# Sensor Selection Based On Boolean Network

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**Abstract:** Process fault diagnosis strategies rely heavily on various types of sensors for temperature, pressure, concentration and etc. Due to the redundancy of the sensors in process systems such as chemical and nuclear plants, sensor selection schemes can deeply influence the diagnostic efficiency. In this paper, a Boolean network with its linear representation is proposed for describing the fault propagation among sensors, the sufficient conditions for both fault detectability and discriminability are given, and a sensor selection method for fault detection and discrimination is then proposed. Finally, the theoretic result is applied to realize the diagnosis oriented sensor selection for a nuclear steam supply system, which not only verifies the feasibility but also show the implementation steps of theoretic results.

**Keyword:** Sensor selection, semi-tensor product, sensor network

## 1 Introduction

The process behavior is inferred by using sensors measuring the important variables in the processes such as chemical and nuclear plants. When a process encounters a fault, the effect of this fault is propagated to all or some of the process variables. The main objective of fault diagnosis is to observe these fault symptoms and determine the root cause for the behavior, and the efficiency of fault diagnosis depends critically on the selection of sensors monitoring the process variables. Directed graph (DG) is such a qualitative model that can be used to infer the fault propagation or cause-effect behaviour in a process system. Sensor selection was treated as different DG-based optimization problems in most of the earlier work. Bagajewicz et al. summarized the sensor selection in a process as mix integer linear programming (MILP) problems focusing on optimizing cost or (and) reliability [1-4]. Bhushan, Narasimhan and Rengaswamy added the criteria of robustness to the MILP problems [5]. Genetic algorithms (GAs) were also applied to solve the optimization problems for sensor selection [6, 7]. The MILP approach has been applied to the sensor selection problem of the fault diagnosis for the integral pressurized water reactors (iPWRs) and the helical coil steam generators [8, 9].

Boolean network (BN), first introduced by Kauffman [10], has been a powerful tool in modelling and analyzing cellular networks. A BN is a network with nodes and directed edges, where the state of a node is quantized to the values of True or False, and is determined through logical rules by the states of other nodes with edges directed to this node. It was shown that BN plays a crucial role in modelling cell regulation. BN can be also applied to other fields such as system sciences as a powerful tool. Cheng and Qi gave the state-space model of BN based on its linear representation, and then revealed some features such as fix points, cycles and controllability [11-13].
Actually, by regarding both the sensors and faults in a process as the nodes in a BN, and by further regarding the cause effect behaviors as the directed edges, BN can be utilized as a qualitative model for the fault propagation in a process system. Based on this idea, a BN model for process fault propagation is proposed, and then the sensor selection problem for fault diagnosis is solved by analyzing the steady state space structure of the corresponding BN model in this paper. Then, the BN-based sensor selection method is applied to realize the fault detection and discrimination of a nuclear steam supplying system, which shows the feasibility of this new approach.

## 2 Semi-tensor product and logics

In this section, some definitions and lemmas about semi-tensor product and logical function [11-13] are introduced and reviewed as follows with some necessary remarks.

**Definition 1.** Suppose $A\in M_{m\times n}$ and $B\in M_{p\times q}$, and let $t$ be the lowest common multiple (LCM) of positive integers $n$ and $p$. The semi-tensor product (STP) of $A$ and $B$ is defined by

$$A\ltimes B=\left(A\otimes I_{t/n}\right)\left(B\otimes I_{t/p}\right), \tag{1}$$

where $\otimes$ is Kronecker product, $I$ is identity matrix.

**Remark 1.** Semi-tensor product is the generalization of traditional matrix multiplication. In the following parts of this paper, symbol "$\ltimes$" is omitted.

**Definition 2.** Matrix $A\in M_{m\times n}$ is called a logical matrix if the columns of $A$, denoted by Col($A$), satisfy Col($A$)$\subset\Delta_m$, where $\Delta_m=\{\delta_m^k \mid k=1,\ldots,m\}$, and $\delta_m^k$ is the $k$th column of $I_m$. The set of $m\times n$ logical matrices is denoted by $L_{m\times n}$, and $\Delta_2$ is usually denoted by $\Delta$.

