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Gyro-kinetic Study of Rosenbluth-Hinton Flow for Fusion Reactor Plasmas

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Abstract - *Zonal flow, an axisymmetric potential fluctuation driven by turbulent Reynolds stress, takes a key role of turbulence regulation with its flow shear. It gets damped by collisionless process, but to a non-zero value called the residual zonal flow level due to polarization shielding. Previous studies were done assuming the Maxwellian equilibrium distribution. In this study, we investigate the residual zonal flow for the Slowing down distribution function using modern gyro-kinetic approach. We find that zonal flow for Slowing down case remains at a higher level than that for equivalent Maxwellian case in the intermediate wavelength range,* $\boldsymbol{k_r} \cdot \boldsymbol{\rho_{\theta i}} \sim 1$.

I. INTRODUCTION

Turbulent transport in tokamak plasma is a key issue in the magnetic confinement fusion research, since it is considered as a main cause of the anomalous transport which reduces the confinement efficiency of fusion devices. Many studies therefore were performed to understand the plasma turbulence and transport driven by it. Especially theoretical[1] and simulation studies[2] on drift wave turbulence such as ion temperature gradient (ITG) mode have revealed some important aspect of nonlinear dynamics of turbulent transport.

Zonal flow, an axisymmetric low frequency potential fluctuation, is now well known to play a key role of turbulence regulation in the tokamak plasma. It is driven by turbulent Reynolds stress[3]. ExB flow can decorrelate the radial length of turbulent eddies by its radial ExB shear and therefore reduce the plasma turbulence. Extensive simulation studies on drift wave turbulence have shown that turbulence gets saturated nonlinearly due to the zonal flow. Thus, understanding the behavior of zonal flow in the fusion plasma is important.

Residual zonal flow is one of the most interesting characteristics of zonal flow. It is a long time asymptotic response of initially excited fluctuation. When the zonal flow is initially excited, it gets damped by collisionless transit magnetic pumping[4]. However Rosenbluth and Hinton[5] have shown that it does not fully decay but asymptotes to some finite residual level due to the polarization shielding. They calculated this level in the long wavelength limit, and the result is now called "Rosenbluth Hinton residual zonal flow (RH flow)". Furthur theoretical progress has been made on this issue. Jenko et.al[6] reported their gyro-kinetic simulation results that the residual zonal flow is enhanced in the short wavelength regime. Xiao and Catto[7] confirmed this result from their theoretical study. Wang and Hahm[8] have derived an analytic formula of the residual zonal flow level which is valid in the arbitrary wavelength regime using modern gyro-kinetic approach[9]. However, all aforementioned previous studies were performed assuming the Maxwellian equilibrium distribution. Considering a burning plasma such as ITER and beyond, alpha particles will be produced due to fusion reaction and one should consider the slowing down distribution as an equilibrium distribution. Therefore, zonal flow study for slowing down distribution is needed.

In the present paper, we investigate the residual zonal flow for slowing down distribution using modern gyro-kinetic approach. We start from the expression of the classical and neoclassical polarization density which result from the pull-back transformation from gyro-center coordinate to particle coordinate and from bounce gyro-center coordinate to gyro-center coordinate, respectively. The classical polarization is calculated via a semi-analytic approach in two limiting cases of the zonal flow radial scales. For the neoclassical polarization part, both trapped and passing particle contributions are considered in three limiting cases. Then we calculate the residual zonal flow for the slowing down distribution function and compare the result with the equivalent Maxwellian case. We find that residual zonal flow for slowing down case is higher than that for equivalent Maxwellian distribution in the intermediate radial wavelength range on the order of ion poloidal gyroradius, $\boldsymbol{k_r} \cdot \boldsymbol{\rho_{\theta i}} \sim 1$.

The remainder of this paper is organized as follows. In Section II, we introduce the modern gyro-kinetic formalism and slowing down distribution. It includes the details of Lie transformation perturbation, polarization density, and slowing down process due to the beam-electron collisions. Then, detailed calculation of classical and neoclassical polarization density is presented in Section III. Section VI illustrates the result of residual zonal flow level for the slowing down distribution. It is compared with the equivalent Maxwellian case. In addition, fusion product's initial beam energy dependency of residual zonal flow is demonstrated on this section. Finally, we summarize our work and discuss its possible application in Section V.

