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Mathematical Analysis for Identifying Neutron Shielding in Neutron Multiplicity Counting

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Abstract - *Passive neutron interrogation and neutron multiplicity counting is becoming a standard procedure in material control and accountability, due to the relative transparency of structure materials to neutron flux, making it very effective in measuring non pure, poorly characterized samples.*
Currently, all applicable neutron multiplicity counting methods assume that both the detection efficiency and the neutron die away time are system parameters, independent of the sample. Thus, if the detection efficiency of the system is reduced due to neutron absorbers in the sample, the measurement will be very biased.
In the present study we develop, through both theory and implementation, new method for detecting a concealment of neutron counts (for instance, due to the presence of an absorbing matrix elements, such as Cd or B in the sample), manifested through a reduced effective detection efficiency. One of the attributes that make the proposed method appealing, is that it does not require any additional operators and may be easily applied to any standard NMC counter.

I. INTRODUCTION

1. Preliminaries

Measuring and quantifying Special Nuclear Materials (SNM), which undergo spontaneous or induced fission, by detecting neutron radiation emitted from it, typically referred to as passive neutron interrogation, is of high importance in nuclear research and industry. Due to the presence of additional neutron sources other than the spontaneous fissile material- mainly (α, n) reactions and induced fission in odd isotopes- higher moments of the count distribution are sampled, and the different sources are quantified by solving an inverse problem [1]. Such general considerations are often referred to as Neutron Multiplicity Counting (NMC).

In recent years, due to the relative transparency of many structure materials (proving to be very useful when measuring poorly characterized, unpure samples), passive neutron interrogation and neutron multiplicity counting, is becoming a standard tool in the nuclear safeguard, safety and security community. In respect, we see many work done in recent years improving the existing methods, in terms of simulation methods[2, 3], spatial corrections [4], uncertainty analysis and quantification [5] and more.

Currently, all applicable NMC methods assume that the detection efficiency (defined as the probability of a neutron to be detected) is a system parameter, independent of the sample. Thus, if the detection efficiency is compromised, the outcome of the measurement is bound to be biased.

The outline of the present study is to introduce, through both theory and implementation, new method for detecting a reduction in the detection efficiency.

One of the attributes that make the proposed method appealing, is that it does not require any additional operation and may be easily applied to any standard NMC counter.

The paper will be arranged in the following manner: in the remainder of the present section we give some general background on NMC and describe the motivation for the present study. In section II. we develop the theoretical background for the method and section III. we give a detailed description of the method. Section IV. is devoted for experimental results and in section V. we conclude the study.

2. Neutron Multiplicity Counting

Neutron measurements of SNM involve detecting the neutrons that are emitted from the sample, which originate in fissions inside the SNM. Passive neutron measurements, which are the subject we are concerned with in this study, refer to neutrons originating from spontaneous fissions in the sample (as opposed to Active measurements, referring to neutrons from induced fissions by an external neutron source [6]).

For most nuclear materials used in the industry today (mainly various isotopes of Uranium and Plutonium), the spontaneous and induced fission neutron yield (per second per mass unit) are well known. Thus, in a well calibrated system, one can, theoretically, measure the rate of neutrons emitted from the sample-S, and then deduce the mass through a simple proportion.Practically, two problems often arrizee: First, in a typical measurement, the neutron flux emitted from a SNM sample is highly affected by additional, background (non-fission) processes, mainly the (α, n) reaction. This leads to the necessity of differentiating the fission neutrons rate, S_f from the background neutrons rate, S_{bk}.

A second problem are the induced fissions. As stated, we are dealing with passive measurements, where no external source is applied in order to induce fissions in the sample.

However, as spontaneous fissions begin to occur, neutrons created by these fissions can induce further fissions in the sample, which leads to the creation of a "fission chain". This phenomena is called Multiplication, and can be quantified by the Leakage Multiplication factor, denoted as M_L. The mathematical modelling of the neutron contribution due to induced fission chains was developed by Bohnel [7] via the so called "super fission" model.

