

Nodal and Pin-by-pin Calculations Comparison with Codes SIMULATE-5 and DYN3D

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Abstract – *The reactor cores have become in the latest years more heterogeneous than in the past and the widely used nodal codes are observing higher errors. Recently, more advanced techniques have been developed, mainly to describe more precisely the leakage on different assembly interface and on the fuel/reflector interface. This study is aimed at quantifying the differences between the advanced nodal code SIMULATE-5, which uses online rehomogenization technique of cross sections during the whole core calculation, and the pin-by-pin code DYN3D. Calculations of 2D problems with and without control rods insertion, burnable absorbers and MOX fuel are presented. The results show that the pin-by-pin code can significantly improve the pin power prediction, but the assembly power error can be even higher, in particular in cores with steep power gradient.*

I. INTRODUCTION

Even with modern computers, 3D whole core heterogeneous calculations are still too time demanding for routine core design and safety analysis calculations. Nowadays, cross sections are homogenized in single assembly 2D calculations and then used in nodal assembly-wise 3D calculations employing usually diffusion solvers.

Since in the latest years the cores have become more heterogeneous than in the past, including for instance, MOX fuel, burnable absorbers and loading with highly depleted assemblies, there is a need for more accurate solvers. One of the main problems of the widely used nodal solvers is that the cross sections are generated in lattice calculation assuming the reflective boundary condition which results in an error on the different assembly interfaces. To decrease this error, the codes employ usually the critical spectrum calculation in the lattice code and then assembly discontinuity factors in the core code [1].

At the moment, in reactor analysis, the nodal codes are based on advanced techniques mostly using spectral cross section rehomogenization [2-3]. However, an alternative option to nodal codes can be the usage of pin-by-pin solvers and the pin-wise homogenization of the cross sections instead of assembly-wise. This paper is aimed at quantifying the differences between the advanced nodal code SIMULATE-5 and the pin-by-pin solver DYN3D. For the comparison two physical systems have been computed: a minicore for addressing the differences between the solvers, a 2D core with highly heterogeneous loading, in order to observe the performances of the codes when computing challenging physical systems.

II. METHODOLOGY AND CODES

In order to exclude cross section library differences as a source of discrepancy between SIMULATE-5 and DYN3D results, the cross sections have been generated with the

same lattice code CASMO-5 (C5) [4]. Two libraries are produced: the nodal and the pin-cell libraries. As nodal homogenization method of the cross sections, C5 adopts the flux-volume weighting. However, for pin-by-pin solvers, the flux-volume weighted homogenization method is not sufficient since it does not preserve the leakage rates between adjacent pin-cells. Therefore, an equivalence technique must be employed as an additional intermediate step between the lattice code and the pin-by-pin solver.

In this study, the superhomogenization (SPH) method has been used as equivalence technique [5]. It is an iterative method which can preserve reaction rates in the pin cells by multiplying the pin-cell cross sections by SPH factors, which are defined as the ratio of heterogeneous and homogeneous scalar flux. The convergence criterion was set to 10^{-4} . The factors can preserve very well the multiplication factor and the pin power but since they are calculated always for single assembly, some errors on the assembly interfaces remain [6].

The lattice calculation is done with reflective boundary conditions assuming zero leakage. The effects of the non-zero leakage in a real reactor core can be taken into account by the critical spectrum calculation. In CASMO-5 the fundamental mode (FUM) method is used [7].

For the generation of the library, CASMO-5 includes default tables of cases adequate for the core simulations in nominal conditions. These tables comprise depletion and branch perturbation cases for considering different core state variables such as boron concentration, moderator and fuel temperatures, and control rod insertion. For SIMULATE-5, the regular binary nuclear library is prepared with the Studsvik code CMSlink5, while for DYN3D an in-house pin-by-pin library format has been used. From the technical point of view, the pin-by-pin library is saved in HDF5 format [8] in order to improve efficiency when searching for values to interpolate. An interpolation routine has been implemented, using the N-dimensional “bicubic” spline, where N corresponds to the number of core state variables

included in the library. Interpolation in burnup axis is performed in logarithmic scale. In the interpolation, an exception is made for the points at the beginning of (0 - 0.1 MWd/kg_{HM}) for which quadratic functions have a better interpolating behavior.

Since benchmarks with measured pin powers lack in literature, the study was done for 2D core which allows comparison of the results with reference multi-assembly CASMO-5 solution.

1. Core analysis codes

A. SIMULATE-5

SIMULATE-5 (S5) is a nodal multi-group code, solving diffusion equation using the analytic nodal method [2]. The core calculation can be performed with 1 or 4 (2x2) radial nodes per assembly. To achieve a higher spatial accuracy, S5 performs rehomogenization of the nuclear data based on more spatially detailed whole core radial and more detailed assembly axial calculation. The default submeshing for the radial calculation is 9x9/assembly.

Two pin power reconstruction techniques are available in S5 - the SMX method, also used in this study, and the Fourier Flux Expansion method. In SMX method, the flux and power in a node are calculated by multiplying S5 flux with pin power form functions obtained during the C5 lattice calculation.

B. DYN3D

DYN3D [9] is a nodal or pin-by-pin code, capable of solving the neutron balance with multi-group diffusion (D) or SP₃ method. In this study, DYN3D is used only as the pin-by-pin code.

The solver of the code uses the nodal expansion method in which the flux is expanded into up to second order polynomials and into exponential functions being the solutions of the homogeneous Helmholtz equation.