**Definition 3.** $W_{[m,n]}\in M_{mn\times mn}$ is called a swap matrix if its column labels are given by (11, …, 1$n$, …, $m$1, …, $mn$), its row labels are given by (11, …, $m$1, …, 1$n$, …, $mn$), and its element in position ($IJ$, $ij$) is given by

$$w_{IJ,ij}=\begin{cases}1, & I=i, J=j,\\ 0, & \text{otherwise.}\end{cases} \tag{2}$$

Moreover, $W_{[n,n]}$ is briefly denoted as $W_{[n]}$.

**Lemma 1.** Let $x\in R^m$, $y\in R^n$, and $A\in M_{p\times q}$. Then

$$W_{[m,n]}xy=yx, \tag{3}$$

$$xA=\left(I_m\otimes A\right)x. \tag{4}$$

**Definition 4.** A $2\times 2^n$ matrix $M_\sigma$ is called the structure matrix of a logical function $\sigma$: $\Delta^n\to\Delta$, if

$$\sigma\left(a_1,a_2,\ldots,a_n\right)=M_\sigma a_1a_2\cdots a_n. \tag{5}$$

where $a_i\in\Delta$, $i$=1, 2, …, $n$.

The power-reducing matrix $M_r$ is determined by

$$a^2=aa=M_r a, \tag{6}$$

and we can see that

$$M_r=\left[\delta_4^1\quad \delta_4^4\right]:=\delta_4\left[1\quad 4\right]. \tag{7}$$

Then, based on the power-reducing matrix and swap matrices, it is easy to obtain the following Lemma 2.

**Lemma 2.** Every logical function $\sigma$: $\Delta^n\to\Delta$ has a structure matrix $M_\sigma\in L_{2\times 2n}$ satisfying equation (5).

**Remark 2.** The structure matrices of logical functions negation "¬", disjunction "∨" and conjunction "∧" are given by

$$M_\neg=\delta_2\left[2\quad 1\right], \tag{8}$$

$$M_\vee=\delta_2\left[1\quad 1\quad 1\quad 2\right], \tag{9}$$

$$M_\wedge=\delta_2\left[1\quad 2\quad 2\quad 2\right], \tag{10}$$

respectively. Moreover, the logical functions of identity "I" and constant "F" have the structure matrices given by

$$M_I=\delta_2\left[1\quad 2\right]=I_2, \tag{11}$$

$$M_F=\delta_2\left[2\quad 2\right], \tag{12}$$

respectively.

For $A_n=a_1a_2\ldots a_n$, where $a_i\in\Delta$, $i$=1,…, $n$. Following lemma 3 gives the relationship between $A_n^2$ and $A_n$.

**Lemma 3.** For $A_n=a_1a_2\ldots a_n$, where $a_i\in\Delta$, $i$=1, …, $n$. Then

$$A_n^2=\Phi_n A_n, \tag{13}$$

where

$$\Phi_n=\prod_{i=1}^{n}\left\{I_{2^{i-1}}\otimes\left[\left(I_2\otimes W_{[2,2^{n-i}]}\right)M_r\right]\right\}. \tag{14}$$

**Remark 3.** By some tedious computation, it can be derived from Lemma 3 that

$$\Phi_n=\mathrm{diag}\left(\delta_{2^n}^1,\cdots,\delta_{2^n}^{2^n}\right), \tag{15}$$

where $\delta_{2^n}^k\in\Delta_{2^n}, k=1,\cdots,2^n$.

