



Nuclear Materials
Modeling Lab.



Effective Thermal Conductivity of UO_2 A Computational approach

Bohyun Yoon, Kunok Chang

Department of Nuclear Engineering, Kyung Hee University



Contents

Research Motivation

Simulation Method

Results

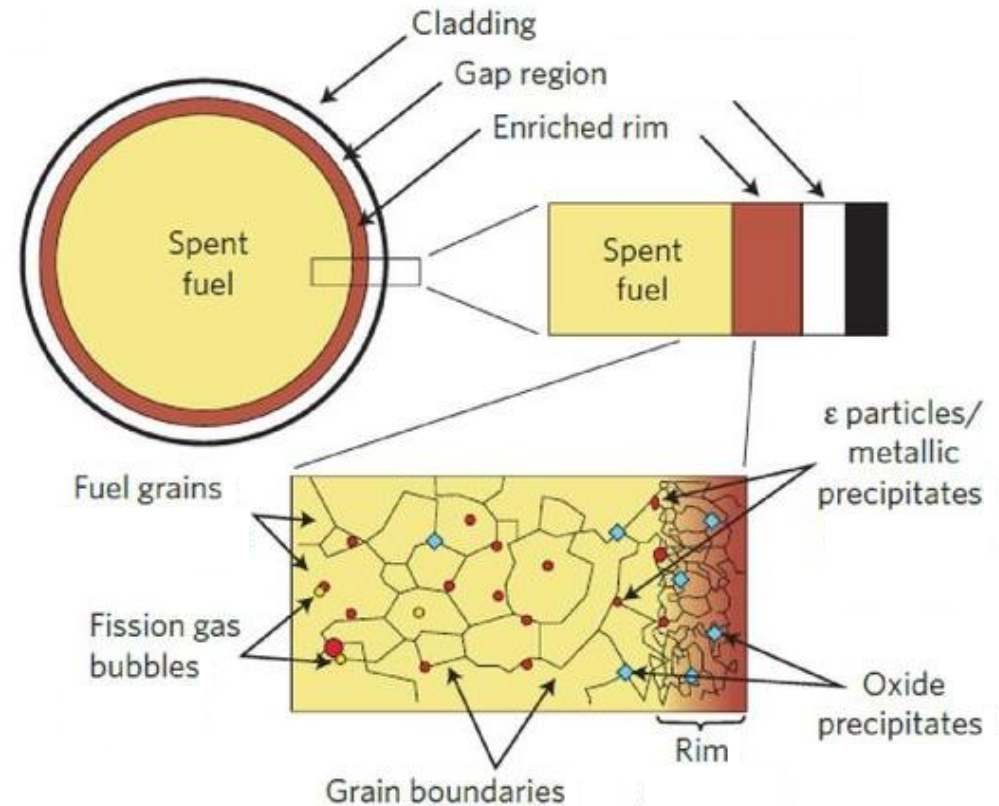
Conclusions

References

Research Motivation

• • •

- Spent fuel microstructure
 - Grain subdivision
 - Defects accumulations
 - Fission product precipitations
 - Fission gas bubbles growth



[1] J. Bruno and R. C. Ewing (2006)

- Thermal conductivity degradation of UO_2 fuel with increasing burn-up, caused by the accumulation of defects in the crystalline lattice.

Effective Thermal Conductivity of Porous Media



- Thermal conductivity models for UO_2 in FRAPCON
 - Based on extensive databases that have been accumulated over decades.
 - Cannot provide property values for the systems outside its range of experience.
 - Needs for developing more generally applicable models based on physical understanding of the thermophysical behavior of system microstructures.
- Models and simulation techniques
 - **Atomistic/particulate models** : The acoustic mismatch model, diffuse mismatch model, and molecular dynamics (MD) simulations preserve the underlying particulate nature of heat carriers.
 - **Continuum models** : The continuum models and finite-element simulations utilize specific constitutive laws and interfacial boundary conditions that coarse the grain atomistic details.