To summarize, if one intends to measure a SNM sample and calculate its mass by passive means, 3 characterizing parameters are needed:

1. S, the total rate of neutrons emitted from it.
2. S_f, the rate of fission-only neutrons emitted from the asmple. For convenience, we will express this parameter using U, the ration between fission neutrons to the total neutron rate ($U = S_f/S$).
3. M_L, the multiplication factor.

Technically, this is typically done by sampling three moments of the neutron count distribution, and then solving a set of three (non linear) equations with three unknowns [1, 8]. When solving the set of equations associated with the three unknowns S, U and M_L, the detection efficiency is assumed to be a system parameter. Theoretically, the detection efficiency can be pre calibrated using well defined neutron source. Practically, for most commercial systems (such as the PSMC and AWCC), the detection efficiency is well known [9, 6].

3. Motivation

The risk of a drop in the detection efficiency was recently described in [10], reporting the difficulties arising in implementation of NMC methods in the Fukushima-Daiichi decommissioning. In particular, the presence of the neutron absorbers, originating from the damaged reactor's regulation system, cause a reduction in the effective detection efficiency. Clearly, as the use of NMC methods grow, the risk of a measurement biasing due to neutron shielding in passive interrogation grows along. Moreover: in current high efficiency detection systems, the sample cavity is separated from the detection system by a thin Cd layer, aimed to prevent neutrons moderated in the moderating media surrounding the detectors to re enter the sample and create induced fissions. There for, a reduction in the neutron count may be caused by the mere presence of a moderator inside the sample cavity, such as polyethylene or even lead. Currently, the most well know method for detecting (and correcting) a biasing in the detection efficiency due to neutron shielding is the "add a source" method [11]. On one hand, the add-a-source method not only detects a reduction of the detection efficiency due to the presence of a neutron absorber, but also corrects the detection efficiency. On the other hand, the add-a-source has three disadvantages: first, the performance of the method depends on the spatial configuration of the tested sample. Second, a well calibrated neutron source must be available, and finally, it is very time consuming since the measurement must be repeated 3 times.

There for, a simple method, that does not require any additional measurement (or materials), only new analysis, but can inform us if the detection efficiency has dropped, might prove to be very useful as a preliminary step to the add-a-source method.

II. THEORETICAL BACKGROUND

1. The SVM method

The SVM method, introduce by the author in [8], is a new method for analyzing the results from a passive neutron interrogation measurement, by correlating the first three central moments (the **M**ean, **V**ariance and **S**kewness) of the number of detections in a given time interval with the measurement parameters. The first three central of the number of detections is sampled in the following manner: the measurement is "broken" into N consecutive gates of duration T (T is, typically 2 or 3 die-away times [8]) and the number of detection in the n^{th} gate is denoted by X_n, $quadn = 1, 2, \ldots N$. Then, the mean (E), variance (V) and skewness (Sk) are sampled by:

$$E = \frac{1}{N}\sum_{j=1}^{N} X_j, \quad V = \frac{1}{N}\sum_{j=1}^{N}(X_j - E)^2, \quad Sk = \frac{1}{N}\sum_{j=1}^{N}(X_j - E)^3$$

In the present context, "measurement parameters" refer to the following:

1. The source intensity- S.
2. The ratio between the total source intensity and the spontaneous fission source intensity U.
3. The neutron die-away time $\frac{1}{\lambda}$.
4. The detection efficiency P_d
5. The k_{th} factorial moments of the spontaneous and induced fission multiplicities $D_{if,k}, D_{sf,k} k = 1, 2, \ldots$ (respectively).
6. the fission probability of a neutron-p_f. Often, the fission probability p_f is replaced by the leakage multiplication factor M_L, defined by $M_L = \frac{1-p_f}{1-p_f D_{if,1}}$

Notice, out of all the measurement parameters, three are unknown: U, S and M_L. Once S and U known, then the spontaneous fission rate is nothing more than $S_f = U \times S$

