Spectral X-ray Radiography for Safeguards at Nuclear Fuel Fabrication Facilities: A Feasibility Study

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Abstract - Accurate, noninvasive accounting of total uranium and uranium-235 concentration at fuel fabrication facilities continues to present a challenging research problem. The methods used by the International Atomic Energy Agency are time consuming and require in-field chemistry, with operation by experts. Spectral X-ray radiography, along with advanced inverse algorithms, is an alternative measurement that could be completed noninvasively, without any in-field chemistry, and with verification measurements completed in minutes. The proposed measurement system and algorithms are presented here for the quantification of total uranium mass. The inverse algorithm uses total variation regularization and adaptive regularization parameter selection with the unbiased predictive risk estimator. Performance of the system is quantified with simulated X-ray data and sensitivity of the output is tested against various measurement system instabilities as well as inhomogeneities in the fuel powder density. Initial findings on the achievable accuracy and precision of this spectral X-ray method indicate strong potential to meet or exceed IAEA’s targets for quantification of uranium oxide samples in the field.

I. INTRODUCTION

The in-field verification measurement technology used by the International Atomic Energy Agency (IAEA) to ensure accurate accounting of nuclear material at fuel fabrication facilities is typically the Combined Procedure for Uranium Concentration and Enrichment Assay (COMPUCEA) [1]. Hybrid K-Edge Densitometry (HKED) is another technique in use by the IAEA, which can measure uranium concentration in samples, but is typically used in reprocessing plants and is installed in a facility permanently [2]. These methods determine uranium concentration by observing the X-ray transmission around the L- and K-edge of the X-ray attenuation coefficients, respectively, and both result in the destruction of the samples via a chemical preparation process. These methods require carefully controlled dissolution chemistry and time-intensive data acquisition on the order of 1000s of seconds. A field-deployable technology that can accurately and nondestructively determine both total uranium mass and uranium-235 relative concentration, while reducing data acquisition times by an order of magnitude, would significantly improve the efficiency and reduce the complexity of IAEA safeguards approaches at fuel fabrication facilities. Pacific Northwest National Laboratory is exploring the viability of spectral X-ray radiography techniques, based on measurement of the transmitted X-ray spectrum, to meet this challenge in international safeguards.

In prior work not related to the uranium oxide assay application, inverse algorithms for X-ray material discrimination were developed for use with data from high-rate spectral X-ray radiography detectors. This was pursued to explore how multi-modal imaging (images that yield structure and material composition data) might be useful as attributes or in templates in an arms control verification context [3]. The key development in the previous work was development of an efficient, robust, and adaptive inverse algorithm to determine material composition from data obtained from spectral X-ray detectors. The algorithms developed showed excellent performance in discriminating and quantifying high-atomic-numbered materials even in layered configurations [4]. Performance was found to be particularly good when the object contained only three or four expected materials, consistent with what would be expected in encapsulated fuel powder verification measurements at nuclear fuel fabrication facilities. Of particular importance was the development of a method to optimally select a parameter in the algorithm that results in accurate material estimations [4]. The adaptive nature of these algorithms could be valuable for IAEA deployed systems since they wouldn’t require expert operators. This is a key improvement over the COMPUCEA and HKED systems, which require operation by expert technicians or inspectors.

Here, we present a model-based feasibility study of the application of inverse algorithms to the noninvasive quantification of total uranium content in fuel powders at fuel fabrication facilities. Spectral X-ray assay was simulated in MCNP [5] and assumes a nominal detector response for a cadmium telluride X-ray detector. The inverse algorithms were applied to the simulated data to determine quantities of uranium (U), oxygen (O), and gadolinium (Gd) noninvasively. We show that material parameters (i.e., mass of select elements) determined from these preliminary explorations exhibit levels of uncertainty similar to those provided by today’s IAEA methods in the field. In order to test the robustness of the algorithm to realistic instabilities in a deployed verification measurement system, the sensitivity of the material estimations to expected measurement system variations was determined.