II. THEORETICAL BACKGROUND

1. Phase-space Lagrangian in Modern Gyro-kinetics

In this section, we present the phase space Lagrangian transformation for the gyro-kinetics[10] and bounce gyro-kinetics[11]. Since charged particles execute two types of quasi-periodic orbit, i.e. gyration and bounce motion, in tokamak magnetic field, one can reduce the phase space Lagrangian of a particle using the appropriate action-angle variables. The reduced Lagrangian for gyration is guiding-center

Lagrangian which contains Finite Lamour Radius(FLR) effect,

$$\gamma_{gc} = \left(\frac{e}{c}\boldsymbol{A} + mv_{\|}\hat{\boldsymbol{b}}\right)\cdot d\boldsymbol{R} + \frac{\mu B}{\Omega}d\Theta - \left(\frac{1}{2}mv_{\|}^2 + \mu B\right)dt$$

where $\boldsymbol{A}$ is vector potential for equilibrium magnetic field $\boldsymbol{B}$, $v_{\|}$ is parallel velocity, μ is magnetic moment of charged particle, $\Omega = \frac{eB}{mc}$ is gyro-frequency and $\hat{b} = \frac{\boldsymbol{B}}{|\boldsymbol{B}|}$.
Next, the reduced Lagrangian for bounce orbit includes Finite Orbit Width(FOW) effect,

$$\gamma_{bgc} = \frac{e}{c}y^2 dy^1 + Jd\xi + \frac{\mu B}{\Omega}d\Theta - H_0(\boldsymbol{y}, J, \mu)dt$$

where $y^1 = \psi(r)$ is the poloidal flux function, $y^2 = \xi - q\theta$ is the binormal angle, J and xi are action and angle variable for bounce motion, respectively, When the small electrostatic fluctuation $\delta\phi$ is introduced on the system, we can imply the Lie transform perturbation method to simplify the perturbed guiding-center Lagrangian. Then, the guiding-center Lagrangian is transformed to gyro-center Lagrangian,

$$\bar{\gamma}_{gy} = \left(\frac{e}{c}\bar{\boldsymbol{A}} + m\bar{v}_{\|}\hat{\boldsymbol{b}}\right)\cdot d\bar{\boldsymbol{R}} - \left(\frac{1}{2}m\bar{v}_{\|}{}^2 + \bar{\mu}B + e\delta\Phi\right)dt$$

and the bounce-guiding center Lagrangian is transformed to bounce gyro-center Lagrangian,

$$\hat{\gamma}_{bgy} = \frac{e}{c}\hat{y}^2 d\hat{y}^1 - \left(H_0(\hat{\boldsymbol{y}}, \hat{J}, \hat{\mu}) + e\delta\Psi\right)dt$$

where $\delta\Phi$ and $\delta\Psi$ are effective potentials, defined below.

$$\delta\Phi = \langle\delta\phi\rangle_g - \frac{e}{2B}\frac{\partial}{\partial\bar{\mu}}\langle\delta\tilde{\phi}^2\rangle_g$$
$$\delta\Psi = \langle\langle\delta\phi\rangle_g\rangle_b - \frac{e}{2B}\frac{\partial}{\partial\bar{\mu}}\langle\langle\delta\tilde{\phi}^2\rangle_g\rangle_b - \frac{e}{2\omega_{b,t}}\frac{\partial}{\partial\hat{J}}\langle\langle\delta\tilde{\phi}\rangle_g{}^2\rangle_b$$
$$\delta\tilde{\phi} = \delta\phi - \langle\delta\phi\rangle_g, \langle\delta\tilde{\phi}\rangle_g = \langle\delta\phi\rangle_g - \langle\langle\delta\phi\rangle_g\rangle_b$$

$\langle...\rangle_g$ and $\langle...\rangle_b$ denote the gyro-average and bounce-average respectively.

