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Modeling Zero Power Reactor Noise and Neutron Count Distribution: A Stochastic Differential Equations Approach

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Abstract - *Reactor noise, caused both by the probabilistic nature of the fission chains and external reactivity noises, is one of the basic topics in nuclear science and engineering, both in theory and practice. Modeling reactor noise (and neutron flux fluctuation in general) is traditionally performed by two main approaches: the stochastic transport equation for the probability generating function and the transfer function response to random perturbations.*

In a recent study, a new modeling approach was introduced, corresponding to an intermediate regime, where noise is modeled by Brownian motion, describing the dynamics by means of Stochastic Differential Equations (SDE). In the present study we further develop the SDE approach by considering a model that preserves the discrete nature of detections, specifically, via the binomial distribution. The new formalism thus results in a non-normal distribution of the neutron count in a given time interval. We provide an explicit formula for the distribution of the neutron count, and provide simplified formulas for its high moments. Comparison between the analytic prediction and experimental results show a very high correspondence, with a bias of less than 0.98% for the first four moments.

I. INTRODUCTION

Reactor noise and neutron flux fluctuation dynamics are one of the basic topics in nuclear science and engineering, both in theory and practice. Applications of the theory of neutron fluctuation may be found both in monitoring and measurements [1], and in non destructive assay of special nuclear materials [2]. Fluctuations in the neutron population size may be attributed to two types of statistical noises: internal noises, governed by the statistical nature of the neutron reactions, and external noises, reflecting stochasticity of other elements of the system, such as temperature fluctuations, mechanical instabilities, electronic noise in the monitoring system and more [3].

Reactor noise and *neutron fluctuations* are a general terms used to describe the modeling and sampling of higher moments of the neutron population in multiplying systems due to both internal and external factors.

In a recent study by the authors, a new approach for modeling reactor noise was introduced [4], where noise is driven by Brownian motion, describing the dynamics by means of a set of *Stochastic Differential Equations (SDE)*. The model formulated in [4] couples two dependent stochastic processes: the size of the neutron population as a function of time, t, and the number of detections in the interval $[0,t]$, as it varies with t. The model is formed by a set of 2 SDE. This model is, in its primal form, non-linear. To obtain explicit formulas for the moments of the detection count, the equation was linearized, by approximating the state dependent diffusion coefficient by a constant coefficient. While it has been proven that this assumption does not effect the computed values of the first two moments (since the analytic expressions obtained for the first and second moments are in complete agreement with the classical results), the steady state solution of the linear model has a normal distribution, and hence all information on higher moments is lost.

In the present study, we introduce a non linear model for detections which also preserves its discrete nature. In this model, the conditional distribution of the detection count over any interval of the form $[0,T]$, given the population, is given by a binomial random variable whose parameters depend on the integral over time of the population, and on the detection efficiency. As we will demonstrate, the steady state distribution of this model can be fully analyzed, resulting in explicit formulas. Moreover, by simple considerations (based on the law of total cumulance), formulas for high moments of the detection count distribution may be derived, under the small detection probability approximation, that are considerably simpler than the full explicit formulas.

This paper is organized as follows. In section II. we give some background on SDE and reactor noise. In section III. the main SDE proposed in this paper is introduced and fully analyzed. In section IV. explicit formulas for the moments are derived using the law of total cumulance. Section V. is devoted to experimental results and to their comparison with the analytic expectations. Section VI. provides conclusion from this work.

II. BACKGROUND

1. General background

Reactor noise originates from two distinct stochasticity factors. One, often referred to as "internal noise", is due to the stochastic nature of the neutron life cycle and of fission chains. The second, referred to as "external noise", corresponds to reactivity fluctuations caused by factors such as temperature changes, mechanical vibration, fluid instabilities, etc. [3]. Since the two types of noise have very different nature, they are often modeled using distinct mathematical tools. Internal noise is often modeled via the Kolmogorov equation for the Probability Generating Function (PGF) [5], whereas

external noise is typically modeled using the transfer function response to a random perturbation.