III. MODEL AND CODES SPECIFICATIONS

Two cores are considered for the comparison of S5 and DYN3D: a minicore and a 2D quarter core with highly heterogeneous loading. The assemblies and the core configuration selected for this study are based on the OECD/NEA PWR MOX/UF₆ benchmark [10].

The assemblies have a lattice of 17x17 rods with 25 guide tubes and no water gap around the fuel region, as shown in Figure 1. There are two types of MOX assemblies (with 4.0 % or 4.3 % of plutonium), composed of fuel rods with weight fractions of 4.5 % (or 5 %, resp.), 3.0 % and 2.5 % of plutonium; and two types of uranium assemblies containing rods with 4.2 % or 4.5 % enrichment. Some rods in the uranium assemblies are doped with the IFBA

absorber (ZrB₂). Control rods can be inserted into the uranium assembly.

The studied minicore is a 2D 6x6 assemblies system composed of 8 fresh MOX and 28 fresh uranium assemblies, 5 of them with control rods inserted. The layout is shown in Figure 2. The minicore has reflective boundary.

The 2D quarter core is composed of two types of MOX assemblies and two types of uranium assemblies. For this core, two run-cases are considered: the all rods out condition and the controlled core with several control rods inserted. The loaded assemblies have burnups ranging from 0.1 to 37.5 MWd/kg_{HM}. The layout of the controlled core is shown in Figure 3. The core is surrounded by a water reflector of the same width as the assembly pitch. The axial boundary conditions are reflective.

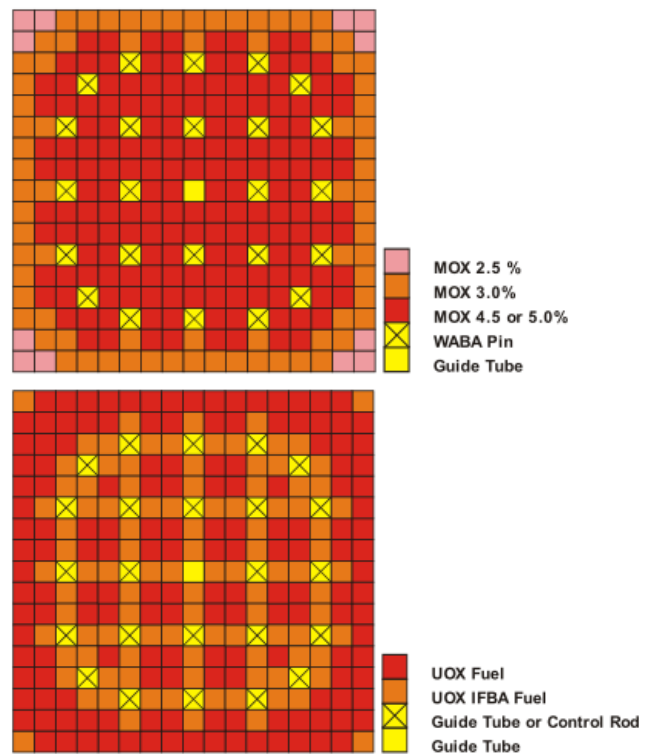


Fig. 1. MOX (above) and uranium (below) assembly layout [10].

	1	2	3	4	5	6
A	U 4.2 %	U 4.2 %	U 4.2 %	U 4.2 %	U 4.2 %	M 4.3 %
B	U 4.2 %	U 4.2 %	U 4.2 %	M 4.3 %	U 4.2 %	U 4.2 %
C	U 4.2 %	U 4.2 %	U 4.2 %	U 4.2 %	U 4.2 %	M 4.3 %
D	U 4.2 %	M 4.3 %	U 4.2 %	M 4.3 %	U 4.2 %	U 4.2 %
E	U 4.2 %	U 4.2 %	U 4.2 %	U 4.2 %	U 4.2 %	U 4.2 %
F	M 4.3 %	U 4.2 %	M 4.3 %	U 4.2 %	U 4.2 %	M 4.3 %

Fig. 2. Minicore configuration (red are assemblies with CR and orange are MOX assemblies).

	1	2	3	4	5	6	7	8
A	U 4.2% CR 35.0	U 4.2% 0.1	U 4.2% CR 22.5	U 4.5% 0.1	U 4.5% 37.5	M 4.3% 17.5	U 4.5% CR 0.1	U 4.2% 32.5
B	U 4.2% 0.1	U 4.2% 17.5	U 4.5% 32.5	M 4.0% 22.5	U 4.2% 0.1	U 4.2% 32.5	M 4.0% 0.1	U 4.5% 17.5
C	U 4.2% CR 22.5	U 4.5% 32.5	U 4.2% CR 22.5	U 4.2% 0.1	U 4.2% 22.5	M 4.3% 17.5	U 4.5% CR 0.1	M 4.3% 35.0
D	U 4.5% 0.1	M 4.0% 22.5	U 4.2% 0.1	M 4.0% 37.5	U 4.2% 0.1	U 4.5% 20.0	M 4.3% 0.1	U 4.5% 20.0
E	U 4.5% 37.5	U 4.2% 0.1	U 4.2% 22.5	U 4.2% 0.1	U 4.2% CR 37.5	U 4.5% 0.1	U 4.2% 17.5	
F	M 4.3% 17.5	U 4.2% 32.5	M 4.3% 17.5	U 4.5% 20.0	U 4.5% 0.1	M 4.3% 0.1	U 4.5% 32.5	
G	U 4.5% CR 0.1	M 4.0% 0.1	U 4.5% CR 0.1	M 4.3% 0.1	U 4.2% 17.5	U 4.5% 32.5		
H	U 4.2% 32.5	U 4.5% 17.5	M 4.3% 35.0	U 4.5% 20.0	Assembly type CR position Burnup [MWd/kg]			

Fig. 3: Quarter core configuration with water reflector.