The correlation is done through the following set of equation:

$$\begin{aligned} SD_{g,1}(U, M_L) &= \frac{E}{P_d T} \\ SD_{g,2}(U, M_L) &= \frac{(V - E)\lambda}{P_d^2(e^{-\lambda T} - 1 + \lambda T)} \\ SD_{g,3}(U, M_L) &= \frac{(Sk - 3V + 2E)2\lambda}{P_d^3(e^{-2\lambda T} + e^{-\lambda T} - 3 + 2\lambda T)}. \end{aligned} \tag{1}$$

where T is the duration of the time interval in which the detections are counted $D_{g,j}(U, M_L) \quad (j = 1 \dots 3)$ are the generalized factorial moments written in terms of the source to noise ratio U and the leakage multiplication factor M_L, given explicitly by (see [8]):

$$\begin{aligned} D_{g,1}(U, M_L) &= (U(D_{sf,1} - 1) + 1)M_L \\ D_{g,2}(U, M_L) &= M_L^2\left(UD_{sf,2} + \frac{M_L - 1}{1 - D_{if,1}}(U(D_{sf,1} - 1) + 1)D_{if,2}\right) \\ D_{g,3}(U, M_L) &= M_L^3(UD_{sf,3} + \frac{M_L - 1}{1 - D_{if,1}}(3UD_{sf,2}U_{if,2} \\ &+ D_{if,3}(U(D_{sf,1} - 1) + 1)) \\ &+ 3\left(\frac{M_L - 1}{1 - D_{if,1}}\right)^2 D_{if,2}^2(U(D_{sf,1} - 1) + 1)) \end{aligned} \quad (2)$$

The SVM formalism and the Multiplicity formalism may be correlated by the following formulas:

$$\begin{aligned} Singles &= \frac{E}{T} \\ Doubles &= \frac{V - E}{e^{-\lambda T} - 1 + \lambda T}\lambda \\ Triples &= \frac{Sk - 3V + 2E}{4e^{-\lambda T} - e^{-\lambda T} - 3 + 2\lambda T}2\lambda \end{aligned}$$

The actual implementation is done in the following manner: first, we divide the measurement into consecutive time intervals of length T (which we will refer to as the *target interval*) and then evaluate the mean, variance and skewness (third central moment) of the number of detections. Then, we solve three equations with three unknowns (S, U and M_L) as defined by 2 and 1. Once the set of equations is solved, the mass is proportional to $S_f = S \times U$, and the proportion coefficient is the reciprocal of the spontaneous fission rate (per gram). For instance, when measuring *Pu* samples, a typical number for the fission rate is approximately 473.5 events per gram per second [1].

2. Analytic derivation of the Kurtosis and the Quadruples rate

As stated, the idea presented in the study is to add one more equation- the fourth central moment- to our set of equations, allowing us to consider one more unknown. In the Multiplicity formalism, the natural choice for the fourth equation would be the Quadruples rate, and in the SVM, the fourth central moment, defined as:

$$K = E\left((x - E(x))^4\right) \quad (3)$$

would be the natural choice. The notation K stand for "Kurtosis"[1]. In the present study, the derivation of the formulas will be done in the SVM formalism. Still, the final results will also be presented in the Multiplicity formalism.

[1]It should be stated that in most literature, the Kurtosis is not defined exactly as is 3, and is normalized such that Kurtosis of a Poisson distribution is equal to the mean.

Our first goal, then, is to write an explicit formula for the fourth central moment of the number of detections in terms of the system parameters.

From a technical point view, obtaining an analytic expression for the for central moment is done in two steps:

1. Derivation of the fourth moment in terms of the generalized factorial moments: In [8], this was done for the first three central moments, and the expansion to the fourth central moment may be achieved using the exact same argumentation.
2. Derivation of the fourth generalized factorial moments $D_{g,4}(U, M_L)$: Once again, for the first three factorial moments this was done in [7], and derivation of the fourth moment can be done in a very similar manner.

From a mathematical and theoretical point of new, there are is nothing really new here, all that is really done is an extension of the work presented in [8] and [7] to the fourth moment. Yet, practically, some of the computations are fairly complicated.