In a recent study introduced by the authors, a new modeling scheme for rector noise was introduced, based on the diffusion scale approximation for the reaction rates [4]. The framework underlying the model is that of a Stochastic Differential Equations (SDE). The SDE approach is closely related to the Fokker-Planck equation introduced in [3]. Yet, the use of SDE (rather than the Fokker-Planck equation) offers a more flexible setting, allowing more elaborate physical configurations.

An SDE is an equation of the form

$$dX_t = b(t, X_t)dt + \sigma(t, X_t)dW_t, \qquad X_0 = x, \tag{1}$$

where the unknown is a stochastic process X, that has continuous sample paths taking values in $\mathbb{R}^d$, for some positive integer d; b and σ are given coefficients; and W is a d-dimensional Brownian Motion (BM). A process X is regarded a solution of the SDE if it satisfies, for every t, $X_t = x + \int_0^t b(X_s)ds + \int_0^t \sigma(X_s)dW_s$, where the last term in this integral equation is an *Ito integral*. The special case where $\sigma = 0$ corresponds to an ordinary differential equation.

The most basic model introduced in [4] describes the neutron population size at time t, here denoted by N_t, in a sub-critical system, subjected to an external (non correlated) source[1], through the SDE

$$dN_t = -\alpha N_t dt + S\,dt + \sqrt{\tilde{\sigma}^2 N_t + S}\,dW_t, \tag{2}$$

where

$$\alpha = \lambda - \lambda_f \overline{\nu}, \quad \tilde{\sigma}^2 = \lambda + \lambda_f(\overline{\nu^2} - 2\overline{\nu}), \tag{3}$$

and λ is the total reaction probability per time unit, λ_f is the fission probability per time unit (both for a single neutron), $\overline{\nu}, \overline{\nu^2}$ are the first and second moments (respectively) of the fission multiplicity and S is the amplitude of the external source (see [4] for full details). Full analysis of equation (4) is difficult, due to the non-linear diffusion coefficient $\sqrt{\tilde{\sigma}^2 N_t + S}$. Thus, a further simplification was suggested, where the diffusion coefficient was replaced by its value under the steady state mean field solution (which is readily seen to be attained by the substitution $N_t = S/\alpha$). This resulted in the equation

$$dN_t = -\alpha N_t dt + S\,dt + \sigma_0 dW_t, \tag{4}$$

where $\sigma_0^2 = \tilde{\sigma}^2 S/\alpha + S$.

One of the challenges in modeling reactor noise is that N_t, which is a basic physical entity, is impossible to measure directly. The only way to experimentally estimate the size of the neutron population is through detectors, which impose two constrains. First, the detection is always performed over a time interval of positive duration. Second, there is randomness associated with the detection. Therefore, a model for reactor noise must couple two (highly correlated) variables: the number of neutrons N_t as it varies within the interval $0 \le t \le T$, and the number of detections, D_T, in the interval $[0, T]$.

[1] A more careful description of the model assumptions is provided in Section 2.

The coupled model proposed in [4] uses the linearization alluded to above as in equation (4), and consists of the following set of SDE,

$$\begin{cases} dN_t = -\alpha_1 N_t dt + \sigma_1 dW_t - dD_t + S\,dt, \\ dD_t = \lambda_d N_t dt + \sigma_2 d\tilde{W}_t. \end{cases} \tag{5}$$

Here,

$$\alpha_1 = \lambda_f + \lambda_\ell - \overline{\nu}\lambda_f,$$

where λ_d is the probability per time unit of a neutron to be detected, λ_ℓ is the probability per time unit of a neutron to be lost without being detected, the term

$$\sigma_1^2 = \frac{S}{\alpha}(\lambda_f + \lambda_\ell + \lambda_f(\overline{\nu^2} - 2\overline{\nu})) + S$$

is the variance of the contribution of all the reactions but the detections, and

$$\sigma_2^2 = \frac{S}{\alpha}\lambda_d$$

is the variance associated with detection. The model (5) was extensively studied in [4]. It has been shown that the first two central moments of the detection count can be explicitly computed, and the analytic expressions were in full agreement with the classical results. Moreover, it was argued that the above model predicts that the so called Feynman-Y function can be sampled using the mean average deviation (MAD) in place of the variance. This prediction was tested in [4], and confirmed with very high accuracy.