# 3 BN for Process Fault Propagation

The nodes of a directed graph describing fault propagation is the faults $f_i\in\Delta$ and sensors $s_j\in\Delta$, $i$=1, …, $m$, $j$=1, …, $n$. The edge from fault $f_i$ to sensor $s_j$ denotes that $f_i$ can be detected by $s_j$. The edge from sensors $s_j$ to $s_k$ reveals the fault-propagation between the process variables measured by these two sensors. It is assumed that there is no edges between any two faults. There may be several edges toward a sensor node, whose logical operation among the start nodes of these edges should be disjunction.

For convenience of discussion, define matrices $E_{SF}=\{e_{SF,ij}\}\in M_{n\times m}$ and $E_{SS}=\{e_{SS,kl}\}\in M_{n\times n}$, where $i$, $k$, $l$=1,⋯, $n$, $j$=1,⋯, $m$. Here, $e_{SF,ij}$=1 if there exists an edge from fault $f_j$ to sensor $s_i$, $e_{SS,kl}$=1 if there is an edge from $s_l$ to $s_k$, otherwise $e_{SF,ij}$=0 and $e_{SS,kl}$=0.
Based on the above assumption and analysis about the features of the DG for process fault propagation, the BN model of this DG can be written as

$$s_k\left(t+1\right)=M_\vee^{m+n-1}\left(\prod_{i=1}^{m}M_{F,i}f_i\right)\left[\prod_{j=1}^{n}M_{S,j}s_j\left(t\right)\right], \tag{16}$$

where $k$=1, …, $n$, $t$ is the times of logic computation,

$$M_{F,i}=\begin{cases}M_I, & e_{SF,ki}\neq 0,\\ M_F, & e_{SF,ki}=0,\end{cases} \tag{17}$$

and

$$\boldsymbol{M}_{\mathrm{S},j}=\begin{cases}\boldsymbol{M}_{\mathrm{I}}, & e_{\mathrm{FF},kj}\neq 0,\\ \boldsymbol{M}_{\mathrm{F}}, & e_{\mathrm{FF},kj}=0,\end{cases} \tag{18}$$

matrices $\boldsymbol{M}_{\vee}$, $\boldsymbol{M}_{\mathrm{I}}$ and $\boldsymbol{M}_{\mathrm{F}}$ are given by equations (9), (11) and (12) respectively, and $\boldsymbol{f}_i$, $\boldsymbol{s}_j\in\Delta$.

Based on equations (16) and (4), we can obtain that

$$\boldsymbol{s}_k(t+1)=\boldsymbol{L}_k\prod_{i=1}^{m}\boldsymbol{f}_i\prod_{j=1}^{n}\boldsymbol{s}_j(t), \tag{19}$$

where

$$\boldsymbol{L}_k=\boldsymbol{M}_{\vee}^{m+n-1}\left\{\left[\prod_{i=1}^{m}\left(\boldsymbol{I}_{2^{i-1}}\otimes\boldsymbol{M}_{\mathrm{F},i}\right)\right]\left[\prod_{j=1}^{n}\left(\boldsymbol{I}_{2^{m+j-1}}\otimes\boldsymbol{M}_{\mathrm{S},j}\right)\right]\right\}. \tag{20}$$

Define

$$\boldsymbol{S}(t)=\prod_{j=1}^{n}\boldsymbol{s}_j(t)=\boldsymbol{s}_1(t)\boldsymbol{s}_2(t)\cdots\boldsymbol{s}_n(t) \tag{21}$$

$$\boldsymbol{F}=\prod_{i=1}^{m}\boldsymbol{f}=\boldsymbol{f}_1\boldsymbol{f}_2\cdots\boldsymbol{f}_m, \tag{22}$$

then the state-space model of BN describing process fault propagation is proposed by Theorem 1.