Simulation Geometry

• • •

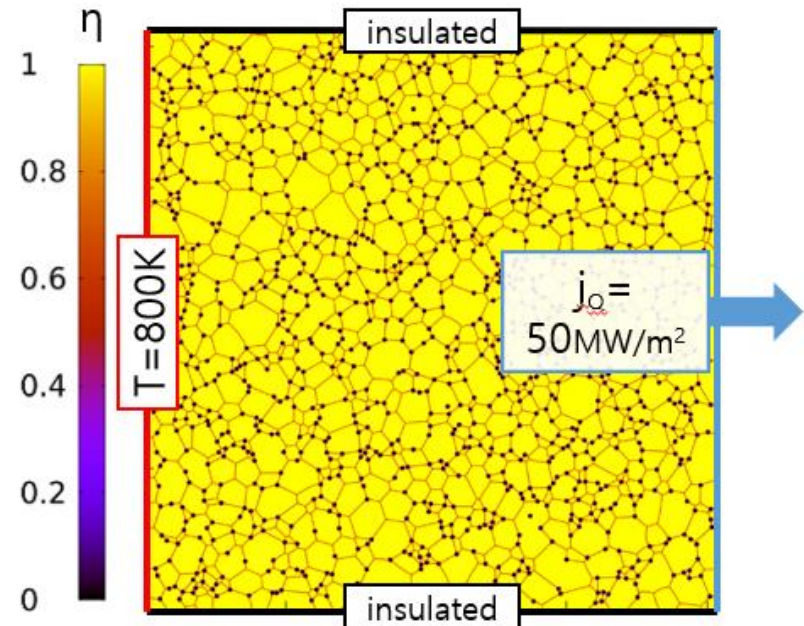
- UO_2 fuel containing defects
 - Helium bubbles (in pores)
 - Grain boundaries
- Microstructure is defined by structural order parameter, η .

$$\begin{cases} \text{UO}_2 \text{ Crystal} : \eta = 1 \\ \text{Helium Bubble} : \eta = 0 \\ \text{Grain Boundary} : 0 < \eta < 1 \end{cases}$$

- Local conductivity depends on the parameter, η .

$$\begin{cases} k_{\text{crystal}} = \frac{1}{0.0375 + 0.0002165T} + \frac{4.715 \times 10^9}{T^2} \exp\left(-\frac{16361}{T}\right) & [2] \text{ J. Harding and D. G. Martin (1989)} \\ k_{\text{local}} = (k_{\text{crystal}} - 0.152) \times \eta^{7.7} + 0.152 \quad (\text{units: W/K/m}) & [3] \text{ P. C. Millett et al. (2008)} \end{cases}$$

Kapitza conductance $\sim 10 \text{ MW/m}_2$
(GB의 thermal resistance 값 결정)



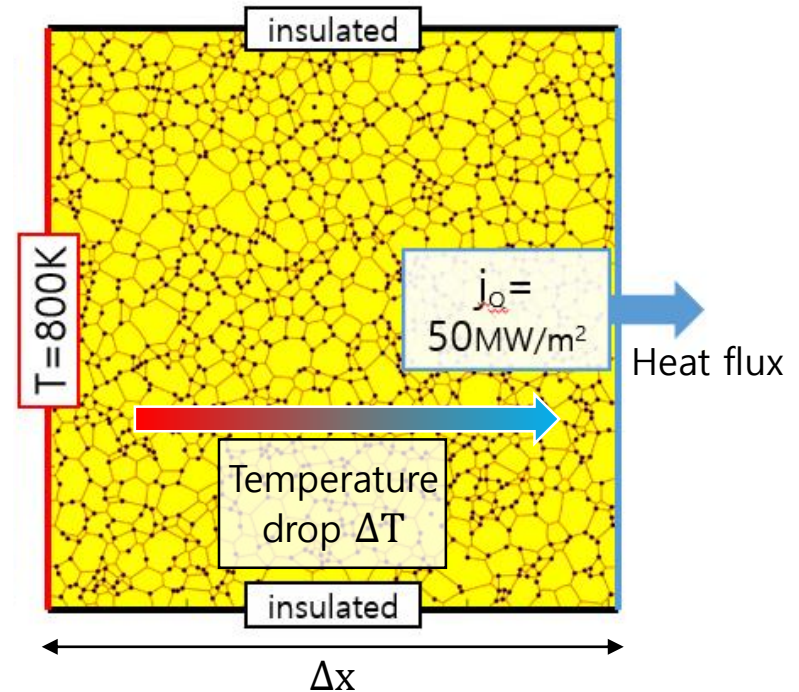
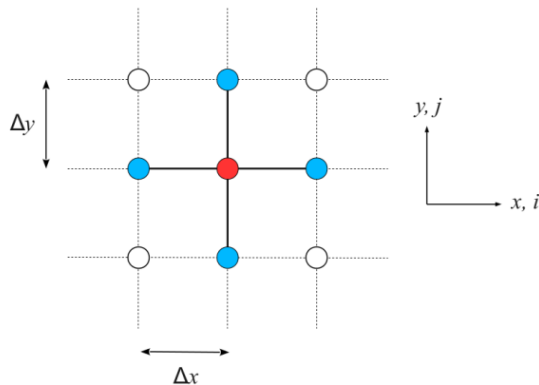
Effective Thermal Conductivity Simulation