For all the run-cases, the cross sections were produced with C5, the MOC calculation was performed in 35 groups and the nuclear data for both S5 and DYN3D were collapsed down to 8 groups. The pin power form functions used for pin power reconstruction in S5 were prepared in 2 groups.

The calculations were performed with no thermal-hydraulic feedback by fixing the state variables as follows: fuel temperature of 900 K, moderator temperature of 580 K, and boron concentration in the moderator of 1000 ppm.

The settings of the solvers are presented in Table I.

Table I. Solvers settings

Code	Solver setting
CASMO 5	Azimuthal angles: 128, polar angles: 3, ray spacing: 0.05 cm
SIMULATE 5	Diffusion solver 4 nodes/assembly and 9x9 submeshing 1 node/assembly and 17x17 submeshing
DYN3D	Pin-by-pin diffusion and SP ₃ solvers

IV. RESULTS

The results of the minicore with reflective boundary conditions are discussed first, followed by the two run-cases of the quarter core with withdrawn and with inserted control rods. Then, to test the interpolation routine, a test case was run with non-nominal temperatures.

The results are presented in terms of comparisons of the multiplication factors, the assembly and the pin power distributions, all with respect to the reference C5 multi-assembly calculations.

1. Minicore

The discrepancy of DYN3D and S5 against C5 in terms of eigenvalue is given in Table II. The differences are smaller than 20 pcm in all cases.

The assembly power discrepancies are shown in Figures 4 – 7. All the calculations show that the highest discrepancy occurs in/around the controlled assembly in position A1, with S5 being almost 3 % off from C5 and DYN3D showing about 1 % and 0.6 % discrepancies when using the diffusion and the SP₃ method, respectively. The usage of the SP₃ instead of diffusion in DYN3D improves the power prediction, in particular in the controlled assemblies. Nevertheless, it also doubles the computation time as reported in Table III.

The power discrepancy for assemblies A1-B2 is shown more in details up to the pin level in Figure 8. The maximum pin power error occurs in the assembly A1 and lays on the value of 4.5 %, 1.5 % and 0.9 % for S5, DYN3D-D and DYN3D-SP₃, respectively.

When focusing around the MOX assembly in position F6 surrounded by several uranium assemblies (Figure 9), DYN3D shows higher discrepancy in the rods close to the MOX/uranium interfaces. The maximum differences are 1.6 % and 0.7 % when solving diffusion and SP₃ equation, respectively. This is caused by the fact that no assembly discontinuity factors are applied to DYN3D calculations, but they are adopted in S5. Nevertheless, there is up to 2.5 % error in S5 using the 9x9 submeshing in the central pin. This error does not occur when the 17x17 submeshing is used.

	1	2	3	4	5	6
A	-2.78	-2.59	-0.34	-0.11	0.14	0.90
B	-2.59	-1.76	-0.73	0.31	0.00	0.25
C	-0.34	-0.73	0.00	-0.10	-0.15	0.63
D	-0.11	0.31	-0.10	0.38	-0.27	-0.31
E	0.14	0.00	-0.15	-0.27	0.00	-0.47
F	0.90	0.25	0.63	-0.31	-0.47	0.00

Fig. 4. Minicore: as. pow. discrepancy [%] of S5 9x9.

	1	2	3	4	5	6
A	-2.78	-2.96	-0.68	-0.11	0.28	0.96
B	-2.96	-2.26	-0.73	0.00	0.14	0.38
C	-0.68	-0.73	0.00	-0.51	0.00	0.56
D	-0.11	0.00	-0.51	-0.10	-0.18	-0.24
E	0.28	0.14	0.00	-0.18	-0.16	-0.28
F	0.96	0.38	0.56	-0.24	-0.28	0.00

Fig. 5. Minicore: as. pow. discrepancy [%] of S5 17x17.

	1	2	3	4	5	6
A	1.02	-0.37	1.36	-0.21	-0.14	0.13
B	-0.37	-0.25	0.00	0.32	-0.07	-0.06
C	1.36	0.00	1.58	-0.10	-0.08	0.22
D	-0.21	0.32	-0.10	0.20	-0.27	-0.23
E	-0.14	-0.07	-0.08	-0.27	1.28	-0.37
F	0.13	-0.06	0.22	-0.23	-0.37	-0.10

Fig. 6. Minicore: as. pow. discrepancy [%] of DYN3D-D.

	1	2	3	4	5	6
A	0.56	0.37	0.00	-0.11	-0.14	0.00
B	0.37	0.25	0.36	0.32	-0.07	-0.13
C	0.00	0.36	0.23	0.10	0.00	0.14
D	-0.11	0.32	0.10	0.20	-0.09	-0.08
E	-0.14	-0.07	0.00	-0.09	-0.16	-0.18
F	0.00	-0.13	0.14	-0.08	-0.18	0.19

Fig. 7. Minicore: as. pow. discrepancy [%] of DYN3D-SP₃.

Table II. Minicore: multiplication factor and average as. pow. difference.

