UNCERTAINTY CHARACTERIZATION FOR DYNAMIC RISK ASSESSMENT

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Abstract: This work proposes a framework to enable dynamic PRA by leveraging information from the well-established static PRA. The framework starts by segmenting accident mitigation process into multiple stages based on static Event Trees and Emergency Operating Procedures. Variations to the initiation of each stage were derived from Fault Trees.
Plant parameters as the exit condition of each mitigation stage were estimated using MARS system code. Because transient simulations through system codes are computationally expensive, the stage's uncertainties were sparsely sampled. A Reduced Order Model (ROM) method was formulated to interpolate and obtain a continuous distribution of the stage's exit condition from these entry condition's samples. It utilizes the Taylor Kriging (TK) method to capture nonlinearities of plant parameters in response to the stage's entry condition variations. Total Variation Regularization (TVR) was used to estimate the Taylor regression order. Numerical error from the time-stepping method, round-offs and ROM construction was formulated and propagated as the next stage's IC error.
A case study on Large Break LOCA (LBLOCA) with uncertainties on Low Pressure Safety Injection (LPSI) actuation timing and capacity is presented. Results show that the proposed ROM method can provide a continuous response and error estimate on plant parameters from finite samples. The methodology enables the plant's safety margin and propagated error to be quantified by using cascading stage ROMs.
Keyword: Uncertainty propagation, Reduced Order Model (ROM), DPRA

1 Introduction

Probabilistic Risk Assessment (PRA) techniques have provided crucial information regarding the quantitative safety level of Nuclear Power Plants. Conventional PRAs were developed with several important assumptions such as the fixed entry condition for each of accident mitigation function and the averaged probability over component lifetime in order to estimate its failure probability. This limitation results in an averaged and simplified risk information. Dynamic PRA (DPRA) relaxes these assumptions to obtain further risk insights due to various uncertainties in mitigation actions. However, a thorough DPRA analysis is challenging since the various uncertainties may lead to an infinite number of cases to be investigated.

Because the accident mitigation process has been documented in a categorized and step-by-step manner in the conventional PRA, it can be leveraged to project possible sets of dynamic scenarios. Mitigation process can be segmented into stages built from Event Tree information. The uncertainties in each stage can be extracted from Fault Trees and Emergency Operating Procedures. In that sense, plant's risk can be estimated by simulating plant's response at each stage and relaying it to the next stage. It is known that transient simulation for accident mitigation is computationally expensive. Therefore this work proposes a method to construct a Reduced Order Model (ROM) to create a full-rank stage's response from sparse samples. Additionally, the ROM's error estimate and its propagation to next stages are analyzed.

2 Methods

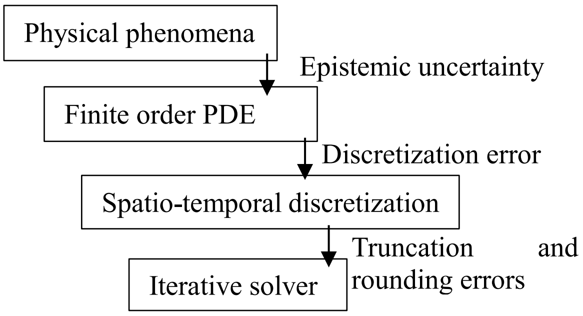


Fig. 1. Uncertainties in system simulation codes

As plant's response is estimated by simulation codes, it is important to ask how accurate the simulation's output is to the actual response. As an approximation to real-world phenomena, the codes have epistemic and aleatory uncertainties as illustrated in Figure 1. Zou et. al. [1] has proven that the largely overlooked aleatory uncertainties may be more significant than the epistemic uncertainty. For that reason, this uncertainty will be analyzed further in the next subsection.

2.1 Numerical Uncertainties

Transient analysis in accident mitigation simulations is more prone to temporal discretization error than steady-state analysis. Existing system analysis codes typically utilize first-order implicit Backward Euler (BDF1) time integration scheme [1] as illustrated in Figure 2. Given a PDE of:

$$\frac{dx}{dt} = f(t,x) \quad (1)$$

The solution's Local Truncation Error (LTE) solved by BDF1 method is:

$$E_{n+1} = x(t_{n+1}) - x_{n+1} = \varepsilon_x + h\varepsilon_f + O(h^2) \quad (2)$$

where x_{n+1} is the code's approximation to exact PDE solution $x(t_{n+1})$ after one time step h, and $O(h^2)$ is the Taylor higher order terms. The ε_f and ε_x are the machine round-off error in computing the tangent and x_{n+1} of this linear time-stepping scheme.