- Steady-state heat equation

$$\nabla \cdot (k(r)\nabla T) = 0$$

→ Steady-state temperature distribution

- Finite difference approximations



- Effective conductivity relation (rearranged Fourier's law)

$$k_{\text{eff}} = \frac{j_Q \times \Delta x}{\Delta T}$$

Numerical Method

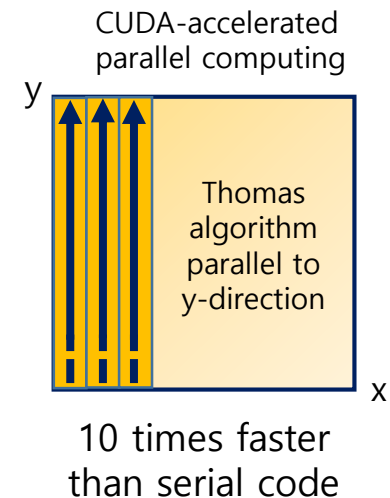
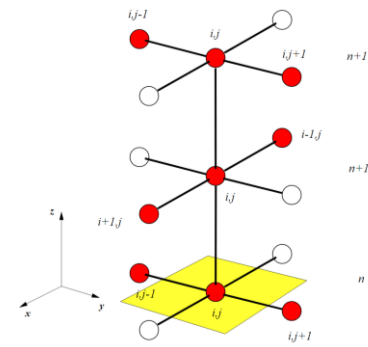
- Crank-Nicolson Scheme (CN)

$$\frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta t} = \frac{1}{2(\Delta x)^2} (\delta_x^2 + \delta_y^2) (T_{i,j}^{n+1} + T_{i,j}^n)$$

- Finite difference method
- Second-order accurate in both time and space
- Numerically stable comparing to simple Euler method

- Alternating Direction Implicit Method (ADI)

$$\begin{cases} \frac{T_{i,j}^{n+1/2} - T_{i,j}^n}{\Delta t/2} = \frac{(\delta_x^2 T_{i,j}^{n+1/2} - \delta_y^2 T_{i,j}^n)}{\Delta x^2} \\ \frac{T_{i,j}^{n+1} - T_{i,j}^{n+1/2}}{\Delta t/2} = \frac{(\delta_x^2 T_{i,j}^{n+1} - \delta_y^2 T_{i,j}^{n+1/2})}{\Delta y^2} \end{cases}$$