	K-eff diff [pcm]	P diff [%]
SIMULATE-5 9x9	18	0.54
SIMULATE-5 17x17	-5	0.59
DYN3D-D	-13	0.34
DYN3D-SP ₃	5	0.16

Table III. Minicore: computation time

	Time [s]
SIMULATE-5 9x9	2
DYN3D-D	60
DYN3D-SP ₃	120
CASMO-5 multi-assembly	2250

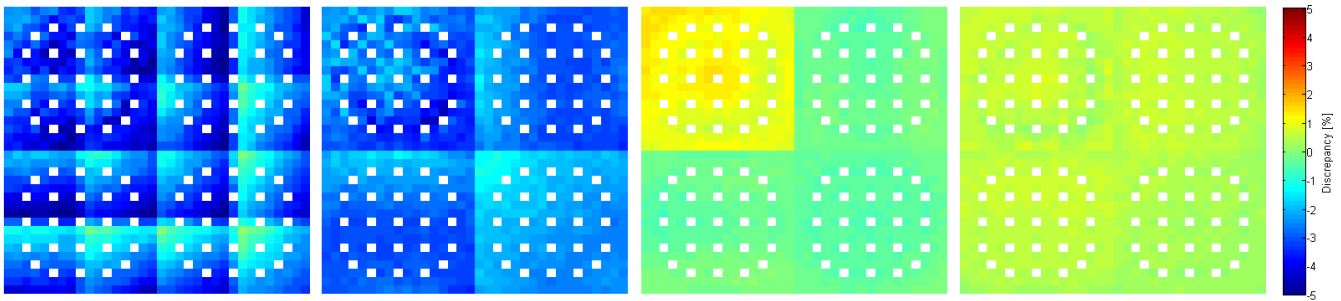


Fig. 8. Minicore: pin power discrepancy [%] of S5 9x9, S5 17x17, DYN3D-D, and DYN3D-SP₃ for assemblies A1-B2.

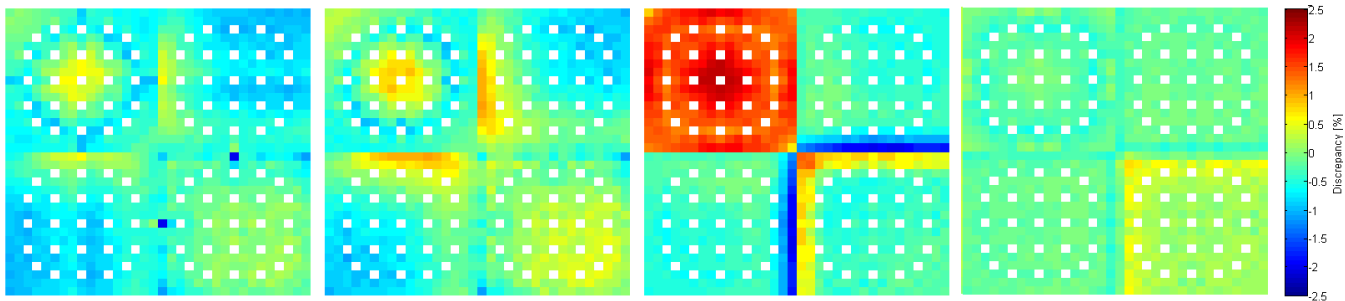


Fig. 9. Minicore: pin power discrepancy [%] of S5 9x9, S5 17x17, DYN3D-D, and DYN3D-SP₃ for assemblies E5-F6.

2. 2D quarter core, ARO

The eigenvalue discrepancy and the average assembly power discrepancy are shown in Table IV. The reference assembly power distribution is provided in the map in Figure 10. From the assembly power discrepancies shown in Figures 11 – 15, it can be noticed that, in case of S5, the highest discrepancy occurs in the assemblies close to the reflector and in the MOX and the higher depleted assemblies in the center. The average assembly power discrepancy is smaller when using the 2x2 node/assembly mesh, with a peak of 1.3 %. In case of DYN3D, the power remains higher than reference at the core periphery and

lower in the center. The average discrepancy is about twice lower when using the FUM option to produce the cross sections in CASMO. The highest power discrepancy occurs in the uranium depleted fuel surrounded by fresh assemblies.

The pin power discrepancies are shown in Figures 16 – 20. The highest pin power discrepancy occurs close to the reflector and is up to 6 % for S5 and up to 3 % for DYN3D-SP₃. In S5 solution with the 9x9 submeshing differences up to 3 % can be observed in certain fuel pins along the MOX/uranium interfaces. This is caused by 9x9 submesh which collects together pins with relatively different neutron spectra [8] for calculating the flux for cross section rehomogenization.

The usage of the diffusion instead of SP₃ in DYN3D does not affect significantly the average assembly discrepancy but increases the error on the MOX/uranium assembly interfaces and in the assemblies along the reflector. The maximum pin power discrepancies are 4% and 2% for the SP₃ and diffusion respectively. (both with cross sections with FUM option).

	1	2	3	4	5	6	7	8
A	1.37	1.76	1.41	1.55	1.03	1.01	1.00	0.43
B	1.76	1.56	1.23	1.25	1.37	0.91	0.95	0.51
C	1.41	1.23	1.32	1.47	1.24	1.08	0.99	0.40
D	1.55	1.25	1.47	1.05	1.33	1.13	0.87	0.38
E	1.03	1.37	1.24	1.33	0.90	1.08	0.61	
F	1.01	0.91	1.08	1.13	1.08	0.74	0.31	
G	1.00	0.95	0.99	0.87	0.61	0.31		
H	0.43	0.51	0.40	0.38				

Fig. 10: ARO core: reference assembly power.