2. Slowing down distribution

When high energy particles are injected in the plasmas, we can use the slowing down distribution as an equilibrium distribution function,

$$F_{SD}(v) = \frac{n_\alpha}{4\pi v_c^3 I_2}\frac{H(v_\alpha - v)}{1 + (\frac{v}{v_c})^3}$$

where v_α is an initial velocity of an energetic particle, v_c is slowing down critical velocity related to the temperature of background plasmas,

$$v_c^3 = 3\sqrt{\frac{\pi}{2}}\frac{m_e}{m_\alpha}Z_{eff}\left(\frac{T_e}{m_e}\right)^{3/2}$$

and $I_n = \int_0^{v_\alpha/v_c}\frac{x^n}{1+x^3}dx$. For a proper normalization, we define the effective temperature of slowing down distribution by the quadratic moment of velocity,

$$\int v^2 F_{SD}(v)d^3v = \int v^2 F_{Max}(v)d^3v$$
$$\frac{T_{Max}}{T_{SD}} = \frac{I_4}{3I_2}$$

III. CLASSICAL AND NEOCLASSICAL POLARIZATION

In the last section, we introduced the modern gyro-kinetic formalism and the slowing down distribution briefly. Here, we derive expressions for the classical/neoclassical polarization density by pull-back transforming the charge density from the gyro-center/bounce gyro-center phase space to the particle phase space, respectively. Details can be found from the following papers. Wang and Hahm[8] presented the generalized expressions for both classical and neoclassical polarization density using the modern gyro-kinetics and bounce gyro-kinetics. By doing so, they fully consider the FLR effect and FOW effect in the expression. Duthoit, Brizard, and Hahm[12] improved the analytic approximations for trapped and passing orbit to calculate the neoclassical polarization density more accurately. Since their analytic expression is too complex for our purpose in this paper, we follow the Wang-Hahm's method when calculating the classical and neoclassical polarization density using the pull-back transform. In this way, both classical and neoclassical polarization density are derived physically and systematically. The schematic description of pull-back transformations is illustrated explicitly in Fig. 2 of Ref. [8].

Now we present the particle charge density in the gyro-kinetic Poisson equation. This is expressed in terms of the distribution in the gyro-center phase space, and in the bounce gyro-center phase space. In this procedure, two consecutive pull-back transformations are performed, one from gyro-center to particle phase space and the other from bounce gyro-center to particle phase space. Each pull-back transformation results in two different polarization densities, the classical polarization density from the gyro-center transformation and the neoclassical polarization density from the bounce gyro-center transformation.

Starting from the particle phase space, the charge density for each species s appears in the Poisson equation as below.

$$\begin{aligned}\boldsymbol{\nabla}^2\phi(\boldsymbol{x}, t) &= 4\pi\sum_s q_s n_s(\boldsymbol{x}) = 4\pi\sum_s q_s\int d^3v f_s(\boldsymbol{x}, \boldsymbol{v}, t)\\ &= 4\pi\sum_s q_s\int d^3\boldsymbol{R}dv_{\|}d\mu F_{gc,s}(\boldsymbol{R}, v_{\|}, \mu, t)\delta(\boldsymbol{R} + \boldsymbol{\rho} - \boldsymbol{x})\\ &= 4\pi\sum_s q_s\int d^6\boldsymbol{Z}F_{gc,s}(\boldsymbol{Z})\delta(\boldsymbol{R} + \boldsymbol{\rho} - \boldsymbol{x})\end{aligned} \tag{1}$$

Here, note that $f_s(\boldsymbol{x}, \boldsymbol{v}, t) = F_{gc,s}(\boldsymbol{Z})$. Via inverse-transform of the kernel in density expressions to gyro-center phase space, the guiding-center phase space distribution function gets Lie transformed to the gyro-center phase space distribution function with the generator of inverse Lie transformation T_{gy}^{-1}. Then the particle density function can be written in terms of the distribution in the gyro-center phase space,

$$\begin{aligned}n(\boldsymbol{x}, t) &= \int d^6\bar{\boldsymbol{Z}}\left(T_{gy}^{-1}F_{gy}(\bar{\boldsymbol{Z}})\right)\delta(T_{gc}^{-1}\boldsymbol{R} - \boldsymbol{x})\\ &\simeq \int d^6\bar{\boldsymbol{Z}}\left(F_{gy}(\bar{\boldsymbol{Z}}) + G_1^{\bar{\mu}}\frac{\partial}{\partial\bar{\mu}}F_{gy}(\bar{\boldsymbol{Z}})\right)\delta(T_{gc}^{-1}\boldsymbol{R} - \boldsymbol{x})\\ &= n_{gy}(\boldsymbol{x}, t) + n_{cl}(\boldsymbol{x}, t)\end{aligned} \tag{2}$$

with

$$n_{gy}(\boldsymbol{x}, t) = \int d^6\bar{\boldsymbol{Z}} F_{gy}(\bar{\boldsymbol{Z}})\delta(T_{gc}^{-1}\boldsymbol{R} - \boldsymbol{x}) \tag{3}$$