STEP 1: Derivation of the fourth moment in terms of the generalized factorial moments

Following the exact same argumentation presented in [8], the fourth central moment of the distribution may be written as:

$$K = I_1 + 7I_2 + 3I_1^2 + 6I_1I_2 + 6I_3 + 3I_2^2 + I_4 \quad (4)$$

where:

$$I_k = P_d^k D_{g,k} \int_0^\infty (1 - e^{-\lambda T} - U_0(t - T)(1 - e^{-\lambda(t-T)}))^k dt; \quad k = 1, \dots, 4$$

The first 3 integrals were computed in [8], resulting with:

$$\begin{aligned} I_1 &= P_d D_{g,1} T, I_2 = P_d^2 D_{g,2} \frac{e^{-\lambda T} - 1 + \lambda T}{\lambda} \\ I_3 &= P_d^3 D_{g,3} \frac{e^{-2\lambda T} + 4e^{-\lambda T} - 3 + 2\lambda T}{2\lambda} \end{aligned}$$

and through direct calculations we obtain:

$$\begin{aligned} I_4 &= P_d^4 D_{g,4} \int_0^\infty (1 - e^{-\lambda T} - U_0(t - T)(1 - e^{-\lambda(t-T)}))^4 dt \\ &= P_d^4 D_{g,4} \frac{e^{-3\lambda T}\left(2 - 9e^{\lambda T} + 18e^{2\lambda T} - 11e^{3\lambda T} + 6\lambda T e^{3\lambda T}\right)}{6\lambda} \end{aligned}$$

Which completes the first step.

STEP 2: Derivation of the fourth generalized factorial moment

In the following, we compute analytic expressions for the fourth generalized factorial moment. Formal non explicit formulas were previously introduced [12]. For sake of completeness, we give the full analysis.

For a multiplying system, the generalized factorial moments are defined as the factorial moments of the number of neutrons emitted in the entire neutron chain, initiated by

a source event. The analytic expression for the generalized factorial moments is defined in the following manner: define $h(x) = \sum_{n=0}^{\infty} x^n a_n$ as the probability generating function of the number of neutrons emitted due to a single *neutron*, and then evaluate the first four factorial moments of $\{a_n\}_{n=0}^{\infty}$ by the following:

$$\begin{aligned}
d_1 = h'(1) &= \frac{1-p_f}{1-p_f D_{if,1}} \quad (5)\\
d_2 = h''(1) &= \frac{p_f D_{if,2}(1-p_f)^2}{(1-p_f D_{if})^3}\\
d_3 = h^{(3)}(1) &= \frac{p_f(1-p_f)^3}{(1-p_f D_{if,1})^4}\left(D_{if,3} + 3D_{if,2}^2 \frac{p_f}{1-p_f D_{if,1}}\right)\\
d_4 = h^{(4)}(1) &= \frac{p_f}{1-p_f D_{if,1}}\{D_{if,4}\left(\frac{1-p_f}{1-p_f D_{if,1}}\right)^4\\
&+ 6D_{if,3}\left(\frac{1-p_f}{1-p_f D_{if,1}}\right)^2 \frac{p_f D_{if,2}(1-p_f)^2}{(1-p_f D_{if,1})^3}\\
&+ 4D_{if,2}\frac{p_f(1-p_f)^4}{(1-p_f D_{if,1})^5} \times\\
&\left(D_{if,3} + 3D_f^2(2)\frac{p_f}{1-p_f D_{if,1}}\right)\\
&+ 3D_{if,2}\left(p_f \frac{D_{if,2}(1-p_f)^2}{(1-p_f D_{if,1})^3}\right)^2\}
\end{aligned}$$

Notice, $d_j, j = 1,\ldots,4$ depend only on the induced fission probability and the induced fission factorial moments.

Next, we define $H(x) = \sum_{n=0}^{\infty} x^n a_n$ as the probability generating function of a number of neutrons emitted in the entire neutron chain, initiated by a single source event. Following the exact same argumentation as in [7], the generalized 4^{th} factorial moment is given by $D_g(4) = H^{(4)}(x)|_{x=1}$. Or, explicitly:

$$\begin{aligned}
D_{g,4}(U,ML) &= UD_{sf,4}d_1^4 + 6UD_{sf,3}d_1^2 d_2 + 3UD_{sf,2}d_2^2\\
&+ 4UD_{sf,2}d_1 d_3 + \left(UD_{sf,1} + 1 - U\right)d_4 \quad (6)
\end{aligned}$$

Implementation of 5 in 6 gives an explicit formula for $D_{g,4} = D_{g,4}(U, M_L)$, which concludes the second step (notice, 5 is written of p_f rather than M_L, but the transformation is trivial).