	1	2	3	4	5	6	7	8
A	0.29	-0.75	0.07	-0.46	0.76	0.78	-0.82	0.23
B	-0.75	-0.26	0.32	0.86	-0.22	0.54	-0.10	-0.40
C	0.07	0.32	0.23	-0.28	0.32	0.72	-0.72	0.99
D	-0.46	0.86	-0.28	1.28	-0.31	-0.27	-0.46	0.27
E	0.76	-0.22	0.32	-0.31	0.44	-1.04	-0.33	
F	0.78	0.54	0.72	-0.27	-1.04	-0.67	0.00	
G	-0.82	-0.10	-0.72	-0.46	-0.33	0.00		
H	0.23	-0.40	0.99	0.27				

Fig. 11: ARO core: as. pow. discrepancy [%] of S5 9x9.

	1	2	3	4	5	6	7	8
A	0.93	-0.12	0.71	-0.07	1.05	0.48	-1.53	0.23
B	-0.12	0.39	0.88	1.17	0.00	0.44	-0.94	-0.80
C	0.71	0.88	0.76	0.00	0.48	0.45	-1.54	0.74
D	-0.07	1.17	0.00	1.38	-0.31	-0.44	-1.14	0.80
E	1.05	0.00	0.48	-0.31	0.33	-1.42	-0.33	
F	0.48	0.44	0.45	-0.44	-1.42	-1.61	0.00	
G	-1.53	-0.94	-1.54	-1.14	-0.33	0.00		
H	0.23	-0.80	0.74	0.80				

Fig. 12: ARO core: as. pow. discrepancy [%] of S5 17x17.

	1	2	3	4	5	6	7	8
A	-0.58	-1.70	-0.43	-0.71	1.17	0.89	-0.70	0.00
B	-1.70	-1.16	0.08	0.48	0.22	1.33	-0.21	-1.19
C	-0.43	0.08	0.23	-0.07	0.81	0.92	-0.70	0.00
D	-0.71	0.48	-0.07	1.52	0.53	0.35	-0.58	-1.06
E	1.17	0.22	0.81	0.53	1.78	-0.28	-0.49	
F	0.89	1.33	0.92	0.35	-0.28	-0.54	-0.33	
G	-0.70	-0.21	-0.70	-0.58	-0.49	-0.33		
H	0.00	-1.19	0.00	-1.06				

Fig. 13: ARO core: as. pow. discrepancy [%] of DYN3D-D with FUM.

	1	2	3	4	5	6	7	8
A	-1.90	-2.27	-1.28	-0.90	0.39	0.60	0.10	0.24
B	-2.27	-1.86	-0.98	-0.24	0.00	0.88	0.42	-0.20
C	-1.28	-0.98	-0.53	-0.27	0.56	0.83	0.30	0.51
D	-0.90	-0.24	-0.27	0.76	0.68	0.89	0.46	0.00
E	0.39	0.00	0.56	0.68	1.45	0.65	0.49	
F	0.60	0.88	0.83	0.89	0.65	0.68	0.65	
G	0.10	0.42	0.30	0.46	0.49	0.65		
H	0.24	-0.20	0.51	0.00				

Fig. 14: ARO core: as. pow. discrepancy [%] of DYN3D-SP3 with FUM.

	1	2	3	4	5	6	7	8
A	-3.07	-2.95	-2.27	-1.16	-0.29	0.70	0.70	0.47
B	-2.95	-2.83	-2.11	-0.80	0.07	0.66	1.05	0.20
C	-2.27	-2.11	-1.29	-0.34	0.40	1.11	1.11	0.76
D	-1.16	-0.80	-0.34	0.47	1.06	1.24	1.27	0.53
E	-0.29	0.07	0.40	1.06	1.56	1.48	1.15	
F	0.70	0.66	1.11	1.24	1.48	1.63	0.98	
G	0.70	1.05	1.11	1.27	1.15	0.98		
H	0.47	0.20	0.76	0.53				

Fig. 15: ARO core: as. pow. discrepancy [%] of DYN3D-SP3 without FUM.

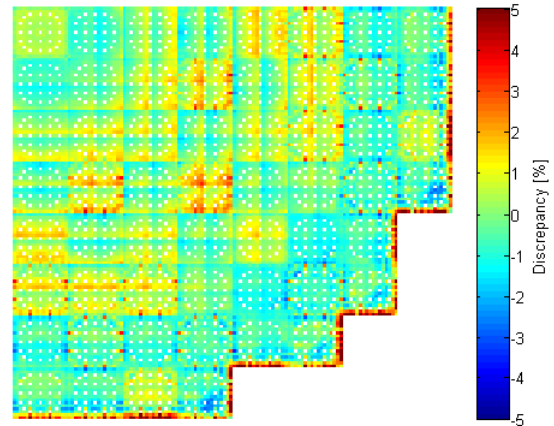


Fig. 16: ARO core: pin power discrepancy [%] of S5 9x9.

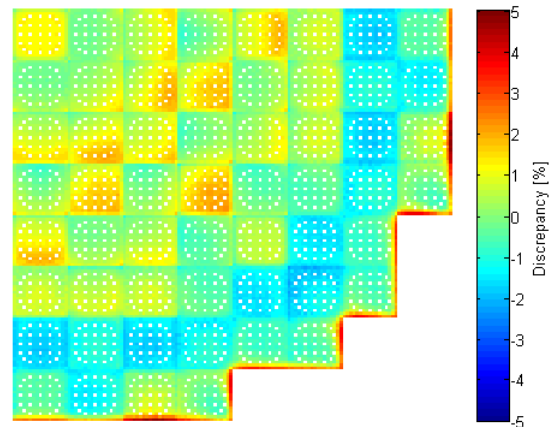


Fig. 17: ARO core: pin power discrepancy [%] of S5 17x17.