Yet, the model (5) has some limitations. The solution process is Gaussian, and thus its steady state, that is also Gaussian, can be calculated easily. In particular, the central third moment is zero. However, it has been shown in previous studies [6, 7] that the third moment is not zero, and thus the model from (5) is oversimplifying with regard to this aspect.

The goal of the present study is to propose a modification of the above model by considering *non-destructive binomial detection*. As we will demonstrate, this model will give rise to an explicit formula for the detection distribution, as well as simple formulas for its high moments. At the same time, the model is subjected to two restrictions. First, the analysis is based on the assumption that the detection efficiency is poor (which, in reactor cores, is often the case). Second, the analysis will only be accurate for systems close to criticality.

2. Model assumptions and definitions

As in most basic models for reactor noise, we restrict the analysis to the single group model (the point reactor in a single energy group). In a sub critical system under the single energy point model, the neutron population is modeled in terms of four parameters:

1. The fission probability per time unit, denoted by λ_f.
2. The absorption probability per time unit, denoted by λ_a.
3. The distribution of the number of neutrons emitted in a fission (or the *neutron multiplicity*), denoted by $\{p(\nu)\}_{\nu=0}^{\nu_{max}}$.

4. The source intensity, here denoted by S. In a stochastic setting, the source intensity is the probability (per time unit) for the external source to emit a neutron.

Denote by $\lambda = \lambda_f + \lambda_a$ the reaction probability per time unit. This parameter can otherwise be characterized as the reciprocal average die-away time of a neutron. Moreover, $p_f = \lambda_f/\lambda$ and $p_a = \lambda_a/\lambda$ give the fission and absorption probabilities, respectively.

We shall need the following standard facts about solutions to SDE of the form

$$X_t = -AX_t + \sigma dW_t$$

where $A > 0$ and $\sigma > 0$ are constants, such as (4). If the initial condition is normal and independent of the driving BM W, then the process X is a Gaussian process, known as an Ornstein-Uhlenbeck process [8]. If, specifically, the initial condition X_0 is distributed as $\mathcal{N}(S/A, \sigma^2/(2A))$ then the process is *stationary*, and one has

$$E(X_t) = \frac{S}{A}, \qquad E(X(t)^2) = \frac{\sigma^2}{2A} + \frac{S^2}{A^2}, \qquad \mathrm{Var}(X_t) = \frac{\sigma^2}{2A}. \tag{6}$$

Moreover, its *autocorrelation function*, defined by $\phi_X(\tau) = E(X_t X_{t+\tau})$, is (independent of t and) given by

$$\phi_X(\tau) = \frac{\sigma^2}{2A} e^{-A\tau}. \tag{7}$$

For the model introduced below, it will indeed be assumed that the initial condition N_0 is distributed as $\mathcal{N}(S/\alpha, \sigma^2/(2\alpha))$, independent of the driving BM, by which the stationary alluded to above, and formulas (6) and (7), are in force.

III. AN SDE FOR NON-DESTRUCTIVE DETECTION

1. A model for the detection count distribution

In equation (4), the solution N_t describes the number of neutrons as a function of time, t. The random variable $\int_0^T N_t dt$, which is measured in units of time, can be interpreted as the accumulated time spent in the system before reacting, summed over all neutrons present in the system within the time interval $[0, t]$. If we denote by ℓ the average lifetime of a neutron and assume that $\ell \ll T$, then the integral

$$X_T = \frac{1}{\ell} \int_0^T N_t dt \tag{8}$$

gives an approximation to the number of neutrons passing through the system in the interval $[0, T]$. Notice that $1/\ell$ is equal to the total reaction rate λ. Since, under the diffusion approximation $\{N_t\}$ is a Gaussian process, and X_T is a linear operation on its trajectories, X_T has a normal distribution. In particular, it is uniquely defined by its mean value and standard deviation. Hence our first task will be to compute these two parameters for X_t.