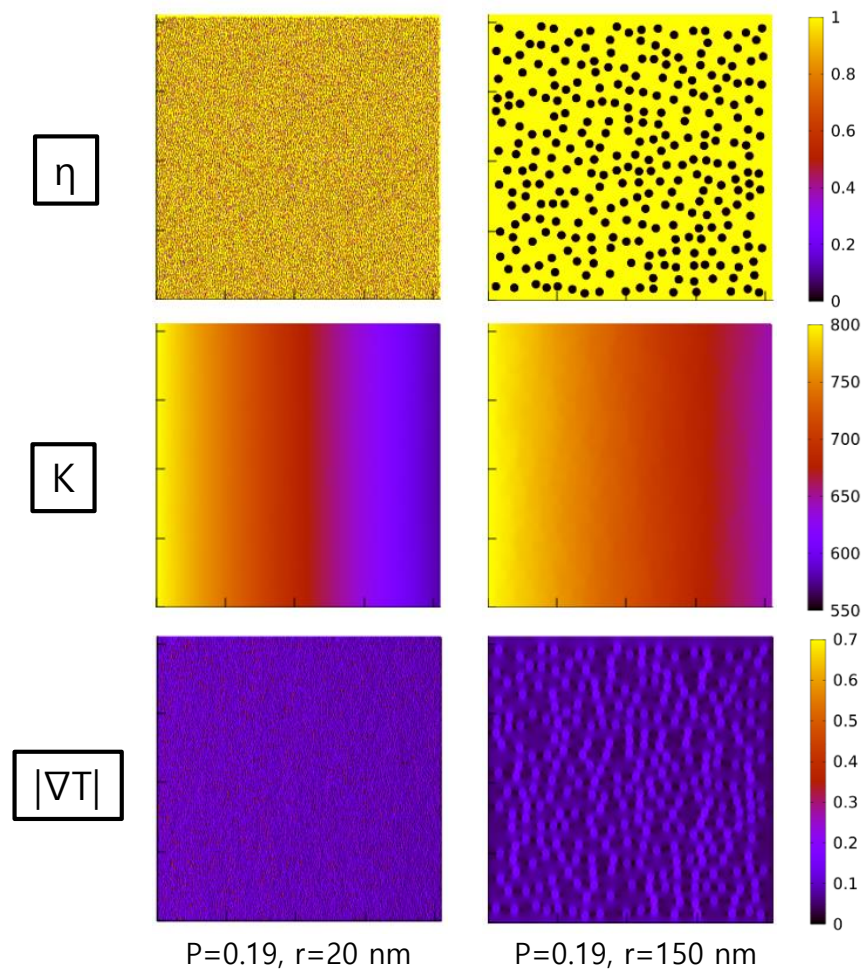


Simulational Result (without grain boundary)

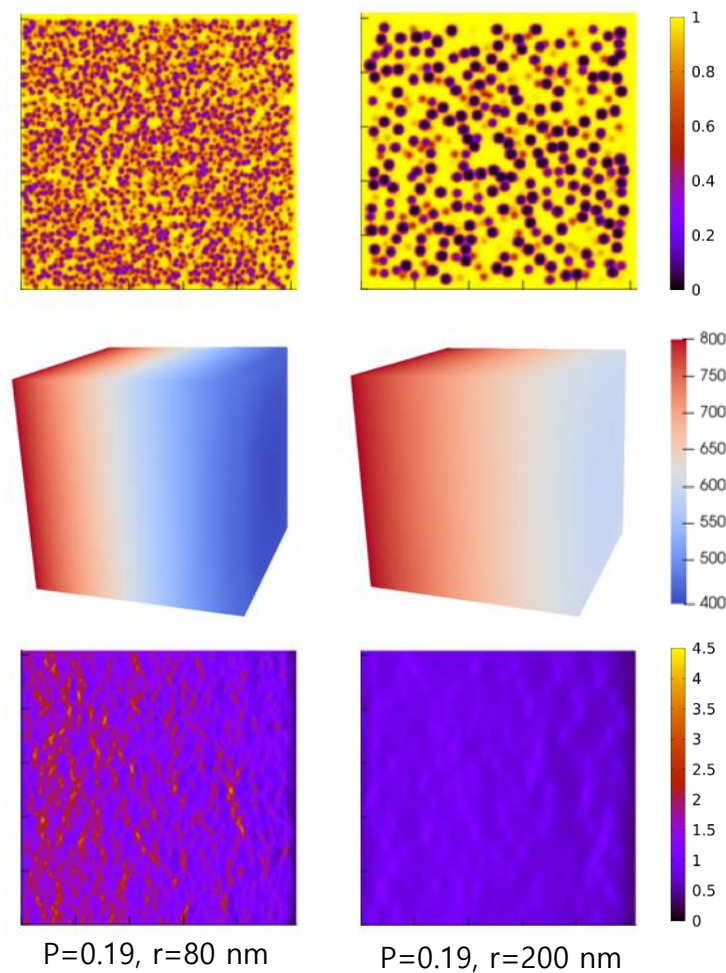
• • •

[4] B. Yoon and K. Chang (2020)