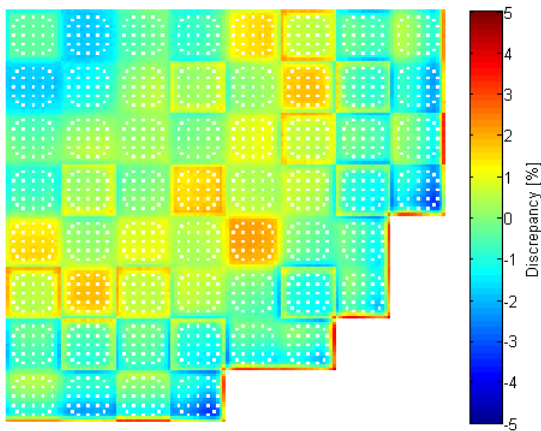


Fig. 18: ARO core: pin power discrepancy [%] of DYN3D-D with FUM.

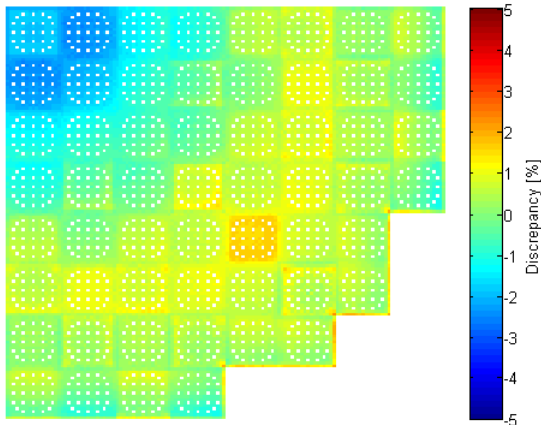


Fig. 19: ARO core: pin power discrepancy [%] of DYN3D-SP3 with FUM.

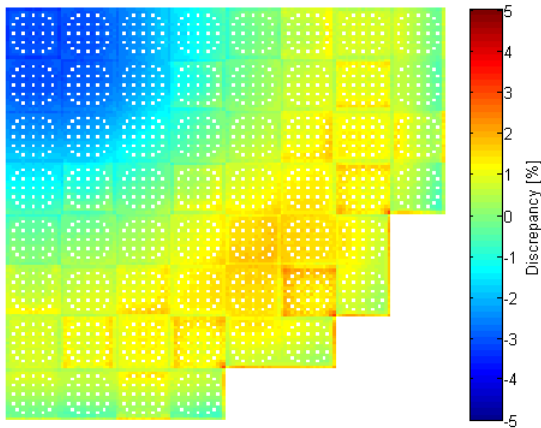


Fig. 20: ARO core: pin power discrepancy [%] of DYN3D-SP3 without FUM.

Table IV. ARO core: multiplication factor and average as. pow. difference

	K-eff diff [pcm]	P diff [%]
Reference k-eff	1.04606	
SIMULATE-5 9x9	-24	0.43
SIMULATE-5 17x17	-35	0.59
DYN3D-D - FUM	16	0.57
DYN3D-SP ₃ - FUM	40	0.61
DYN3D-SP ₃ - no FUM	51	0.96

3. 2D quarter core, controlled

The reference assembly power for the case with inserted control rods is given in Figure 21. The assembly power discrepancy is shown in Figures 22 – 26 and the pin power discrepancy in Figures 27 – 31. The same trend as for the previous case can be observed. In case of S5, the average discrepancy is lower when using the 9x9 submeshing and 2x2 node/assembly mesh than when using the 17x17 submeshing and 1 node/assembly. However, the pin power error is smoother in the second case. In the first case, the error in certain pins along the MOX/uranium interface and in/around the controlled assemblies is bigger.

In case of DYN3D-SP3, the average power error is again smaller when applying the FUM option (0.40 % compared to 0.55 %). When using the diffusion solver, there is up to 3 % assembly difference in the controlled assemblies.

	1	2	3	4	5	6	7	8
A	0.41	0.90	0.56	1.56	1.31	1.12	0.49	0.30
B	0.90	0.91	0.84	1.34	1.78	1.06	0.78	0.39
C	0.56	0.84	0.65	1.61	1.64	1.28	0.54	0.32
D	1.56	1.34	1.61	1.28	1.65	1.43	0.98	0.41
E	1.31	1.78	1.64	1.65	0.64	1.36	0.85	
F	1.12	1.06	1.28	1.43	1.36	1.03	0.46	
G	0.49	0.78	0.54	0.98	0.85	0.46		
H	0.30	0.39	0.32	0.41				

Fig. 21: Controlled core: reference assembly power.

	1	2	3	4	5	6	7	8
A	0.00	-1.37	0.53	0.00	0.90	1.05	-0.41	-0.33
B	-1.37	-0.55	0.71	1.17	-0.23	0.84	0.00	-1.03
C	0.53	0.71	0.90	0.00	0.24	0.84	-0.37	0.62
D	0.00	1.17	0.00	1.52	-0.37	-0.42	-0.70	-0.49
E	0.90	-0.23	0.24	-0.37	0.76	-1.50	-0.95	
F	1.05	0.84	0.84	-0.42	-1.50	-1.25	-0.22	
G	-0.41	0.00	-0.37	-0.70	-0.95	-0.22		
H	-0.33	-1.03	0.62	-0.49				

Fig. 22: Controlled core: as. pow. discrepancy [%] of S5 9x9.