For the mean, interchanging the order of expectation and integration gives

$$\begin{aligned} E(X_T) &= E\left(\frac{1}{\ell}\int_0^T N_t dt\right) = \frac{1}{\ell}\int_0^T E(N_t)dt = \frac{1}{\ell} \times \frac{ST}{\alpha} \\ &= \frac{ST}{1-k} \end{aligned}$$

For the standard deviation, we first center the population process about its mean by denoting $\tilde{N}_t = N_t - \frac{S}{\alpha}$ and write

$$Y_T = \frac{1}{\ell}\int_0^T \tilde{N}_t dt$$

Then, we notice that $\tilde{N}_t$ satisfies the SDE

$$d\tilde{N}_t = -\alpha \tilde{N}_t dt + \sqrt{\frac{S}{\alpha}\lambda_f \overline{\nu(\nu-1)} + 2S}\, dW_t.$$

Hence the variance of X_t is given by

$$\begin{aligned} Var(X_t) &= E\left[Y_t^2\right] = E\left[\frac{1}{\ell^2}\int_0^T \tilde{N}_t dt \int_0^T \tilde{N}_s ds\right] \\ &= \frac{1}{\ell^2}\int_0^T\int_0^T E\left[\tilde{N}_t \tilde{N}_s\right] dtds \\ &= \frac{1}{\ell^2}\int_0^T\int_0^T \phi_{\tilde{N}}(t-s) dtds \\ &= \frac{1}{\ell^2} 2\int_0^T\int_0^t \phi_{\tilde{N}}(t-s) dtds \end{aligned}$$

where in the last line we used the fact that $\phi_{\tilde{N}}(t) = \phi_{\tilde{n}}(-t)$. By equation (7), $\phi_{\tilde{N}}(t) = (2\alpha)^{-1}[\frac{S}{\alpha}\lambda_f \overline{\nu(\nu-1)} + 2S]e^{-\alpha t}$, and we can compute the integral explicitly, resulting in

$$\begin{aligned} Var(X_t) &= \frac{\frac{St}{\alpha}\lambda_f \overline{\nu(\nu-1)} + 2St}{(1-k)^2}\left[1 - \frac{1-e^{-\alpha t}}{t\alpha}\right] \\ &= \frac{St(P_f \overline{\nu(\nu-1)} + 2(1-k))}{(1-k)^3}\left[1 - \frac{1-e^{-\alpha t}}{t\alpha}\right] \end{aligned} \tag{9}$$

As in the previous section, we denote by D_t the number of detections in the interval $[0, t]$. Recall our interpretation of X_T as a count of neutrons within the time interval $[0, T]$. Each of these X_T neutrons may or may not create a detection, and detection follows a fixed probability per neutron. Hence the conditional law of D_T given X_T has a binomial distribution with parameters (X_T, P_d). The three elements of the model are thus described by the set of equations

$$\begin{cases} dN_t = -\alpha N_t dt + \sigma_1 dW_t + S\, dt, \\ X_T = \frac{1}{\ell}\int_0^T N_t dt, \\ D_T \sim \mathrm{Bin}(X_T, P_d). \end{cases} \tag{10}$$

By the law of total probability, denoting by $[x]$ the integer part of $x \in \mathbb{R}$,

$$\begin{aligned} &P(D_t = n) = \textstyle\sum_{\ell=0}^{\infty} P(D_t = n | [X_t] = \ell) P([X_t] = \ell) \\ &= \textstyle\sum_{\ell=n}^{\infty} \binom{\ell}{n} P_d^n (1-P_d)^{(\ell-n)} \frac{1}{\sqrt{2\pi}\sigma_{X_t}} \int_\ell^{\ell+1} e^{\frac{\xi - S/(1-k)}{2\sigma_{X_t}^2}} d\xi \end{aligned} \tag{11}$$

where σ_{X_t} is the square root of the variance whose formula appears in equation (9). Equation (11) provides a novel formula for the probability distribution of the neutron detection count. To the best of out knowledge, it is the first explicit formula which is not under the assumption that the neutron count distribution is normal.