Finally, through algebraic considerations, we obtain that:

$$\begin{aligned}
K(X) &= 6E(X) - 11Var(X) + 6Sk(X) + 3Var^2(X) + \quad (7)\\
&P_d^4 S D_{g,4}(U,M_L) \times\\
&\frac{e^{-3\lambda T}\left(2 - 9e^{\lambda T} + 18e^{2\lambda T} - 11e^{3\lambda T} + 6\lambda T e^{3\lambda T}\right)}{6\lambda}
\end{aligned}$$

As we can see, the expression for $K(X)$ is constructed of two parts: the first part is an algebraic combination of the first three central moments, and the second, which we denote by Q, and refer to as the *Quadruples* rate, is given by:

$$\begin{aligned}
Q &= P_d^4 S D_{g,4}(U,M_L) \times \quad (8)\\
&\frac{e^{-3\lambda T}\left(2 - 9e^{\lambda T} + 18e^{2\lambda T} - 11e^{3\lambda T} + 6\lambda T e^{3\lambda T}\right)}{6\lambda}
\end{aligned}$$

These formulas form the theoretical basis of the method introduced in this study, which will be described in details in the next section.

III. DETECTING NEUTRON SHIELDING USING THE QUADRUPLES RATE

1. Theory

Once the first four central moment are sampled, Q may be realized in two different manners. First, we can directly sample Q using the equality

$$Q = K(X) - \left(6E(X) - 11Var(X) + 6Sk(X) + 3Var^2(X)\right) \quad (9)$$

We will denote the value of Q obtained via equation 9 as Q_m ("m" for "measured)".

A second realization may be obtained in the following manner: Once the first three central moments or sampled, we can solve equations 2 and 1 and then insert the values of of S, U and M_L (together with the system parameters P_d and λ) in equation 8. We will denote the value of Q obtained in this form as Q_c ("c" for "calculated").

Since the value of Q_c is explicitly dependent of P_d, a comparison between Q_c and Q_m links the 4^{th} central moment of the count distribution and the detection efficiency P_d. Unfortunately, calculating P_d directly from the Kurtosis (or, equivalently, Q) is not applicable: since P_d by definition is less the 1, and Q is proportional to P_d^4, the equation is ill posed from a numeric point of view.

On the other hand, we can use the sampled value of the Kurtosis for a "go/no go test" regarding P_d: Is the effective efficiency of the system equal to the declared one? The concept is as follows: If a SNM sample is placed in a matrix consisting of neutron absorbing materials (Such as Cd of B), a constant fraction of neutrons emitted from the sample will be absorbed prior to being detected by the system. This is equivalent to reducing the effective efficiency of the system. Theoretically, Q_c and Q_m should be equal (up to measurement uncertainties). However, if the effective efficiency of the system differs from the declared value (as is the case when the sample is placed in a neutron absorbing material), the values are expected not to be equal. Thus, a strong discrepancy between Q_c and Q_m can serve as an indication for a deviation from the declared detection efficiency - the presence of neutron absorbing materials in the sample.

Two remarks are due: first, the fact the P_d is in the fourth power is now favorable, since it amplifies the reduction when transforming from P_d to Q. For instance, if the the P_d is reduced by 20%, then Q will be reduced by approximately 60% (assuming all other parameters are not changed dramatically), and if P_d is reduced by 30%, then Q will be reduced by approximately 80%. Second, from a theoretical point of view, it might be the case that equation 8 is an algebraic combination of equations 2. While we did not prove analytically that this is not the case, the conditions for this to happen are extremely strong. For instance, the Jacobian matrix of the set of equations defined by 2 and 8 combined mast have a zero determinant every where.