**Theorem 1.** The state-space model of the BN for process propagation given by (19) can be written as

$$\boldsymbol{S}(t+1)=\boldsymbol{L}_{\mathrm{F}}\boldsymbol{S}(t), \tag{23}$$

where

$$\boldsymbol{L}_{\mathrm{F}}=\boldsymbol{L}\boldsymbol{F}, \tag{24}$$

$$\boldsymbol{L}=\boldsymbol{L}_1\prod_{k=2}^{n}\left[\left(\boldsymbol{I}_{2^{m+n}}\otimes\boldsymbol{L}_k\right)\boldsymbol{\Phi}_{m+n}\right], \tag{25}$$

$\boldsymbol{L}_k$ and $\boldsymbol{\Phi}_{m+n}$ is given by (20) and (14) respectively.

**Proof:** From Lemma 3,

$$\left[\boldsymbol{F}\boldsymbol{S}(t)\right]^2=\boldsymbol{\Phi}_{m+n}\boldsymbol{F}\boldsymbol{S}(t). \tag{26}$$

Based on equations (21) and (19),

$$\begin{aligned}\boldsymbol{S}(t+1)&=\boldsymbol{L}_1\boldsymbol{F}\boldsymbol{S}(t)\boldsymbol{L}_2\boldsymbol{F}\boldsymbol{S}(t)\prod_{k=3}^{n}\boldsymbol{L}_k\boldsymbol{F}\boldsymbol{S}(t)\\ &=\boldsymbol{L}_1\left[\left(\boldsymbol{I}_{2^{m+n}}\otimes\boldsymbol{L}_2\right)\boldsymbol{\Phi}_{m+n}\right]\boldsymbol{F}\boldsymbol{S}(t)\prod_{k=3}^{n}\boldsymbol{L}_k\boldsymbol{F}\boldsymbol{S}(t)\\ &=\cdots\\ &=\boldsymbol{L}_1\prod_{k=2}^{n}\left[\left(\boldsymbol{I}_{2^{m+n}}\otimes\boldsymbol{L}_k\right)\boldsymbol{\Phi}_{m+n}\right]\boldsymbol{F}\boldsymbol{S}(t),\end{aligned} \tag{27}$$

which completes the proof of this theorem.

**Remark 4.** From equation (23), it is easy to see that $\boldsymbol{L}_{\mathrm{F}}=\boldsymbol{L}\boldsymbol{F}$ is a $2^n\times 2^n$ matrix which is called sensor network state transition matrix. Fault $\boldsymbol{F}$ is regarded as a constant vector which is the parameter of state transition matrix.

**Remark 5.** From (21), (22) and $\boldsymbol{f}_i$, $\boldsymbol{s}_j\in\Delta$, it can be seen that $\boldsymbol{F}\in\Delta_{2^m}$ and $\boldsymbol{S}\in\Delta_{2^n}$.

# 4 Detectability and discriminability

Based on the definition of semi-tensor product, it is easy to prove the following Lemma 4.

**Lemma 4.** Consider $\boldsymbol{A}_n=\boldsymbol{a}_1\boldsymbol{a}_2\ldots\boldsymbol{a}_n$, where $\boldsymbol{a}_i=[Q_i\ 1-Q_i]^{\mathrm{T}}\in\Delta$, $Q_i\in\{0,1\}$, and $i=1, 2, \ldots, n$. If $\boldsymbol{A}_n=\delta_{2^n}^{k}$ ($k=1,\cdots,2^n$), then

$$k=2^n-\sum_{i=1}^{n}Q_i2^{n-i}. \tag{28}$$

Based on Lemma 4, we have the following Theorem 2 that gives fault detectability and discriminability of the BN.