The derivation of equation (11) was carried out under two assumptions. To test how these assumptions are manifested in technical conditions for the applicability of equation (11), we shall compute the first two central moment of D_T and compare with the classical formulas. To this end, apply the laws of total expectation and total variance, to get

$$E(D_t) = E(E(D_t|X_t)) = P_d E(X_t) = P_d \frac{S\,t}{1-k} \tag{12}$$

$$\begin{aligned} Var(D_t) &= Var(E(D_t|X_t)) + E(Var(D_t|X_t)) && (13)\\ &= P_d^2 Var(X_t) + E(X_t) P_d (1-P_d) \\ &= P_d^2 \frac{S\,t(P_f \overline{\nu(\nu-1)} + 2(1-k))}{(1-k)^3} \left[1 - \frac{1-e^{-\alpha t}}{t\alpha}\right] && (14) \end{aligned}$$

$$+ \frac{S\,t}{1-k}(P_d - P_d^2) \tag{15}$$

Comparing the above formula with the classical formula for the variance [5] shows that for the approximation to be accurate, two conditions must be met. First, $2(1-k) \ll P_f \nu(\nu-1)$, so that the first term in the denominator on (14) vanishes, and $P_d^2 \ll P_d$, so that the term (15) takes the form $S\,t P_d/(1-k)$. Thus, we may conclude that there are two necessary conditions for equation (11) to be applicable: First, the system should be close to critical, and second, the detection efficiency should be sufficiently small (which was the original assumption introduced earlier in this section).

In the next section we compute higher moments of the detection count distribution, and in Section V., verify the formulas by comparison to experimental results.

IV. COMPUTING THE DETECTION COUNT DISTRIBUTION MOMENTS

In this section we use the model introduced above to formulate simplified expressions for the third and fourth moments of the neutron detection count distribution in an interval of fixed duration T. Computing the higher moments of this distribution was the subject of several studies in recent years. In [7], the third central moment was computed, aiming at an alternative sampling of the Feynman-Y curve, and in [6], analytic expressions for the third and fourth moments were introduced, in the context of uncertainty analysis of the Feynman-α method. We restrict our analysis to the third and fourth moments, yet the exact same considerations may be used for any moment. The analysis is divided into two sections. The first gives some background on cumulants and the law of total cumulance, and the second derives formulas for the third and fourth moments.

1. The moments and cumulants of a random variable

For a random variable X, the *cumulant* generating function is given by

$$K_X(u) = \log E\left(e^{uX}\right), \quad u \in \mathbb{R}.$$

When this function has a convergent Taylor expansion about the origin, we can write it as

$$K_X(u) = \sum_{n=0}^{\infty} u^n \frac{k_n(X)}{n!}.$$

The coefficient $k_n(X)$ is called the *n-th cumulant of X*.

For a random vector $X = (X_1, X_2, \ldots, X_N)$, the joint cumulant generating function is defined as

$$K_{\boldsymbol{X}}(u_1, u_2, \ldots, u_N) = \log E\left(e^{\sum_{j=1}^{N} u_j X_j}\right).$$

We adopt the following multi-index notation: for $1 \le m_1, m_2, \ldots, m_n \le N$, $k(X_{m_1}, X_{m_2}, \ldots, X_{m_n})$ is the coefficient of $u_{m_1} \times u_{m_2} \times \cdots \times u_{m_n}$ in $K_{\boldsymbol{X}}(u_1, u_2, \ldots, u_N)$. Notice that there may be multiple tuples $(X_{m_1}, \ldots, X_{m_n})$ that refer to the same coefficient. For instance $k(X_1, X_1, X_2) = k(X_1, X_2, X_1) = k(X_2, X_1, X_1)$.