	1	2	3	4	5	6	7	8
A	-0.24	-1.14	0.53	0.13	1.13	1.14	-0.41	1.00
B	-1.14	-0.55	0.83	1.10	0.00	0.93	0.00	-0.51
C	0.53	0.83	0.90	0.06	0.37	0.69	-0.55	1.54
D	0.13	1.10	0.06	1.44	-0.37	-0.63	-1.11	0.99
E	1.13	0.00	0.37	-0.37	0.15	-1.96	-0.83	
F	1.14	0.93	0.69	-0.63	-1.96	-2.31	-0.43	
G	-0.41	0.00	-0.55	-1.11	-0.83	-0.43		
H	1.00	-0.51	1.54	0.99				

Fig. 23: Controlled core: as. pow. discrepancy [%] of S5 17x17.

	1	2	3	4	5	6	7	8
A	0.74	-2.23	2.33	-0.26	1.53	1.07	1.64	-0.67
B	-2.23	-1.66	0.36	0.67	0.28	1.23	-0.26	-2.04
C	2.33	0.36	2.93	0.00	0.61	0.62	1.47	-0.63
D	-0.26	0.67	0.00	1.33	-0.12	-0.49	-1.43	-2.21
E	1.53	0.28	0.61	-0.12	2.99	-1.54	-1.65	
F	1.07	1.23	0.62	-0.49	-1.54	-1.95	-1.30	
G	1.64	-0.26	1.47	-1.43	-1.65	-1.30		
H	-0.67	-2.04	-0.63	-2.21				

Fig. 24: Controlled core: as. pow. discrepancy [%] of DYN3D-D with FUM.

	1	2	3	4	5	6	7	8
A	-0.49	-1.56	0.36	-0.38	0.53	0.18	0.00	-1.01
B	-1.56	-1.22	-0.24	0.15	0.00	0.38	-0.39	-1.28
C	0.36	-0.24	0.77	-0.06	0.43	0.31	0.37	-0.63
D	-0.38	0.15	-0.06	0.63	0.18	0.14	-0.31	-0.98
E	0.53	0.00	0.43	0.18	1.42	-0.22	-0.35	
F	0.18	0.38	0.31	0.14	-0.22	-0.20	-0.22	
G	0.00	-0.39	0.37	-0.31	-0.35	-0.22		
H	-1.01	-1.28	-0.63	-0.98				

Fig. 25: Controlled core: as. pow. discrepancy [%] of DYN3D-SP3 with FUM.

	1	2	3	4	5	6	7	8
A	-2.22	-2.12	-1.07	-0.38	-0.08	-0.09	-0.61	-1.68
B	-2.12	-1.99	-1.07	-0.15	0.00	-0.09	-0.39	-1.53
C	-1.07	-1.07	-0.31	0.06	0.24	0.39	0.18	-0.63
D	-0.38	-0.15	0.06	0.55	0.55	0.49	0.41	-0.74
E	-0.08	0.00	0.24	0.55	0.94	0.66	0.24	
F	-0.09	-0.09	0.39	0.49	0.66	0.78	0.22	
G	-0.61	-0.39	0.18	0.41	0.24	0.22		
H	-1.68	-1.53	-0.63	-0.74				

Fig. 26: Controlled core: as. pow. discrepancy [%] of DYN3D-SP3 without FUM.

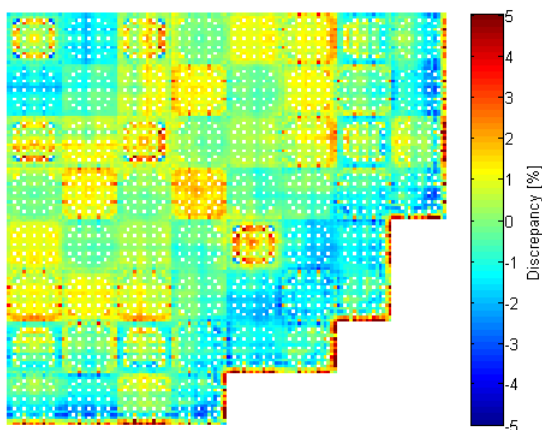


Fig. 27: Controlled core: pin power discrepancy [%] of S5 9x9.

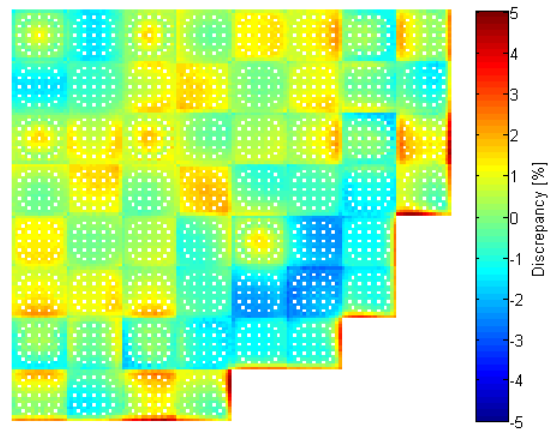


Fig. 28: Controlled core: pin power discrepancy [%] of S5 17x17.