• 2D Simulation



• 3D Simulation



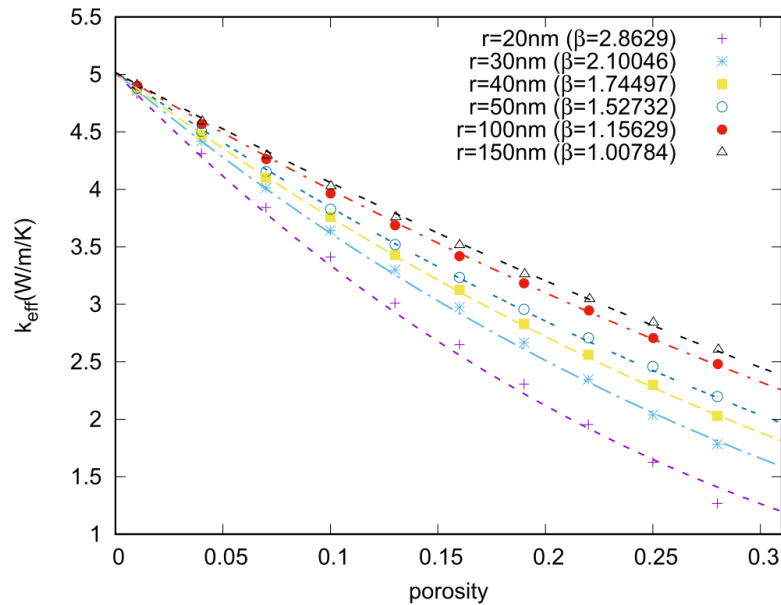
Pore Size Effect



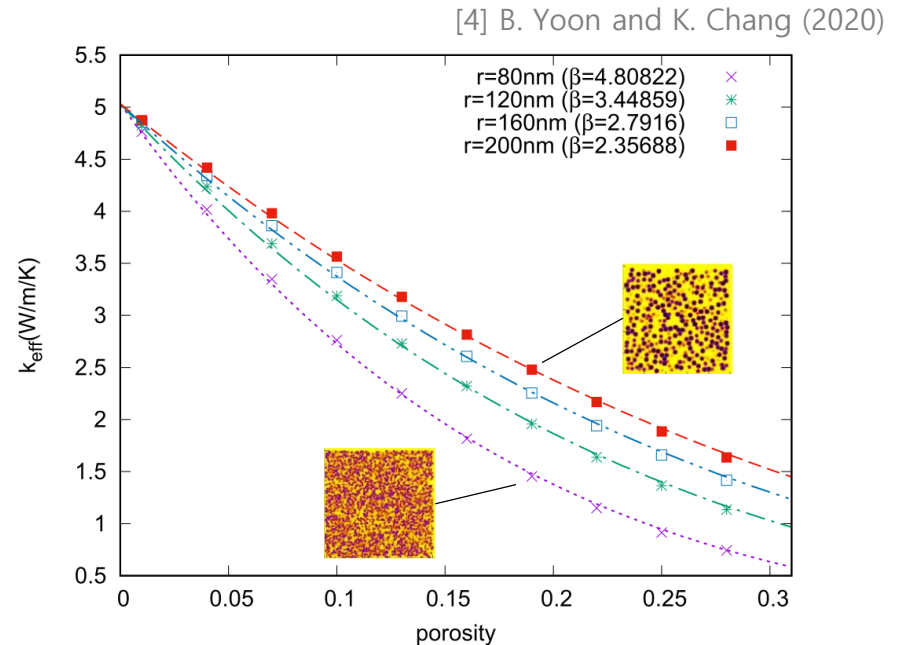
- Schulz Model ^{[5] B. Schulz (1981)}

$$k_{eff} = k_0(1 - P)^\beta$$

- P : porosity