$$n_{cl}(\boldsymbol{x}, t) = \int d^6\bar{\boldsymbol{Z}} G_1^{\bar{\mu}} \frac{\partial}{\partial\bar{\mu}} F_{gy}(\bar{\boldsymbol{Z}})\delta(T_{gc}^{-1}\boldsymbol{R} - \boldsymbol{x}) \tag{4}$$

where the first term is the gyro-center density (denoted by n_{gy}) and the last term is the classical polarization density (denoted by n_{cl}), the difference between the particle charge density and the gyro-center density. Note that we only consider the 1st order contribution of the generator of inverse Lie transformation, especially the magnetic moment part $G_1^{\bar{\mu}}$ compared to the $G_1^{\bar{\mu}}$ term. Other moment contribution $G_1^{\bar{\boldsymbol{R}}}, G_1^{\bar{v}_\parallel}$ are of higher order in the spatial ordering[13].

Next, the gyro-center density can be also written in terms of the distribution in the bounce gyro-center phase space. Using the fact that guiding-center phase corresponds to the lowest order gyro-center phase space in the gyro-kinetic ordering, one can use an identity that $F_{gy}(\bar{\boldsymbol{Z}}) = F_{bgc}(\boldsymbol{Z_{bgc}})$[8]. Then the gyro-center density function can be written in terms of the distrbution in the bounce gyro-center phase space,

$$\begin{aligned} n_{gy}(\boldsymbol{x}, t) &= \int d^6\boldsymbol{Z_{bgc}} F_{bgc}\left(\boldsymbol{Z_{bgc}}\right)\delta\left(T_{gc}^{-1}T_{bgc}^{-1}\boldsymbol{R_{bgc}} - \boldsymbol{x}\right) \\ &= \int d^6\hat{\boldsymbol{Z}}\left(T_{bgy}^{-1}\hat{F}\left(\hat{\boldsymbol{Z}}\right)\right)\delta\left(T_{gc}^{-1}T_{bgc}^{-1}\boldsymbol{R_{bgc}} - \boldsymbol{x}\right) \\ &\simeq \int d^6\hat{\boldsymbol{Z}}\left(\hat{F}\left(\hat{\boldsymbol{Z}}\right) + G_1^{\hat{J}}\frac{\partial}{\partial\hat{J}}\hat{F}\left(\hat{\boldsymbol{Z}}\right)\right)\delta\left(T_{gc}^{-1}T_{bgc}^{-1}\boldsymbol{R_{bgc}} - \boldsymbol{x}\right) \\ &= n_{bgy}(\boldsymbol{x}, t) + n_{nc}(\boldsymbol{x}, t) \end{aligned} \tag{5}$$

with

$$n_{bgy}(\boldsymbol{x}, t) = \int d^6\hat{\boldsymbol{Z}}\hat{F}\left(\hat{\boldsymbol{Z}}\right)\delta\left(T_{gc}^{-1}T_{bgc}^{-1}\boldsymbol{R_{bgc}} - \boldsymbol{x}\right) \tag{6}$$

$$n_{nc}(\boldsymbol{x}, t) = \int d^6\hat{\boldsymbol{Z}} G_1^{\hat{J}}\frac{\partial}{\partial\hat{J}}\hat{F}\left(\hat{\boldsymbol{Z}}\right)\delta\left(T_{gc}^{-1}T_{bgc}^{-1}\boldsymbol{R_{bgc}} - \boldsymbol{x}\right) \tag{7}$$

where the first term is the bounce gyro-center density (denoted by n_{bgy}) and the last term is the neoclassical polarization density (denoted by n_{nc}), the difference between the gyro-center density and the bounce gyro-center density. As we did for the classical polarization density case, we neglect the other moment contribution $G_1^{\hat{\boldsymbol{R}}}, G_1^{\hat{v}_\parallel}$, since compared to the $G_1^{\hat{J}}$ term they are of higher order in the spatial ordering[13].