II. METHODS

The response of a spectral transmission X-ray system to an arbitrary object can be described using a modified form of Beer’s law, paired with a detector response model $S(E)$ [6]:

$$d(\rho, E_{\text{bin}}, t) = \int_0^\infty S_i(E)\phi_0(E) \exp \left[ -\sum_k \mu_k(E)\rho_k \right] dE. \quad (1)$$
Here, \( d \) is the detector output for an energy bin \( \ell \) and \( S_{\ell} \) is the energy-dependent bin sensitivity of energy bin \( \ell \). The X-ray flux at the image plane is \( \phi, \phi_0 \) is the unattenuated flux, \( \mu_k \) is the mass attenuation coefficient of the \( k \)'th material and \( \rho_k \) is its areal density in a given pixel. The density vector \( \rho \) indicates that the transmitted flux depends on a vector of any number of materials between the X-ray source and the detector.

From Eq. (1), known assay-system parameters, and sample material composition, an expected output from a spectral X-ray radiography system can be calculated. This is the forward problem. The inverse problem is to determine the material composition from the data output of the radiography system. For a typical problem, this can be done using simple least-squares fitting. However, since the material attenuation coefficients are not strongly unique (other than around the K-edges), e.g., Fig. 2, it has been shown that least-squares solutions are quite sensitive to image data noise. Therefore, regularization was required to accurately determine material composition [4] by minimizing the cost function \( F(\rho) \).

\[
F(\rho) = \frac{1}{2} \left\| \frac{d_{\rho}(\rho) - d_{\text{obs}}}{\sqrt{d_{\text{obs}}}} \right\|^2 + \alpha \sum \sqrt{(D_i \rho)^2 + (D_j \rho)^2 + \beta}.
\] (2)

Here, the first, least-squares term comparing the modeled detector output \( d(\rho) \) to the observed data \( d_{\text{obs}} \) was normalized by the square root of the forward problem calculation from the previous iteration, \( n - 1 \). This was done so that the variance in the high-count (e.g., background) pixel data does not overweight the norm, and to simplify the regularization parameter \( \alpha \) selection method. The density vector \( \rho \) was expanded to include all materials of interest for all image pixels. Accordingly, the vector \( d \) contains the data for all detector pixels and energy bins. The minimization of \( F \) was done over this entire vector.

The second term was the total variation regularization, which smooths the solution by penalizing for sharp variations between neighboring pixels. This was weighted by \( \alpha \), which varies the strength of the regularization. The operator matrices \( D_i \) and \( D_j \) determine the backwards finite difference of the \( \rho \) vector in the row \( i \) and column \( j \) dimension of the image data, respectively [7]. These operator matrices were defined so that the variation across image borders or between the different materials is not taken into account in the calculation of the total variation. The term \( \beta \) ensures that the function is differentiable so that analytical first and second derivatives can be found for optimization. This term was set to unity in this study.

The minimum of Eq. (2) was found using the Gauss-Newton algorithm, with the application of a non-negativity constraint using the projected gradient, reduced Hessian algorithm [8]. The Gauss-Newton algorithm is iterative and approximates \( F \) as quadratic at each iteration and minimizes that approximation. To accomplish this, the first derivative (gradient) and second derivative (Hessian), or an approximation thereof, is required. The Gauss Newton algorithm uses the Jacobian of the misfit term to approximate its second derivative \( H \), i.e., \( H \approx J^T J \). Though Newton-type algorithms are not resistant to falling into local minima, it was found this wasn’t a problem in practice.

The value for \( \alpha \) was selected adaptively within the optimization algorithm using the unbiased predictive risk estimator (UPRE) method, as in [4]. The predictive risk \( p_\alpha \) is defined as

\[
p_\alpha = d(\rho_\alpha) - d(\rho_{\text{true}})
\] (3)

where, \( \rho_\alpha \) is the regularized density vector solution from the algorithm and \( \rho_{\text{true}} \) is the true density vector. Since the true density is not known, the UPRE, a statistical estimator of \( p_\alpha \), was minimized instead. The UPRE was minimized over \( \log(\alpha) \) to a tolerance of 0.1.