- k_0 : conductivity of the corresponding nonporous material



2D simulation



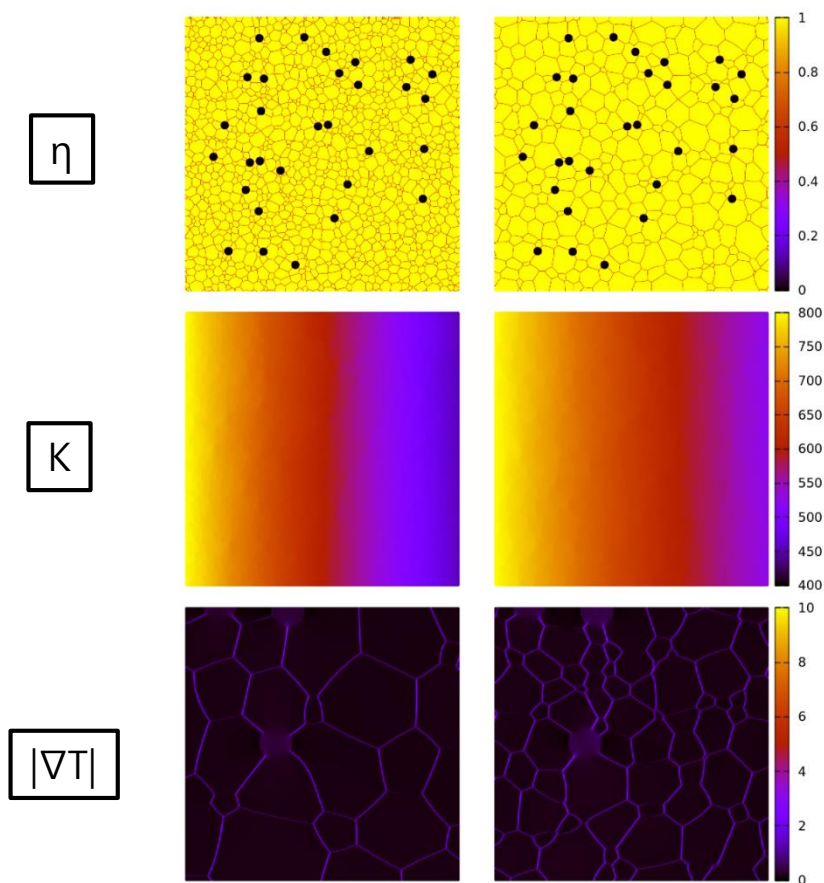
3D simulation

Simulational Result (with grain boundaries)

[6] B. Yoon and K. Chang (2021)

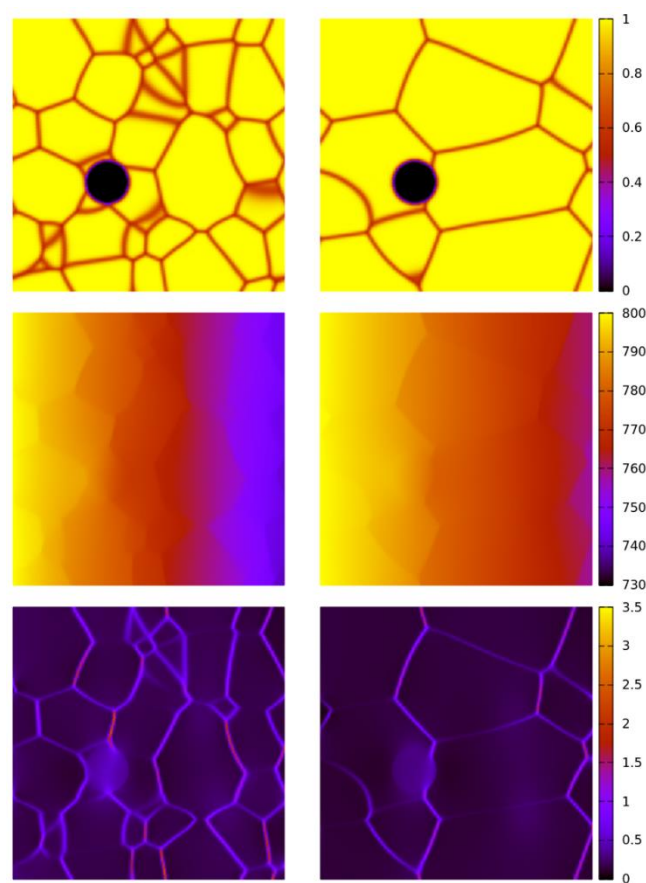
- 2D Simulation (20.48^2 um^2)