With the generator of the inverse Lie transform given in Ref. [9], we can express each polarization density analytically. By flux-surface averaging to keep only the zonal component , we obtain,

$$\begin{aligned} [n_{cl,k}]_\psi \simeq n_k e^{iS(\bar{\psi})} \int &\left(1 - |\langle e^{i\delta\zeta}\rangle_g|^2\right) \\ &\times\left(-\frac{T\partial}{B\bar{\mu}}\right) F_{SD}(\bar{Z}) 2\pi d\bar{v}_\parallel \frac{Bd\bar{\mu}}{m} \end{aligned} \tag{8}$$

$$\begin{aligned} [n_{nc,k}]_\psi \simeq n_k e^{iS(\hat{\psi})} \int &|\langle e^{i\delta\zeta}\rangle_g|^2 \left(1 - |\langle e^{i\Delta\zeta}\rangle_{b,t}|^2\right) \\ &\times\left(-\frac{T\partial}{\omega_{b,t}\partial\hat{J}}\right) F_{SD}(\hat{Z}) \frac{d\hat{J}}{qR_0}\frac{Bd\hat{\mu}}{m} \end{aligned} \tag{9}$$

where $\delta\zeta$ is the particle's excursion from the gyro-center, $[...]_\psi$ denotes the flux-surface averaging, and
$\delta\zeta = \boldsymbol{k}\cdot\boldsymbol{\rho}$, $\Delta\zeta$ is the particle's excursion from the bounce gyro-center,

$$\Delta\zeta = \boldsymbol{k}\cdot\boldsymbol{\rho_\theta} \simeq \sqrt{2\epsilon}k_r\rho_{\theta i}\sqrt{\kappa}\times\begin{bmatrix} cn\left(\xi_b|\kappa\right) & for\ trapped\ particle \\ dn\left(\xi_t|\kappa^{-1}\right) & for\ passing\ particle \end{bmatrix}$$

and n_0 is initially perturbed density, $n_k = \dfrac{e\phi_k(0)}{T}n_0$.

In this section, we have systematically derived the classical and neoclassical polarization density. Note that this result has been obtained with the consideration of full FLR/FOW effect. Detailed calculation of each polarization density is presented in next subsections.

1. Calculation of classical polarization density

As demonstrated before, classical polarization density is the difference between the particle density and the gyro-center density. The main reason for this is that polarization due to the gyration motion of ions and electrons changes the dielectric response of plasma. Thus, classical polarization density is sensitively affected by FLR effect. Since we consider the slowing down distribution, its velocity space composition is quite different from the equivalent Maxwellian distribution. It results in a difference of classical polarization density between slowing down case and equivalent Maxwellian case.

Because the generator term derived from the slowing down distribution function is too complex to solve directly, we only consider the two limiting cases, the long wavelength range $k_r\rho_\perp < 1$, and short wavelength range $k_r\rho_\perp > 1$. Then, we construct a connection formula using the method employed in Ref. [8]. As a result, classical polarization density for slowing down distribution is expressed as,

$$[n_{cl,k}]_\psi = n_k\left\{\frac{1}{1 + k_r^2\rho_c^2}\frac{1}{k_r^2\rho_c^2} + \frac{k_r^2\rho_c^2}{1 + k_r^2\rho_c^2}\frac{1}{\Lambda\left(k_r\rho_c; \dfrac{v_\alpha}{v_c}\right)}\right\}^{-1} \tag{10}$$

where $\Lambda\left(k_r\rho_c; \frac{v_\alpha}{v_c}\right)$ is a normalized function,

$$\Lambda\left(x; a\right) = \int_0^a \frac{1 + H_{-1}(2xt)}{1 + t^3}\frac{dt}{I_2}$$

and ρ_c is thermal gyro-radius at the critical temperature, $\rho_c = v_c/\Omega$. It is shown in the Fig.1 that there's no difference between slowing down case and equivalent Maxwellian case in the long wavelength regime. However, the value for the slowing down case gets much higher than that for the equivalent Maxwellian case in the range $k_r\rho_{\theta i} \simeq 1$.

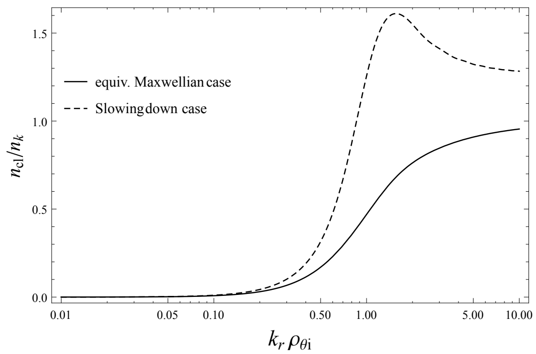

Fig. 1. n_{cl}/n in the wavelength range, $0.01 \leq k_r\rho_{\theta i} \leq 10$.