2. How good is good enough?

As stated earlier, the main idea is to compare two different realizations of Q: Q_m and Q_c. On the other hand, even if the values would theoretically be equal, from a practical point of view, the actual numeric values will always be subjected to experimental uncertainties, creating a biasing between the values. Thus, the "go / no go" condition will not be of the form $Q_m = Q_c$, but rather a condition on $\mathcal{D} = |Q_m - Q_c|/Q_m < \epsilon$. If $\mathcal{D} < \epsilon$, then we deduce that the sample *is not* shielded, and if $\mathcal{D} > \epsilon$, then we deduce that the sample *is* shielded. We will refer to ϵ as the *tolerance* of the procedure. As in any " go / no go" classification, choosing the tolerance balances between to type of false result: "false negative"- the procedure decides that the sample is not shielded when it is, and "false positive", when we deduce that the sample is shielded, when in fact it is not. Clearly, there is always a trade-off between the false positive and the false negative: As ϵ increases, we reduce the chance of a false positive, but at the price of increasing the risk of a false negative (and vise verse). Thus, one of the aims of this study, is to optimize the tolerance ϵ, and quantify the trade-off between the false positive and the false negative.

IV. EXPERIMENTAL RESULTS

The present section is devoted for experimental implementation of the method introduced, followed by a discussion on the optimal value of the tolerance ϵ. The experimental results will be divided into two different settings. In section 1., we have taken a set of 5 standard measurements, and sampled the difference between Q_c and Q_m in two situations: first we have sampled Q_c and Q_m without any additional manipulation, and then we have sampled Q_c and Q_m after an artificial "shielding" was inflicted, by randomly removing a certain fraction of the detections.

In section 2., we implement the method on a set of 7 additional measurements, where the sample was covered by a Polyethylene cup, creating an observable reduction in the count rate.

1. Experiment 1: non shielded measurements

A. Experimental Setting 1

The initial validation of the formulas presented above was tested on a set of 5 measurements, acquired using 3 standard neutron coincidence counters: JCC31 [13], AWCC [1] and the PSMC [9]. The measurements were taken at the PERLA facility in the JRC laboratory, Ispra, Italy. The characteristics of each system and measured samples are listed in the table I. All samples, except sample number 1, consist of pure Plutonium with varying isotopic composition. Sample number 1 also contains a small fraction (3.6%) of Gallium.

For each of the following measurements, we have emulated a shielding effect, by randomly deleting a fraction F of the counts. This was done for $F = 0$ (no reduction at all) and $F = 0.1, 0.2, 0.3, 0.4, 0.5$. For each value of F, the effective detection efficiency is given by $P_d \times F$.

B. Experimental results 1

As described, for each value of F, we have computed $\mathcal{D}$ as defined in section 2. Theoretically, for $F = 0$, $\mathcal{D} = 0$ as well. In practice, this is never the case. The measurement is always subjected to numeric, systematic and statistical uncertainties. On the other hand, we should expect that as F grows, the biasing will also grow. The experimental results for all samples are shown in table II and figure 1.

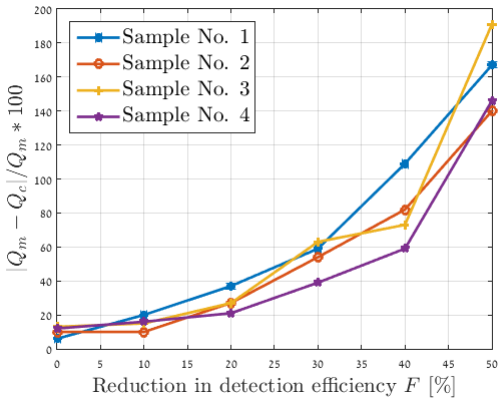

Fig. 1. The value of $\mathcal{D}$ with respect to the fraction of detection lost F.

As we can see, this prediction the $\mathcal{D}$ increases with F is met in all the samples. Still, the true question is not whether or not we will observe an increase in $\mathcal{D}$, but is the increasement large enough to establish a value of ϵ that will strictly determine if the sample is shielded or not.