The algorithm was implemented in Matlab (Mathworks, Inc., Natick, MA).
1. Fuel powder verification measurement

Nuclear fuel powder is produced in fuel fabrication facilities before being sintered into nuclear fuel pellets. This offers an opportunity to inspect a lower-density sample (~3 g/cm³), where the attenuation around the K-edges of the sample can be observed with typical X-ray radiography systems. In this work it was assumed that the uranium oxide (UO₂) powder to be assayed was placed in a thin-walled plastic or quartz container. The average density of the powder was well below the theoretical density because there was no specific effort made to pack or compact the powder. The density of the simulated UO₂ fuel powders here was 3 g/cm³, with an option for Gd-loading (Gd₂O₃) of 5 w/o. Fuel powder thicknesses of 1, 3, and 5 mm were simulated to explore if there is an optimal thickness that balances sufficient X-ray transmission and counting statistics with a material thickness sufficient to produce strong contrast in the K-edge signal.

Two types of fuel powder objects were simulated. The first assumes a uniform mixture of fuel powder such that there is no variation in powder density within the object. This offers a best-case with which to compare more realistic simulations. The second object includes more realistic density variation where the density has a 20% or 40% variation around the nominal 3 g/cm³ density, either normally or uniformly distributed, as shown in Fig. 1 for the 100-117 keV energy window. This simulates the expected variation that could be observed in a loosely tamped powder sample and was simulated by splitting the fuel volume into right parallelepipeds of dimension 0.3 × 0.1 × 0.1 cm³.

The X-ray source was a cone 300 kVp Bremsstrahlung source with a half-angle of 8.1°, 11 cm from the middle of the inspected object. The source spectrum was simulated in MCNP by transporting electrons onto a tungsten target and tallying the photon production. The X-ray flux was determined from a radiography tally at 20 cm from the middle of the object. The radiography tally was a pixel array of 80 x 80 pixels² with 0.1 cm pixel pitch. An example of the radiograph is shown in Fig. 1, for a single energy range on the cadmium telluride detector. Noise was added to these results assuming a maximum count of 10⁸ in the background pixels, which corresponds to a data acquisition time of minutes.

The detector response $S(E)$ in Eq. 1 was modeled in MCNP for a 5-mm thick cadmium telluride detector by tallying the energy deposition in a multi-pixel detector medium. This response $S(E)$ indicates the probability that an incident X-ray of a given energy will be detected in an energy bin $E$ and shows similarity with experimentally determined response functions [6]. The simulated detector had 6 energy bins, placed to observe the material specific K-edges in the attenuation coefficients $\mu$ and the low-energy regions, where $\mu$ shows greater uniqueness for varying materials, as shown in Fig. 2. The material attenuation coefficients $\mu$ (cm²/g) were obtained from MCNP, which uses the ENDF cross-section database.

The inverse algorithm requires a set of materials to search for in the data, determined from the user-provided set of material attenuation coefficients $\mu$. This allows flexibility to search for any number of materials in the data, though increasing the number of materials also increases the ill-posedness (instability) of the problem. Nonetheless, accuracy in quantifying three or four materials has been demonstrated previously [4]. Here, a three-material set of {uranium, oxygen, gadolinium} was used for the uniform fuel powder verification measurement, to allow flexibility to possible gadolinium loading, which is typical in nuclear fuel fabrication. For the density varying simulation, a two-material set was used {uranium, oxygen}, which reduces the ill-posedness of the problem. This was done since it was expected that the varying density estimations would present a more difficult problem, though it was subsequently found that a three-material set could also be used here.