2. Calculation of neoclassical polarization density

The neoclassical polarization density is a difference between the gyro-center density and the bounce gyro-center density. Since it results mainly from the banana orbit, it is strongly affected by FOW effect. However, because we start from the gyro-center phase space when transforming to the bounce gyro-center phase space, FLR effect is also considered in the calculation.

Neoclassical polarization density is calculated in three different wavelength regimes, long wavelength regime ($k_r\rho_i < k_r\rho_{\theta i} << 1$), intermediate wavelength regime ($k_r\rho_i << 1 << k_r\rho_{\theta i}$), and short wavelength regime ($1 << k_r\rho_i < k_r\rho_{\theta i}$) following the Wang-Hahm's approach[8]. Then, we construct the connection formula to complete the full version of neoclassical polarization density as we did for the classical polarization density. In this procedure, both trapped and passing particle contribution is included. We use deeply trapped limit for the trapped particle and strongly passing limit for the passing particle for simplicity. As a result, the neoclassical polarization density is ,

$$[n_{nc,k}]_\psi = n_k \left\{ \frac{1}{\chi_{nc}^{long}} + \frac{1}{1+k_r^2\rho_c^2}\frac{1}{\chi_{nc}^{m}} + \frac{k_r^2\rho_c^2}{1+k_r^2\rho_c^2}\frac{1}{\chi_{nc}^{short}} \right\}^{-1} \tag{11}$$

where χ_{nc}^{long}, χ_{nc}^{in}, and χ_{nc}^{short} are the electric susceptibility in the range of long, intermediate, and short wavelength, respectively,

$$\begin{aligned}
\chi_{nc}^{long} &= 1.63\epsilon^{3/2}k_r^2\rho_{\theta c}^2 \\
\chi_{nc}^{m} &= \frac{I_0(v_0/v_c)}{I_2(v_0/v_c)} - \frac{2\sqrt{2\epsilon}}{\pi}\frac{1}{I_2(v_0/v_c)}\sqrt{\frac{\pi}{2}}\left(1-\frac{1}{1+(v_0/v_c)^3}\right)\Gamma'_{tr} \\
&\quad -\left(1-\frac{2\sqrt{2\epsilon}}{\pi}\right)\frac{1}{I_2(v_0/v_c)}\sqrt{\frac{\pi}{2}}\left(1-\frac{1}{1+(v_0/v_c)^3}\right)\Gamma'_p \\
\chi_{nc}^{short} &= \frac{1}{\pi k_r\rho_c}\left[\frac{1}{I_2(v_0/v_c)}\left(1-\frac{1}{1+(v_0/v_c)^3}\right)\right. \\
&\quad - \frac{2\sqrt{2\epsilon}}{\pi}\frac{3}{I_2(v_0/v_c)}\sqrt{\frac{\pi}{2}}G(v_0/v_c)\Gamma_{tr} \\
&\quad \left. -\left(1-\frac{2\sqrt{2\epsilon}}{\pi}\right)\frac{3}{I_2(v_0/v_c)}\sqrt{\frac{\pi}{2}}G(v_0/v_c)\Gamma_p\right]
\end{aligned} \tag{12}$$

As expected for the slight change, it is shown in Fig.2 that the peak of neoclassical polarization density for slowing down distribution gets a little shifted to short wavelength range compared to that for equivalent Maxwellian distribution.

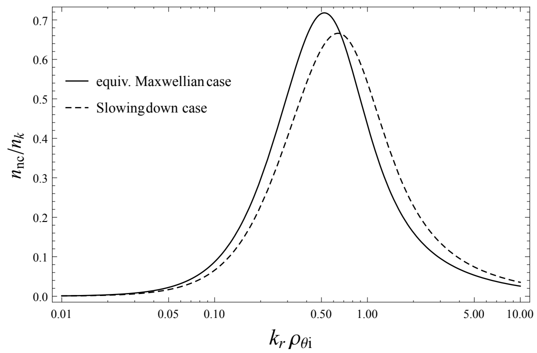

Fig. 2. n_{nc}/n in the wavelength range, $0.01 \leq k_r\rho_{\theta i} \leq 10$.