The answer, naturally, depends on F. For small values of F (10%), the shielded and non shielded signals are hard to discriminate. But for $F = 0.2$ and more, by choosing $\mathcal{D} > 0.15$ as the "no go" criteria, we have a 100% success rate both in terms of false positive and false negative.

2. Experiment 2: shielded measurements

A. Experimental Setting 2

Once the initial testing proved satisfying, we have implemented the proposed method on a set of 7 measurements in which the sample was shielded. All mesurements were done using a standard PSMC at the JRC laboratory, in Ispra, Italy. Measurements were conducted on a set of 4 Pu samples, all of them having (approximately) the same mass of 6.6[gr Pu], differing only in the isotopic composition.The effective ^{240}Pu mass of each sample is given in table III.

The samples were measured in 3 different configurations:

1. plain measurements of SNM samples.
2. the samples were placed inside a small polyethylene cup.
3. the samples were placed inside a large polyethylene cup.

The small polyethylene cup had a diameter of 10 [cm] and a total wight of 0.5 [kg], and the large cup had a diameter of

sample No.	total Pu mass [g]	^{240}Pu effective mass [4]	Counter type	detection efficiency P_d	detector die-away time [μsec]
1	4.79	1.108	PSMC	54%	50
2	20.57	5.41	AWCC	33%	50
3	49.7	6.51	AWCC	33%	50
4	6.7	1.4	JCC31	16%	50

TABLE I. Experimental setting for the un-shielded measurements.

	$F = 0$	$F = 0.1$	$F = 0.2$	$F = 0.3$	$F = 0.4$	$F = 0.5$
1	6	20	37	59	109	167
2	10	10	27	54	82	140
3	13	15	27	63	73	191
4	12	16	21	39	59	146

TABLE II. Experimental results for the un-shielded measurements.

15 [cm] and a total wight of 1.6 [kg]. As mentioned earlier, althoguh no absorber was added to the samples, the presence of a moderator around the sample has created a shielding due to the Cd lining between the sample cavity and the detector rings. The small cup reduced the count rate by roughly 9%, and the large cup has reduced the count rate by 38%.

B. Experimental results 2

Table IV summarizes the differences between Q_M and Q_c (in percentage) for all 7 measurements.

As we can see, the results are even more distinct than in the previous section: using a 10% tolerance yields full distinction between the shielded and non shielded measurements, with a 0% false positive / negative.

V. CONCLUDING REMARKS

In the study, we have introduced a new mathematical method for detecting a shielding of the sample measured in a neutron coincidence counter. The detection is done by observing a drop in the detection efficiency, translated into a biasing between two different realizations of Q. Theoretically, the method can be generalized into a correction of the detection efficiency- but in practice this might create very big biasing in the corrected efficiency, since the corresponding equations are numerically ill posed.

The method was tested in two settings: in the first setting, the sample was not shielded, but the count rate was reduced artificially, and in the second, the sample was covered by a polyethylene cup, creating a reduction in the count rate.

In the first setting, results indicate, that a 20% biasing between Q_C and Q_m is a clear indicator of a neutron shielding, and a clear distinction between a shielded and a non shielded measurement appears once the drop in the count rate is 20% and more. In the second setting, which is more similar to an actual shielding, results indicate that even a 10% shielding can be detected.

One possible reason for the difference between the results is that the second setting was only measured using a PCMS counter, which has a very high efficiency to begin with (54%).

Since the method does not require any change of the system configuration, and can be easily implemented on any existing facility (assuming that the data acquisition is done in LIST mode), it can serve as an effective tool to determine whether additioanl measurements are required using the "Add-A-Source" method.

sample	1	2	3	4
Effective ^{240}Pu mass [gr]	1.58	1.18	0.98	39

TABLE III. Effective ^{240}Pu mass of the measured samples.

	Non shielded measurement	small cup	large cup
Sample No.1	5%	17%	91%
Sample No.2	2%	24%	87%
Sample No.3	7%	28%	85%
Sample No.4	1%	18%	not measured

TABLE IV. Measured values of $\mathcal{D}$ (%) for all 7 measurements.

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