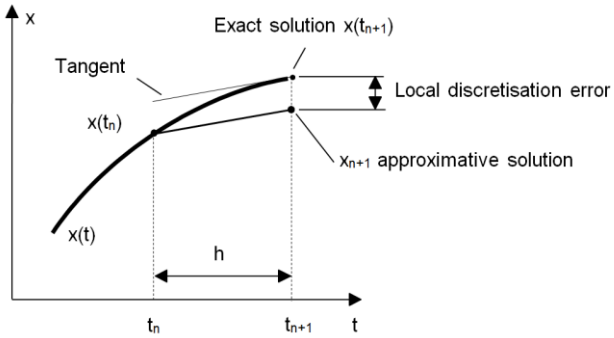


Fig. 2. Implicit time integration method

Accumulated LTEs after m time steps form a Global Truncation Error (GTE) given by:

$$E_{n+m} = E_{n+m-1}\left(1 - h\frac{df}{dx}\right) + h\varepsilon_f + O(h^2) + \varepsilon_x \quad (3)$$

The df/dx term and round-off errors may self-correct GTE. However, in the worst case scenario the terms may have a uniform sign, in which the maximum possible error can be bound as:

$$|E_n| \le \frac{\left(\varepsilon_f + \frac{Mh}{2} + \frac{\varepsilon_x}{h}\right)}{K}\left(e^{Kt_n} - 1\right) \quad (4)$$

$$K = \max_{(t,x)\epsilon R}\left|\frac{df}{dx}\right| < \sim \quad (5)$$

$$M = \max_{(t,x)\epsilon R}\left|\frac{df}{dt} + f\frac{df}{dx}\right| < \sim \quad (6)$$

If there is an error portion in the Initial Condition (IC), at t=0, equation (4) expands into:

$$|E_n| \le \frac{\left(\varepsilon_f + \frac{Mh}{2} + \frac{\varepsilon_x}{h}\right)}{K}\left(e^{Kt_n} - 1\right) + E_0(1 + Kh)^n \quad (7)$$

Equation (7) expresses the upper bound on numerical error due to relay of mitigation stages' output by using ROM. The ROM's error is represented by E_0 which is formulated in the next subsection.

2.2 ROM Creation and Error Estimate

ROM approximates a continuous stage response from observations obtained from system code simulation. In that sense, it may introduce further error to the system code's output. To avoid such degradation, we established the following criteria the ROM has to fulfill:

1) Provides unbiased estimates at sampled points
2) Error variance at unsampled points is minimized
3) Provides error estimate at every points

The Taylor Kriging (TK) method [2] among other interpolation-based methodologies satisfies the aforementioned criteria. TK response function is composed of a regression model and stochastic term as illustrated in Figure 3.

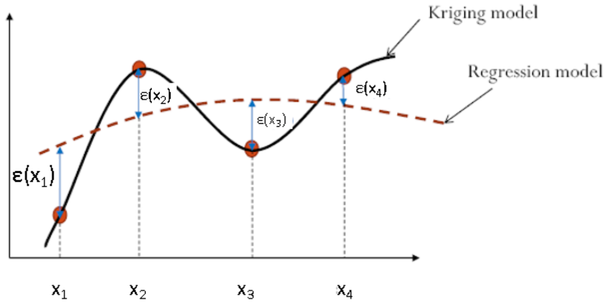


Fig. 3. TK response function

The regression part is approximated by Taylor polynomial to capture a nonlinear trend between sample points. The TK response $\hat{y}(x)$ at an unsampled point x is a linear weighted combination of responses at sample points x_α given by:

$$\hat{y}(x) = \sum_{\alpha} \lambda_\alpha y(x_\alpha) \tag{8}$$

Where the weights are inversely proportional to difference between x and x_α , calculated by the following set of equations:

$$\begin{bmatrix} U \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 & F^T \\ F & S \end{bmatrix}^{-1} \begin{bmatrix} f \\ C \end{bmatrix} \tag{9}$$

$$F = \begin{bmatrix} f_1(x_1) & \cdots & f_M(x_1) \\ \vdots & \ddots & \vdots \\ f_1(x_N) & \cdots & f_M(x_N) \end{bmatrix} \tag{10}$$

$$S = \begin{bmatrix} C_{\varepsilon 11} & \cdots & C_{\varepsilon 1N} \\ \vdots & \ddots & \vdots \\ C_{\varepsilon N1} & \cdots & C_{\varepsilon NN} \end{bmatrix} \tag{11}$$

$$U = [u_1 \cdots u_M]^T \tag{12}$$

$$\lambda = [\lambda_1 \cdots \lambda_N]^T \tag{13}$$

$$f = [f_1 \cdots f_M]^T \tag{14}$$

$$C = [C_{\varepsilon 1x} \cdots C_{\varepsilon Nx}]^T \tag{15}$$

$$f_l(x) = (x - x_0)^l,\ 0 \le l \le M+1 \tag{16}$$

Where M is the order of Taylor polynomial and N is the number of samples. The variable C_ε is the covariance among sample data in the S matrix and covariance between unsampled data and sampled data in C array. TK's error variance is given by:

$$\hat{\sigma}_e^2 = \sum_{\alpha=1}^{N} \hat{\lambda}_\alpha C_{\varepsilon \alpha x} - \sum_{i=1}^{N} \hat{\lambda}_\alpha \sum_{l=1}^{M} \hat{u}_l f_l(x_\alpha) \tag{17}$$

To minimize TK's error variance, the degree of nonlinearity M needs to be properly estimated. Because the uncertainty sources are not explicitly represented in the PDEs, M cannot be determined analytically. A numerical approach is pursued instead by using the Total Variation Regularization (TVR) method [3].

2.3 Total Variation Regularization

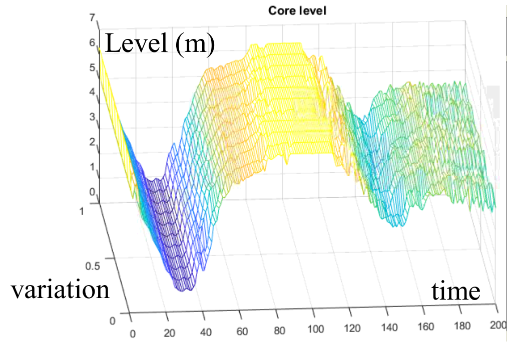
The regularization method fits a function which minimizes discrepancy between it and the sample data while penalizing any irregularities to prevent overfitting. Therefore TVR is suitable to estimate the TK's regression term. The TVR implementation in estimating a derivative u to a function f on the interval [0,L] minimizes an objective function F(u):

$$F(u) = \frac{1}{2}\int_0^L \left| \int u - f \right|^2 + \alpha \int_0^L |u'| \tag{18}$$

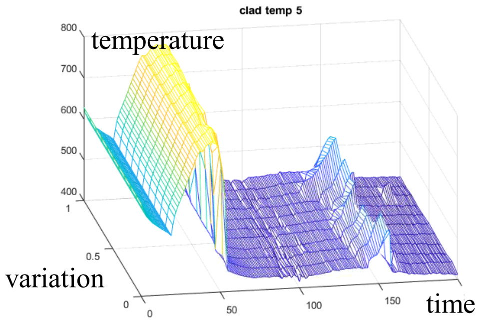
Where α is the regularization parameter which balances penalty term and data fidelity term. The derivatives are estimated from sample data until the ratio of M order derivative to M-1 order drops significantly.

3 Results

To test the proposed ROM methodology, a Large-Break LOCA (LBLOCA) accident was simulated with mitigation uncertainties arising from Low Pressure Safety Injection (LPSI) actuation timing and capacity. Figure 4 shows the core level and clad temperature when LPSI actuation was delayed between 0 and 10 seconds. Figure 5 shows the same variables when LPSI injection capacity was sampled between 0 and 100%. The figures suggest that the LBLOCA phases, i.e. blowdown, refill and reflood occur at different times among the LPSI uncertainty samples as hypothesized.