IV. RESIDUAL ZONAL FLOW LEVEL

In the previous sections, we derived the analytic expressions of the classical and neoclassical polarization density. Now, we express the residual zonal flow level in terms of classical and neoclassical polarization density [5],

$$R_{ZF} = \frac{\phi_k(t=\infty)}{\phi_k(t=0)} = \frac{[n_{cl,k}]_{flux}}{[n_{cl,k}]_{flux} + [n_{nc,k}]_{flux}} \tag{13}$$

Since classical polarization density for the slowing down distribution function gets much higher than that for the equivalent Maxwellian distribution function for wavelength on the order of poloidal gyro-radius, we expect some difference in the wavelength range, $k_r\rho_{\theta i} \simeq 1$. This is shown in Fig.3 that the residual zonal flow level for two different distribution functions in the Ion Temperature Gradient(ITG) turbulence with adiabatic electron shows the significant difference in the range $k_r\rho_{\theta i} \simeq 1$.

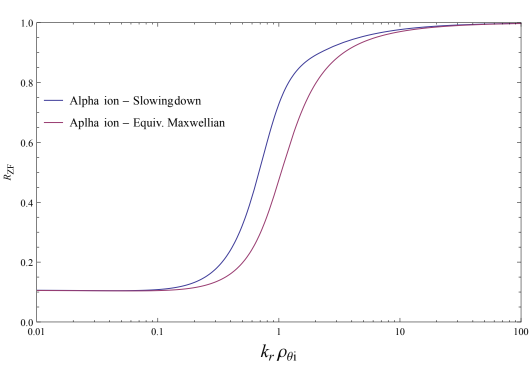

Fig. 3. Residual ZF level compared with slowing down case and equivalent Maxwellian case.

The residual level of zonal flow not only depends on the radial wavelength but also depends on the initial particle energy when it is for slowing down distribution. It is shown in Fig.4 that normalized to the case of alpha particle energy ($E_\alpha = 3.5MeV$), the relative amplitude of residual zonal flow has a local extremal value at the wavelength range $k_r\rho_{\theta i} \simeq 1$, and it increases as the initial particle energy decreases.

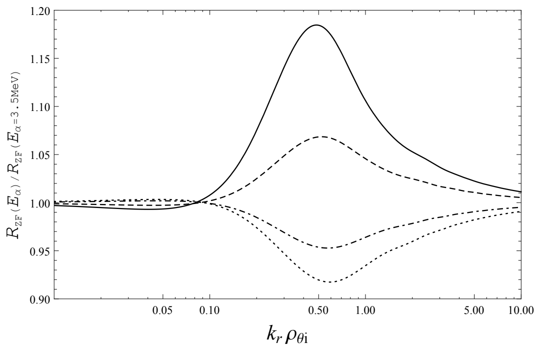

Fig. 4. Relative amplitude of residual level compared with $E_\alpha = 3.5MeV$. Each line shows the result of $E_\alpha = 1.75MeV$(solid) $E_\alpha = 2.625MeV$(dashed), $E_\alpha = 4.375MeV$(dot-dashed), and $E_\alpha = 5.25MeV$(dotted) respectively.

V. CONCLUSIONS

We systematically calculate the classical and neoclassical polarization density for the slowing down equilibrium distribution via modern gyro-kinetics and bounce gyro-kinetics, respectively. In this procedure, we use flux-surface averaged quantities to see the zonal contribution to each polarization density. Also, not only the FOW but also the FLR effect are considered in the neoclassical polarization density. Because of the velocity space composition of the slowing down distribution, classical polarization density for the slowing down distribution gets much enhanced in the wavelength range, $k_r\rho_{\theta i} \simeq 1$.

Next, we calculate the residual zonal flow level for the arbitrary wavelength using the expressed polarization densities. We compare this result with the equivalent Maxwellian case. It is shown that the residual level gets much higher for the wavelength range $k_r\rho_{\theta i} \sim 1$ due to the enhancement of the classical polarization density. Also, we investigate the initial particle energy dependence of residual zonal flow for slowing down distribution function and observe that the local extremum of relative zonal flow intensity occur around $k_r\rho_{\theta i} \simeq 1$, and it increases as the initial particle energy decreases.

As mentioned above, residual zonal flow takes a key role for understanding the zonal flow response in the turbulent plasma. Since our focus is on the practical issues of fusion reactors, our results can be useful for the prediction of zonal flow and turbulence in the burning plasma like ITER and DEMO.

VI. ACKNOWLEDGEMENTS

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