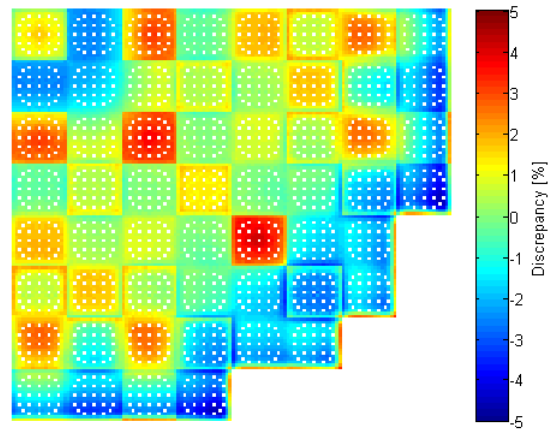


Fig. 31: Controlled core: pin power discrepancy [%] of DYN3D-D with FUM.

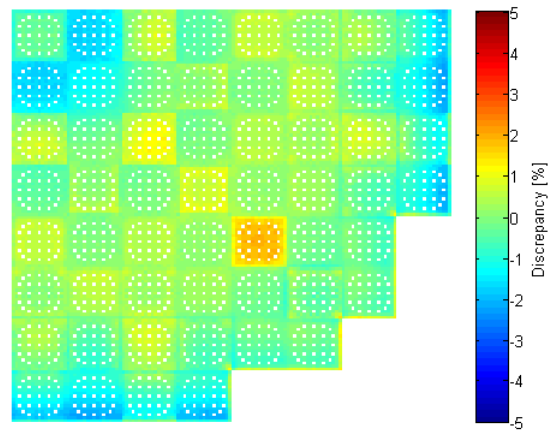


Fig. 29: Controlled core: pin power discrepancy [%] of DYN3D-SP3 with FUM.

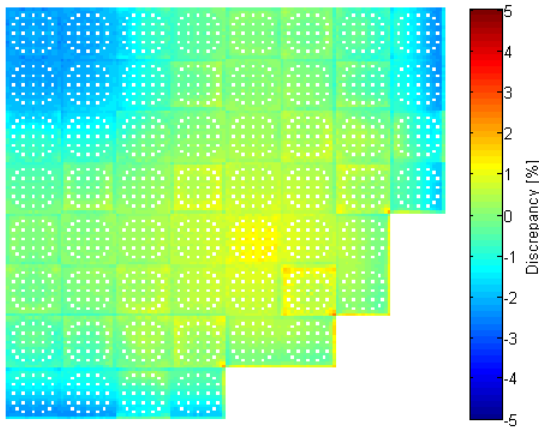


Fig. 30: Controlled core: pin power discrepancy [%] of DYN3D-SP3 without FUM.

Table V. Controlled core: multiplication factor and average as. pow. difference

	K-eff diff [pcm]	P diff [%]
Reference k-eff	1.00921	
SIMULATE-5 9x9	-17	0.56
SIMULATE-5 17x17	-37	0.66
DYN3D-D - FUM	-25	1.01
DYN3D-SP ₃ - FUM	55	0.40
DYN3D-SP ₃ - no FUM	53	0.55

4. Interpolation routine test

The test for the interpolation routine for the cross sections described in Chapter 2 is presented in this section. The library was produced for the following state variables:

- Moderator temperature [K]: 560, 580, 600
- Fuel temperature [K]: 560, 900, 1320
- Boron concentration [ppm]: 0.1, 1000, 2000
- Burnup: 0, 0.1, 0.5, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.5, 15, ... 70 MWd/kg with 2.5 step

For the test, a minicore solution in ARO conditions has been produced with S5 using the thermalhydraulic feedback and the obtained moderator and fuel temperatures were used also in C5 and DYN3D.

The eigenvalue discrepancy given in Table VI is very small for both S5 and DYN3D and the nodal power discrepancies in case of DYN3D is significantly smaller than of S5, as is shown in Figures 31-32.

This preliminary test show that the interpolation routine works sufficiently well.

One interpolation takes about 0.015 ms. The fuel temperature parametrization have negligible impact on precision and such interpolation takes only 0.005 ms.

Table VI. Multiplication factor

	K-eff diff [pcm]
SIMULATE-5	-17
DYN3D	-17

	1	2	3	4	5	6
A	-1.08	-0.88	-0.59	-0.20	0.10	0.84
B	-0.88	-0.78	-0.50	0.30	0.10	0.40
C	-0.59	-0.50	-0.20	0.00	0.20	0.62
D	-0.20	0.30	0.00	0.40	0.10	0.10
E	0.10	0.10	0.20	0.10	0.10	0.00
F	0.84	0.40	0.62	0.10	0.00	0.74

Fig. 32. As. pow. discrepancy [%] of S5.

	1	2	3	4	5	6
A	-0.58	-0.49	-0.39	-0.10	0.10	0.43
B	-0.49	-0.39	-0.20	0.31	0.20	0.20
C	-0.39	-0.20	-0.10	0.00	0.10	0.42
D	-0.10	0.31	0.00	0.31	0.00	0.00
E	0.10	0.20	0.10	0.00	0.00	0.00
F	0.43	0.20	0.42	0.00	0.00	0.22

Fig. 33. As. pow. discrepancy [%] of DYN3D-SP₃.

V. CONCLUSIONS

The DYN3D results employing cross sections generated without any critical spectrum correction show high discrepancies in particular for the cores with strong power gradient with depleted fuel and reflector. The errors in the presented cases seem to be reduced by employing the fundamental mode correction when generating the pin-by-pin library. The remaining discrepancy could be caused by no assembly discontinuity factors in DYN3D and/or by the fact that S5 uses the analytical nodal method which is known to be more precise in cores with steep flux gradient than the nodal expansion method adopted in DYN3D.

The usage of the SP₃ instead of diffusion in DYN3D improves the power prediction, in particular for controlled assemblies. Nevertheless, it also doubles the computation time.

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