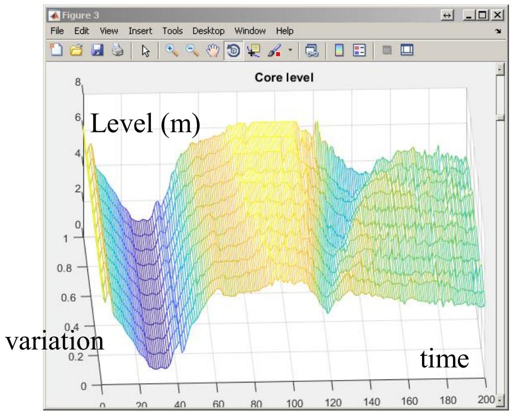
(a) Core level



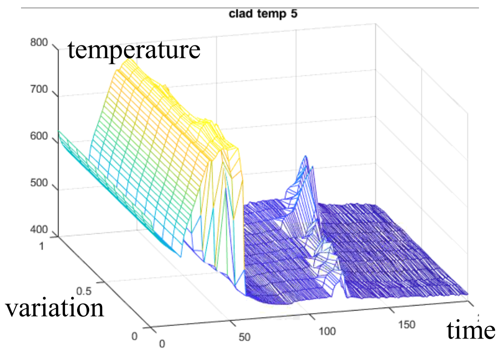
(b) Clad temperature

Fig. 4. Plant response due to variation in LPSI actuation timing

The timing difference of plant physics indicates that the interpolation neighbourhood must be carefully selected based on the interpolation location. In the stage-based DPRA methodology, the next stage has its own dynamic uncertainty which implies that its initial condition may come from various times of the preceeding stage's response. Consider for example, that it is required to estimate the minimum core level during SIT depletion phase, which directly correlates to the clad temperature spike at t>100 s. If the sample data were taken from the same time step, it would include data from different phases, i.e. refill and reflood. It is only appropriate to cluster the interpolation neighborhood from similar physics by selecting a certain sampling direction. This sampling direction is presented as Local Anisotropies (LAs) in the data. The LA for our example (minimum core level during SIT depletion) is shown in Figure 6.



(a) Core level



(b) Clad temperature

Fig. 5. Plant response due to variation in LPSI injection capacity

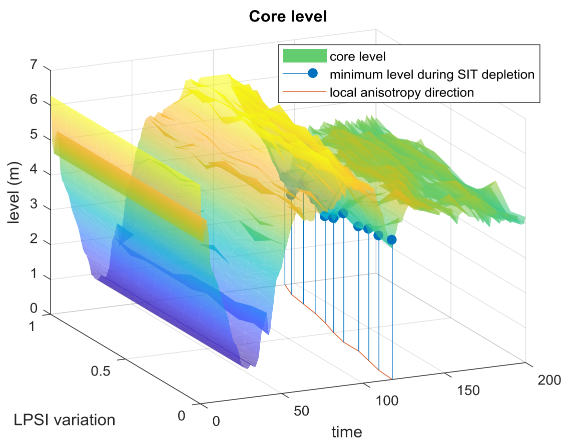


Fig. 6. Local anisotropy in estimating minimum core level during SIT depletion

The semivariogram for these data samples followed a power model as shown in Figure 7. It implies that there is a significant regression trend between the samples. This trend was approximated by Taylor polynomial which derivatives were found using the TVR method. The ratio of successive derivatives is given in Figure 8. This data suggests that M=2, and that the best Taylor polynomial's pivot location was at LPSI 0% capacity, where the derivative ratio was the least.

