,\%$
	1,1			$17.091 \pm 0.58\,\%$
3	3	T	2.025	$3.301 \pm 58.73\,\%$
	2,1			$2.108 \pm 22.79\,\%$
	1,1,1			$1.997 \pm 13.48\,\%$

IV. CONCLUSIONS

An alternative method has been proposed recently for the determination of sample parameters from fast neutron measurements using fission chambers operating in current mode. The mathematical model of the original proposal assumed that the emitted neutrons are detected instantly which, however, does not hold for practical cases. To overcome this limitation, a new, extended model has been developed and presented accounting for a random time delay between the emission and subsequent detection of neutrons; the considered delay was independent and identically distributed for all neutrons. Expressions have been derived for the first low order auto and cross cumulants of the detector signals. It has been shown, that when all the detectors are identical (as in the mathematical model of the traditional pulse counting method), the cumulants can also be expressed with the traditional singles, doubles and triples detection rates.

In order to verify the newly proposed theory, specific values have been chosen for the parameters of the source and the detectors. Using these values, simulations have been performed to produce detector signals. The signal cumulants and the traditional detection rates could then be determined both from the theoretical formulas and by analyzing the simulated signals, and their values were compared. It has been shown that the presence of the random time delay between the emission and detection has a significant effect on the values of the cumulants, and the estimated cumulants were properly described only by the formulas accounting for the delay. It has been further demonstrated, that the values of the singles, doubles and triples detection rates can still be determined from the estimated cumulants of the simulated signals.

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