Errors on material composition estimated from the algorithm were determined with the relative error e, where the material estimation vector $\rho_{est,k}$ for material $k$ is compared to the true $\rho_{true,k}$.

$$e_k = \frac{\rho_{est,k} - \rho_{true,k}}{\rho_{true,k}}$$

The mean and standard deviation of the $e$ vector are reported as measures of success of the algorithm material estimations. The results include material estimations from 20 independent noise realizations of the image data ignoring a 5-pixel-wide border of the object, to avoid any edge artefacts introduced by the total variation regularization. This results in the $e$ vector containing >34,000 elements (i.e., 20 realizations × 1716 pixels) over which the mean and standard deviations were calculated. When $\rho_{true,k} = 0$, such as when a material estimated from the algorithm was not actually present, the denominator in Eq. (4) was set to 1.

For the object simulations that have density variation, the error reported is on the total mass determined from the algorithm compared to the actual for each material. Here, Eq. (4) was modified to consider the scalar total estimated mass and actual total mass. Notably, the thickness or density of the bulk material need not be assumed for calculation of material mass since the algorithm determines material areal density (g/cm²), which can be used to calculate the mass directly with a known pixel size and imaging geometry.

2. Verification measurement system variation

Beyond the expected variability in the fuel powder sample density that was introduced in the previous section, it was expected there will be other variabilities in the measurement system, e.g., in the source spectrum or detector response. These variabilities will also affect the estimated material composition since it will affect the accuracy of the detector response $S(E)$ and input spectrum $\phi(E)$ in the forward model, Eq. (1).

Three verification measurement system variations were selected based on what are expected to be the most significant in a deployed system: 1) Bremsstrahlung X-ray spectrum endpoint, 2) energy resolution, as measured by the full-width at half maximum (FWHM), of each detector pixel and 3) detector pixel gain. Examples of these system variations are shown in Fig. 3.

Figure 3a shows how inaccuracy in the Bremsstrahlung X-ray spectrum endpoint will affect the energy spectrum that is used to inspect the object. In particular, an increased endpoint...
© 1% with a standard deviation of 0.4 keV for a 7.0 keV FWHM at 60 keV. The second, shown in Fig. 3b, was the pixel gain, which will cause sharper energy bin boundaries with lower FWHM. The simulated FWHM also varies with energy such that

\[
\text{FWHM}(E) = b + 0.04E
\]

where \( b \) is defined based on a nominal FWHM, e.g., \( b \approx 4.6 \) keV for a 7.0 keV FWHM at 60 keV. The second, shown in Fig. 3b, was the pixel gain, which will extend or compress the energy bins on the detector due to an over or underestimate on the gain, respectively. Notably, both the pixel gain and pixel FWHM can vary across the pixels of the detector so that the gain or FWHM may not be the same between neighboring pixels.

The level of these variations expected in the measurement system, based on device documentation and literature [9, 10], are shown in Table I.

<table>
<thead>
<tr>
<th>Endpoint energy</th>
<th>Nominal variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pixel gain</td>
<td>1% (~2.3 keV)</td>
</tr>
<tr>
<td>Pixel FWHM</td>
<td>0.4 keV</td>
</tr>
</tbody>
</table>

Table I: Approximate variations that could be expected from a typical spectral X-ray verification measurement system.

shift affects all pixels on the detector in the same way. In contrast, the detector response may vary from one pixel to the next. Therefore, the variation in detector gain and FWHM was simulated by sampling these parameters pixel-by-pixel from a normal distribution with 0 mean and few standard deviations near those shown in Table I.

In order to isolate the sensitivity to each given measurement system variation, the simulated measurement case used for these studies was the 3-mm thick UO2 fuel density, without image noise. A two-material set (uranium, oxygen) was used here. The sensitivities were tested with \( \alpha = 10^{3.7} \), which was found to be optimal according to the UPRE parameter selection method.

III. RESULTS AND ANALYSIS

Results from the inverse algorithm are shown in Fig. 4 for the uniform 3-mm material thickness and the relative error of the estimations, Eq. (4), are summarized in Table II. The results show bias on the mean estimated uranium content of \( \pm 0.01% \) with a standard deviation of \( < 0.1% \) for all cases. Importantly, the algorithm can also determine gadolinium (Gd) content directly with a minimal impact on the uranium accuracy, allowing greater flexibility for measurement of typical fuel compositions.