**Theorem 2.** Consider BN (23) for process fault propagation. Suppose that there is only one fault occurs at a time. For each $i=1, 2, \ldots, m$, it is assumed that there is a positive integer $q_i$ such that

$$\left(\boldsymbol{L}\delta_{2^m}^{k_i}\right)^{q_i+1}=\left(\boldsymbol{L}\delta_{2^m}^{k_i}\right)^{q_i}, \tag{29}$$

where

$$k_i=2^m-2^{m-i}. \tag{30}$$

Define

$$\boldsymbol{P}=\left[\boldsymbol{p}_1\quad\boldsymbol{p}_2\quad\cdots\quad\boldsymbol{p}_m\right], \tag{31}$$

where

$$\boldsymbol{p}_i=\left(\boldsymbol{L}\delta_{2^m}^{k_i}\right)^{q_i}\boldsymbol{S}_0,\ i=1, 2, \ldots, m. \tag{32}$$

Then, for an initial sensor state $\boldsymbol{S}_0=\boldsymbol{S}(0)\in\Delta_{2^n}$, the faults are detectable at $\boldsymbol{S}_0$ if

$$\boldsymbol{p}_i\neq\delta_{2^m}^{2^m}\quad\text{for } i=1, 2, \ldots, m, \tag{33}$$

Faults are discriminable at $\boldsymbol{S}_0$ if conditions (33) and

$$\mathrm{Rank}\left(\boldsymbol{P}\right)=m, \tag{34}$$

are satisfied, where the value of function Rank($\boldsymbol{P}$) is the rank of matrix $\boldsymbol{P}$.

**Proof:** For a given $i\in\{1, 2, \ldots, m\}$, i.e. $\boldsymbol{F}=\delta_{2^m}^{k_i}$ with $k_i$ given by (30). Then, from BN model (23),

$$\boldsymbol{S}(t)=\left(\boldsymbol{L}\boldsymbol{F}\right)^t\boldsymbol{S}_0=\left(\boldsymbol{L}\delta_{2^m}^{k_i}\right)^t\boldsymbol{S}_0. \tag{35}$$

If there exists a positive integer $q_i$ so that (29) is well satisfied, then there is the steady response of the BN to fault $i$ at initial sensor state $\boldsymbol{S}_0$ is

$$\boldsymbol{S}(q_i)=\left(\boldsymbol{L}\delta_{2^m}^{k_i}\right)^{q_i}\boldsymbol{S}_0. \tag{36}$$

If there is no sensor response to fault $i$, the steady response of the BN should be $\delta_{2^m}^{2^m}$. Therefore, if

$$\left(\boldsymbol{L}\delta_{2^m}^{k_i}\right)^{q_i}\boldsymbol{S}_0\neq\delta_{2^m}^{2^m} \tag{37}$$

holds for each $i\in\{1, 2, \ldots, m\}$, i.e. condition (33) is satisfied, then the faults are detectable.

Moreover, suppose

$$\boldsymbol{p}_r\neq\boldsymbol{p}_s \tag{38}$$

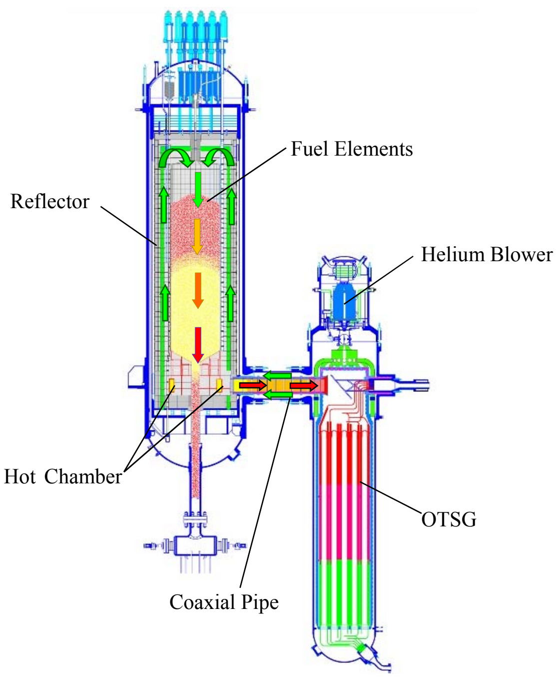


Fig. 1. Schematic diagram of the HTR-PM's NSSS

**Table 1. Available sensor nodes for selection**

| Nodes | Description |
|---|---|
| $s_1$ | reactor neutron flux |
| $s_2$ | primary helium flowrate |
| $s_3$ | average temperature of the primary flow |
| $s_4$ | average temperature of the secondary flow |