Two facts on cumulants that will be useful for us are as follows. First, there is a relation between moments and cumulnats. The n-th cumulant is always a polynomial function in the first n moments. For instance, if we denote by $\mu_n(X)$ the n-th centered moment of the random variable X, then

$$\begin{aligned} k_1(X) &= E(X), \quad k_2(X) = \mu_2, \quad k_3(X) = \mu_3(X), \\ k_4(X) &= \mu_4(X) - 3\mu_2^2(X). \end{aligned} \tag{16}$$

The second useful fact is the so called *law of total cumulance*, that may be considered a generalization of the law of total expectation. For a random vector $X = (X_1, X_2, \ldots, X_N)$ and a random vector Y, one has [9]

$$k(X_1, X_2, \ldots, X_N) = \sum_{\pi} k\left(k\left(X_{\pi_1}|Y\right), k\left(X_{\pi_2}|Y\right), \ldots, k\left(X_{\pi_n}|Y\right)\right),$$

were the sum extends over all partitions π, of the form $(\pi_1, \ldots, \pi_n)$ (with arbitrary $n \le N$) of the set $\{1, ..., N\}$ of indices, and the expression $k(X_{\pi_m}|Y)$ gives the cumulant associated with the conditional distribution of the random variables whose indices are in the set π_m, given Y. The cases that will be used here are

$$\begin{aligned} k_3(X) &= k(X,X,X) = k(k_3(X|Y)) && (17)\\ &\quad + k_3(k(X|Y)) + 3k(k(X|Y), k_2(X|Y)) \\ k_4(X) &= k(X,X,X,X) = k_1(k_4(X|Y)) \\ &\quad + 4k(k_3(X|Y), k_4(X|Y)) + 3k_2(k_2(X|Y)) + k_4(k_1(X|Y)) \\ &\quad + 6k(k_2(X|Y), k_1(X|Y), k_1(X|Y)). \end{aligned}$$

2. Explicit formulas for the third and fourth central moments

In the present section, we use the law of total cumulance in the form (17) to compute the third and fourth moments of the neutron detection count distribution. Since the conditional law of D_T given X_T has a binomial distribution, setting $X = D_T$ and $Y = X_T$ in (17) provides expressions for the third and fourth central moments. Assuming $P_d^2 \ll P_d$, it is easy to show that

$$k_2(D_T|X_T) \approx k_3(D_T|X_T) \approx k_4(D_T|X_T) \approx E(D_T|X_T) = \frac{S \times P_d}{1-k}$$

and equations (17) give

$$\begin{aligned} k_3(D_T) &= k_1(P_d X_T) + k_3(P_d X_T) + 3k_2(P_d X_T) \\ k_4(D_T) &= k_1(P_d X_T) + 7k_2(P_d X_T) + 6k_3(P_d X_T) + k_4(P_d X_T). \end{aligned}$$

Since X_T has a normal distribution, and the third and fourth cumulants of a normal distribution are both zero, the above relations become

$$k_3(D_T) = k_1 + 3k_2(P_d X_T) \quad (18)$$
$$k_4(D_T) = k_1(P_d X_T) + 7k_2(P_d X_T) \quad (19)$$

By (12) and the second equality in (13), we have

$$k_1(P_d X_T) = P_d E(X_T) = E(D_T)$$
$$k_2(P_d X_T) = P_d^2 Var(X_T) \approx Var(D_T) - E(D_T)$$

resulting in the following approximations for the third and fourth central moments,

$$\mu_3(D_T) \approx 3Var(X_T) - 2E(X_T) \quad (20)$$
$$\mu_4(D_T) \approx E(D_T) + 7\left(Var(D_T) - E(D_T)\right) + 3Var^2(D_T) \quad (21)$$

(the last term in (21) is due to equation (16)).