Results from the variable density simulations are shown in Fig. 5 for the 20% normally distributed and 40% uniformly distributed density cases. These results are also summarized in Table III. Here, the relative error was reported on the total mass determined from the algorithm compared to the true value in the simulated sample. The effective mass can be calculated from the algorithm output by multiplying the pixel-wise material estimations in g/cm² by the area projected in the sample plane on the detector plane. Given the simulated image geometry in this work the 0.1 × 0.1 cm² pixel area was 0.038 × 0.038 cm² in the object plane. Note, since the object has a finite thickness of 0.3 cm this projected area was the average of the projected area from the front and back of the fuel powder volume.
The results show bias on the mean estimated uranium mass of less than 0.2% with a standard deviation of less than 0.002%, even with a large density deviation of 40%. The 40% density variation may overestimate that which will be seen in practice. The bias on the material estimation was likely due to the X-rays passing through multiple material density voxels, as they may be approaching the sample at some small angle, due to the divergence of the beam. Also, the standard deviation was lower here since the mass result is essentially a sum of the density estimations from a number of pixels, effectively cancelling out the positive and negative variations seen in the material estimations from pixel to pixel.

Sensitivity of the estimated uranium estimation with respect to variations in the measurement system are shown in Table IV. The variation levels shown are the mean and standard deviation of the normal distribution sampled to simulate the given variation. The uranium error mean and standard deviations quantify both the bias and degradation in precision of the measurement from these system variations.

![Material estimations from the uniform 3 g/cm³, 3-mm thick fuel powder verification measurement without (a) and with (b) Gd loading of 5 w/o.](image)

**Fig. 4:** Material estimations from the uniform 3 g/cm³, 3-mm thick fuel powder verification measurement without (a) and with (b) Gd loading of 5 w/o. Accuracies are summarized in Table II.

**TABLE II:** Relative error (ε) on material estimations for simulated measurements of samples assumed to have uniform density.

<table>
<thead>
<tr>
<th>Est. U density (g/cm²)</th>
<th>Est. O density (g/cm²)</th>
<th>Est. Gd density (g/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
</tbody>
</table>

TABLE III: Relative error on total uranium mass using simulations with varying density.

<table>
<thead>
<tr>
<th>Density distribution</th>
<th>Estimated U mass error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% normal</td>
<td>0.068 ± 0.001</td>
</tr>
<tr>
<td>20% uniform</td>
<td>0.086 ± 0.001</td>
</tr>
<tr>
<td>40% uniform</td>
<td>0.182 ± 0.001</td>
</tr>
</tbody>
</table>

![Material estimations from the UO₂ fuel sample with density variation of (a) 20% normally distributed and (b) 40% uniformly distributed.](image)

**Fig. 5:** Material estimations from the UO₂ fuel sample with density variation of (a) 20% normally distributed and (b) 40% uniformly distributed. The results are summarized in Table III.
TABLE IV: The sensitivity of the uranium (U) estimated from the algorithm to variations in the measurement system.

<table>
<thead>
<tr>
<th>Variation</th>
<th>Level</th>
<th>$e_U$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectrum endpoint</td>
<td>300 ± 2 keV</td>
<td>0.01 ± 0.23</td>
</tr>
<tr>
<td></td>
<td>300 ± 3 keV</td>
<td>0.01 ± 0.35</td>
</tr>
<tr>
<td>Pixel FWHM</td>
<td>7.0 ± 0.4 @ 60 keV</td>
<td>0.00 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>7.0 ± 0.8 @ 60 keV</td>
<td>−0.03 ± 0.09</td>
</tr>
<tr>
<td>Pixel gain</td>
<td>1.0 ± 0.005</td>
<td>−0.18 ± 0.93</td>
</tr>
<tr>
<td></td>
<td>1.0 ± 0.01</td>
<td>−0.6 ± 1.8</td>
</tr>
</tbody>
</table>

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