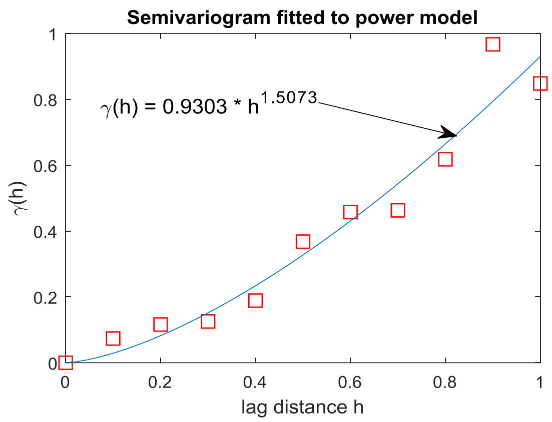


Fig. 7. Experimental semivariogram fitted to a power function model

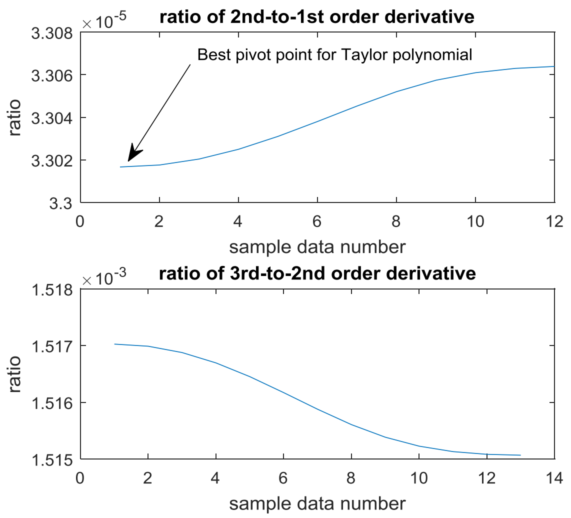


Fig. 8. Ratio of successive derivatives calculated from TVR results

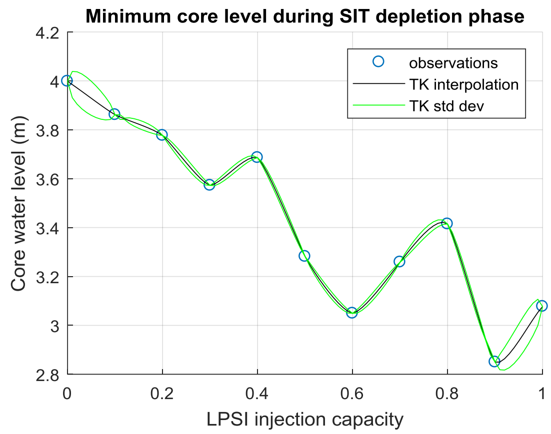
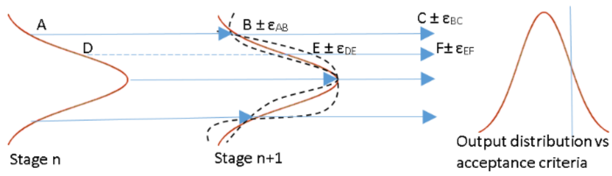


Fig. 9. TK ROM on minimum core level during SIT depletion phase

Having obtained the Taylor's order and pivotal point, the TK continuous response along the LA of minimum core level is shown in Figure 9. The figure reveals that TK gives an unbiased response at sample points. The standard deviation band signifies the error that can be propagated to the next stage's simulation. The error band diminishes at sample locations. Therefore next stage's IC taken from these points will have numerical error of system code given in Equation 4 instead.

4 Summary



$$\varepsilon_{AB} = f(truncation, roundoff)$$
$$\varepsilon_{BC} = f(\varepsilon_{AB}, truncation, roundoff)$$
$$\varepsilon_{DE} = TK\ \ ROM\ \ error$$
$$\varepsilon_{EF} = f(\varepsilon_{DE}, truncation, roundoff)$$

Fig. 10. Uncertainty and error propagation scheme in stage-based DPRA

We outlined a stage-based DPRA by leveraging static PRA information. To support this approach, we have proposed the TK ROM at each stage and formulated its numerical error. The propagation of uncertainties and error among stages is summarized in Fig. 10. The extent of numerical error depends on ROM sampling location. The error can be minimized by a proper estimation of

Taylor's order and pivot point in TK methodology, and proper identification of Local Anisotropy in the sample data. The first two requirements have been automated by using TVR, while automation of the latter is still under active research. Case study results on LBLOCA scenarios proved has shown the successful application of the proposed TK ROM method

References

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