Equations (20) and (21) are somewhat surprising for two reasons. First, they are extremely simple. The simplicity is even more surprising when compared to the precise expressions obtained in [7] and [6], which are very lengthy. Thus the approximation we have obtained, justified by the fact that P_d is typically several orders of magnitude less then one, is very useful. The price paid for the simplicity is not expected to be noticed experimentally, for the reasons just mentioned.

The second surprising aspect of (20) and (21) is crucial. These equations indicate that from a practical point of view, once the first and second moments have been sampled, *no significant new information can be obtained by sampling the third and fourth moments*. When sampling the first and second moment through the Feynman-Y formula, we can measure the α eigenvalue. Thus in principle higher moments of the detection count distribution do carry further information about the system. However, from a practical point of view, equations (20) and (21) indicate that such information corresponds to observables that are simply too small to measure.

V. EXPERIMENTAL RESULTS

We now turn to the comparison of equations (20) and (21) to experimental results (since under the conditions that $|1 - k| \ll 1$ and $P_d \ll 1$, equations (12) and (13) coincide with the classical results, there is no point in validating these equations).

To validate (20) and (21), we have sampled all first four central moment of detection signal from the MINERVE reactor, in two sub critical states: $\rho = -270pcm$ (Acq16) and $\rho = -120pcm$ (Acq16) (both with a 10% error bar). In each experiment two detectors were active, resulting in a total of four signals. Each measurement was roughly 1.5 hours. The detection count rate was less that 50,000 CPS and the estimated dead time was 160nanosecond. Thus, dead time losses are negligible.

Figures 1 and 2 show the detection count distribution in two of the signals in a time interval of duration $T = 10^{-3}$.

At a first glance, it may seem that both distributions are normal. However, one can verify numerically that there is

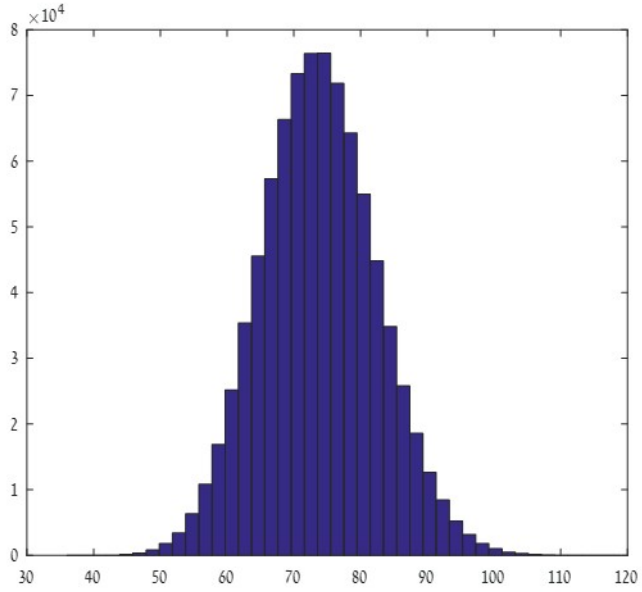

Fig. 1. Count distribution in measurement Acq16.

a certain "skewness" of the distribution to the right. This skewness can be quantified by sampling the cubic root of the central third moment and computing to what degree it is scalable with the standard deviation. We have followed this scheme. Table I lists the standard deviation ($\sigma(D_T)$) and the cubic root of the third central moment (that we also call the normalized third central moment, $\sqrt[3]{|\mu_3(D_T)|}$) for each of the signals analyzed.

Examining table I shows that in all four measurements, $\sqrt[3]{|\mu_3(D_T)|}$ is approximately 50% of the standard deviation. Thus, all four signals indicate a significant deviation from the normal distribution. Moreover, in all four signals the third central moment, μ_3, was positive, meaning that the distribution is skewed to the right, as expected.