**Table 2. Fault nodes to be detected or discriminated**

| Nodes | Description |
|---|---|
| $f_1$ | abnormal reactivity injection to the reactor |
| $f_2$ | malfunction of the primary helium blower |
| $v_3$ | heat transfer degradation of OTSG two sides |

for $r \neq s$ and $r, s \in \{1, 2, \ldots, m\}$, i.e. condition (34) is satisfied. It is easy to see from inequality (38) that the steady responses of sensor state to different faults are different from each other, which means that the faults are discriminable. This completes the proof of this theorem.

**Remark 6.** It can be seen that there is a $l_i \in \{1, \cdots, 2^n\}$, $i$=1, 2, …, $m$, so that

$$\delta_{2^n}^{l_i} = \left(\boldsymbol{L}\delta_{2^m}^{k_i}\right)^{q_i} \boldsymbol{S}_0 . \quad (39)$$

Define

$$\Pi_i = \left\{ r \in \mathrm{N} \middle| \; 2^n - l_i = \sum_{r=1}^{n} Q_r 2^{n-r}, Q_r \neq 0 \right\} \quad (40)$$

which is the collection of sensors having respond to fault $i$.

Let

$$\Theta_1 = \bigcap_{i=1}^{m} \Pi_i . \quad (41)$$

If $\Theta_1$ is not empty, then we can use sensor $\boldsymbol{s}_\lambda$ with $\lambda \in \Theta_1$ for fault detection, and use sensors with their number in the set

$$\Theta_2 = \bigcup_{i=1}^{m} \Pi_i - \{\lambda\} \quad (42)$$

for fault discrimination.

# 5 Application to a nuclear plant

The BN model and its linear representation for process fault propagation given by Theorem 1, the sufficient conditions for fault detectability and discriminability given by Theorem 2 and the sensor selection method presented in Remark 5 are applied to realize a fault-diagnosis oriented sensor selection of a nuclear steam supply system (NSSS).

## 5.1 Background

The modular high temperature gas-cooled reactor (MHTGR) adopts helium as coolant and graphite as both moderator and structural materials. From Fig. 1, this NSSS is composed of an MHTGR, a helical-coil once-through steam generator (OTSG), a helium blower and some pipes. The cold helium enters to the helium blower mounted on top of the OTSG, and is then pressurized before flowing in the cold gas duct. The cold helium enters into the channels in the side-reflector from bottom to top for cooling the reflector, and then passes through the pebble-bed from top to bottom where it is heated to a high temperature about 750℃. The hot helium leaves the hot gas chamber inside the bottom reflector, and flows into the OTSG primary side where it is cooled by the secondary water/steam flow.

Fault diagnosis of the NSSS is meaningful to the safe, stable and efficient operation of MHTGR-based nuclear plant. Sensor selection is crucial and necessary for satisfactory diagnosis. The sensors to be selected are those measuring the reactor neutron flux, primary helium flowrate, secondary feedwater flowrate and the average coolant temperatures of the primary and secondary sides. The faults to be detected or discriminated are abnormal reactivity injection, error of the primary helium blower and heat transfer degradation between the two sides of the OTSG.