- 3D Simulation (2.56^3 um^3)



$r=300 \text{ nm}, d=600 \text{ nm}$

$r=300 \text{ nm}, d=1200 \text{ nm}$



$r=300 \text{ nm}, d=600 \text{ nm}$

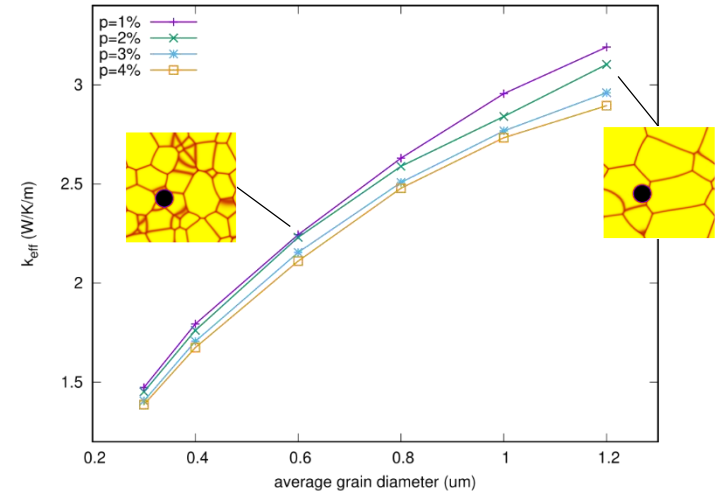
$r=300 \text{ nm}, d=1200 \text{ nm}$

Effective Thermal Conductivity of Porous Polycrystalline UO_2

• • •

- As expected, the effective conductivity decreases with increasing porosity and decreasing grain diameter.
- We found that under the given porosity and grain size, thermal conductivity decreases as the pore radius decreases.

[6] B. Yoon and K. Chang (2021)

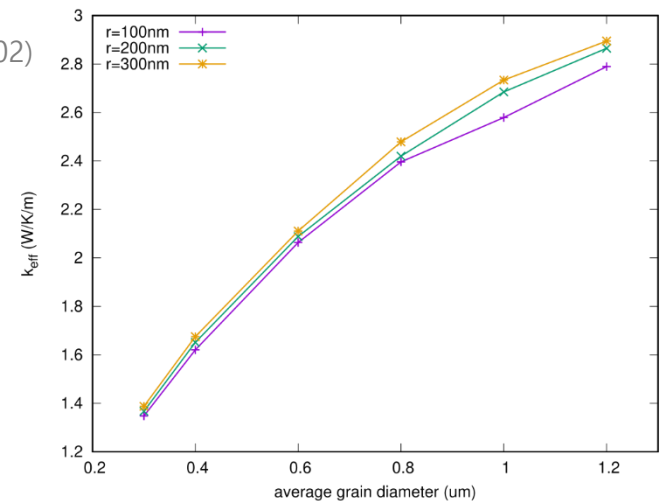


[7] H.-S. Yang et al. (2002)

- Schulz Model + Yang's polycrystal model (Multiplier model)

$$k_{eff} = k_0 \frac{(1 - P)^\beta}{1 + k_0 / (G_k d)}$$

- P : porosity
- d : average grain diameter
- k_0 : conductivity of corresponding nonporous material
- G_k : kapitza conductance