Table II shows the sampled values of the first four central moments, along with the values predicted by formulas (20) and (21).

	Asq16 det1	Acq16 det2	Acq19 det	Acq19 det2
σ	8.769	9.042	6.87	6.517
$\sqrt[3]{\lvert\mu_3\rvert}$	4.38	4.490	3.525	3.630

TABLE I. Normalized second and third central moments.

	$E(D_T)$	$Var(D_T)$	$\mu_3(D_T)$	$\mu_4(D_T)$	Eq. (20)	Eq. (21)
Acq16 det1	39.05 ± 0.03	40.7 ± 0.1	44.0 ± 0.9	5042 ± 20	44.28	5045
Acq16 det2	40.28 ± 0.05	42.39 ± 0.2	46.68 ± 0.9	5446 ± 30	46.62	5447
Acq19 det1	73.74 ± 0.1	76.93 ± 0.2	83.37 ± 1.8	17843 ± 100	83.29	17851
Acq19 det2	77.92 ± 0.1	81.76 ± 0.1	89.05 ± 1.6	20173 ± 89	89.44	20161

TABLE II. Sampled values of $\mu_3(D_T)$ and $\mu_4(D_T)$, and the values predicted by equations (20) and (21).

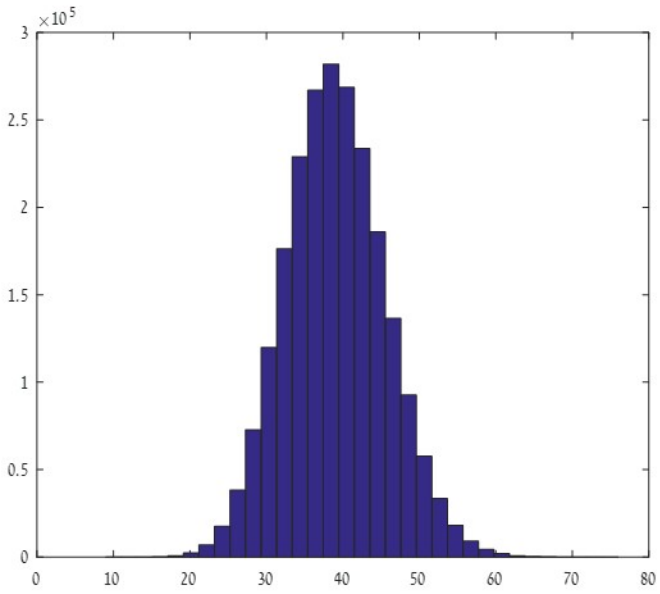

Fig. 2. Count distribution in measurement Acq19.

Table II shows excellent agreement between the sampled values of $\mu_3(D_T)$ and $\mu_4(D_T)$, and the values predicted by (20) and (21). The maximal error is 0.98% (estimating $\mu_3(D_T)$ in measurement Acq 16 det2) and the average error is 0.2%.

VI. CONCLUDING REMARKS

In this work we introduced a new modeling scheme for the neutron detection count distribution in a nuclear reactor core, based on the SDE approach from [4], and demonstrated its accuracy and applicability.

It has been established that the model is valid under two conditions: the detection efficiency is sufficiently small, and the system is nearly critical. Both conditions are typically met when reactor noise experiments are held.

Through stochastic analysis of the model, an explicit formula for the detection count distribution was derived, as well as formulas for its third and fourth central moments. These moments were compared with experimental results, showing very good accuracy, with an average error of 0.2% and a maximal error of 0.98%.

Under the aforementioned conditions, it was shown that the third and fourth moments of the detection count distribution are functions of the first and second moments. We conclude from this that sampling the third and fourth central moments does not add any significant information on the system parameters beyond what can be attained from the first and second moments.

The model introduced here accounts for neutron fluctuations due to the inherent stochastic nature of the fission chain. In future work it is intended to use this approach to address far broader noise settings.

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