## 5.2 Linear representation of BN model

The BN model for the fault propagation is

$$\begin{cases} s_1(t+1) = f_1, \\ s_2(t+1) = f_2, \\ s_3(t+1) = f_3 \vee s_1(t) \vee s_4(t), \\ s_4(t+1) = f_3 \vee s_2(t) \vee s_3(t). \end{cases} \quad (43)$$

From Theorem 1, the linear representation of model (43) can be rewritten as

$$S(t+1) = LFS(t), \quad (44)$$

where

$$\begin{aligned} L = \delta_{16}[\ & 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \\ & 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \\ & 1\ 1\ 1\ 1\ 1\ 1\ 2\ 2 \\ & 1\ 3\ 1\ 3\ 1\ 3\ 2\ 4 \\ & 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5 \\ & 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5 \\ & 5\ 5\ 5\ 5\ 5\ 5\ 6\ 6 \\ & 5\ 7\ 5\ 7\ 5\ 7\ 6\ 8 \\ & 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9 \\ & 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9 \\ & 9\ 9\ 9\ 9\ 9\ 9\ 10\ 10 \\ & 9\ 11\ 9\ 11\ 9\ 11\ 10\ 12 \\ & 13\ 13\ 13\ 13\ 13\ 13\ 13\ 13 \\ & 13\ 13\ 13\ 13\ 13\ 13\ 13\ 13 \\ & 13\ 13\ 13\ 13\ 13\ 13\ 14\ 14 \\ & 13\ 15\ 13\ 15\ 13\ 15\ 14\ 16\ ]. \end{aligned}$$

### 5.3 Verification of fault discriminability

From equation (30), $k_1$=4, $k_2$=6 and $k_3$=7. Furthermore, we can obtain that

$$\begin{aligned} L_{F1} = LF_1 = L\delta_8^4 = \delta_{16}[\ & 5\ 5\ 5\ 5 \\ & 5\ 5\ 6\ 6 \\ & 5\ 7\ 5\ 7 \\ & 5\ 7\ 6\ 8\ ], \\ L_{F2} = LF_2 = L\delta_8^6 = \delta_{16}[\ & 9\ 9\ 9\ 9 \\ & 9\ 9\ 10\ 10 \\ & 9\ 11\ 9\ 11 \\ & 9\ 11\ 10\ 12\ ], \\ L_{F3} = LF_3 = L\delta_8^7 = \delta_{16}[\ & 13\ 13\ 13\ 13 \\ & 13\ 13\ 13\ 13 \\ & 13\ 13\ 13\ 13 \\ & 13\ 13\ 13\ 13\ ]. \end{aligned}$$

It is easy to verify that $q_1$=$q_2$=3 and $q_3$=1, i.e.

$$L_{F1}^4 = L_{F1}^3, \quad L_{F2}^4 = L_{F2}^3, \quad L_{F3}^2 = L_{F3}.$$

Consider initial state $S_0$=$\delta_{16}^{16}$, which means that all the sensor states are $\delta_2^2$. Then, we have

$$P = \delta_{16}[5 \quad 9 \quad 13],$$

which satisfies condition (33) and (34) of Theorem 2, and certainly leads to the fault discriminability.

### 5.4 Sensor selection

From (39), $l_1$=8, $l_2$=12 and $l_3$=13. Then from (40),

$$\begin{cases} \Pi_1 = \{1,3,4\}, \\ \Pi_2 = \{2,3,4\}, \\ \Pi_3 = \{3,4\}, \end{cases}$$

which means that we can use sensor $s_4$ for fault detection and sensors $s_1$, $s_2$ and $s_3$ for discrimination.

## 6 Conclusions

Since there are hundreds of sensors for temperature, pressure, concentration and etc. in the complex process systems such as nuclear and chemical plants, and due to fault propagation effect among these sensors, a proper sensor selection scheme is the basis for efficient process fault diagnosis. Sensor selection was treated as optimization problems under certain criteria. However, the precondition of doing optimization is fault detectability and discriminability. In this paper, a Boolean network (BN) model in a linear representation is proposed for describing the fault propagation among sensors. Based on the analysis of the steady state-space structure of the BN model, sufficient conditions for both fault detectability and discriminability are given. According to these sufficient conditions, a sensor selection method for fault detection and discrimination is also proposed. Finally, the above result is applied to realize the fault-diagnosis oriented sensor selection for the MHTGR-based NSSS, which verifies the result and shows the steps of implementation.

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