Effective Thermal Conductivity Model of Porous Polycrystal

• • •

- Multiplier Model

$$k_{eff} = k_0 \frac{(1 - P)^\beta}{1 + k_0 / (G_k d)}$$

The value of exponent β in multiplier model

	2D	3D
r=100nm	2.46884	2.41845
r=200nm	1.20273	1.88383
r=300nm	0.77723	1.66590

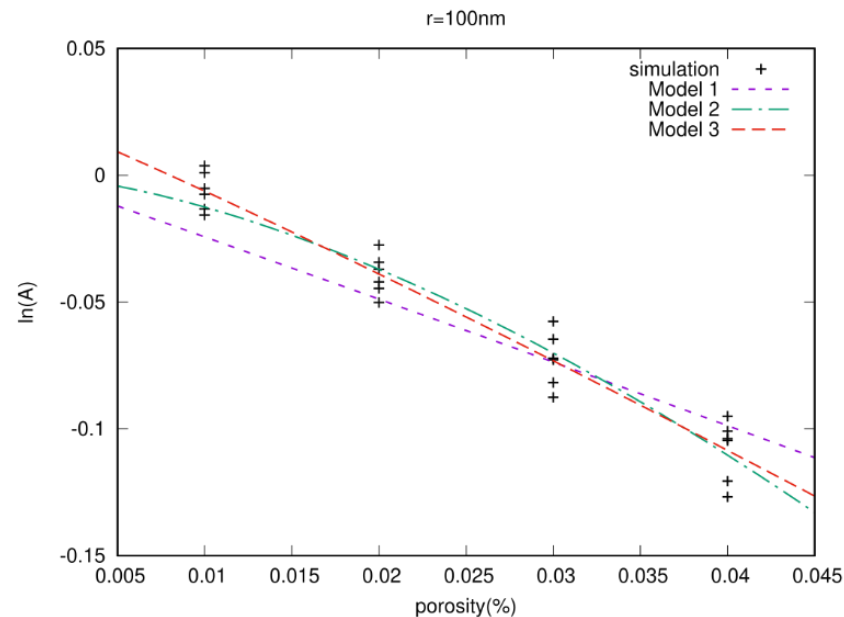
- The exponent β is 1.5 in Nikolopoulos and Ondracek's analysis of experimental data for spherical pores. [8] P. Nikolopoulos and G. Ondracek (1983)
- We found that the pore size affects the exponent β in both single crystal and polycrystal models.

Modified Effective Thermal Conductivity Models

• • •

The effective thermal conductivity models of UO_2

Model 1	$\ln \left[\frac{k_{\text{eff}} \times (1 + k_0 / (G_k d))}{k_0} \right] = (\beta + 1) \times \ln(1 - p)$
Model 2	$\ln \left[\frac{k_{\text{eff}} \times (1 + k_0 / (G_k d))}{k_0} \right] = (\beta + 1) \times \ln(1 - p)^n$
Model 3	$\ln \left[\frac{k_{\text{eff}} \times (1 + k_0 / (G_k d))}{k_0} \right] = (\beta + 1) \times \ln(1 - p)^n + \alpha(1 - p)$



Root Mean Square Deviations of Three Models

• • •

- Root mean square deviation

$$RMSD = \sqrt{\frac{1}{N} \sum |k_{simulation} - k_{model}|^2}$$

- $k_{simulation}$: effective conductivity value obtained from simulation
- k_{model} : effective conductivity value predicted by model 1, 2 and 3
- N : number of effective conductivity values being compared

Root mean square deviations of 2D system

Pore radius	Model 1	Model 2	Model 3
100nm	0.0560	0.0381	0.0339
200nm	0.0471	0.0349	0.0331
300nm	0.0598	0.0509	0.0507

Root mean square deviations of 3D system

Pore radius	Model 1	Model 2	Model 3
100nm	0.0341	0.0235	0.0217
200nm	0.0355	0.0179	0.0157
300nm	0.0334	0.0249	0.0209

Conclusion



- In the mesoscale regime, we have computationally investigated the effect of porosity, pore size, and grain boundaries on the effective conductivity of UO_2 by means of Crank-Nicolson and ADI method with implementation of CUDA GPU parallelization in 2D and 3D systems.
- We found that under the given porosity and grain size, thermal conductivity decreases as the pore radius decreases.
- We proposed empirical models for effective conductivity of porous polycrystal UO_2 based on microstructure generated by phase-field method in the presence of pores and grain boundaries.

References



- [1] J. Bruno and R. C. Ewing, "Spent Nuclear Fuel," *Elements*, vol. 2, pp. 343-349, 2006.
- [2] J. Harding and D. G. Martin, "A recommendation for the thermal conductivity of UO₂," *Journal of Nuclear Materials*, vol. 166, no. 3, pp. 223-226, 1989.
- [3] P. C. Millett et al., "Phase-field simulation of thermal conductivity in porous polycrystalline microstructures," *Journal of Applied Physics*, vol. 104, no. 3, p. 033512, 2008.
- [4] B. Yoon and K. Chang, "Effect of the pore radius on the effective conductivity of UO₂ in 2D and 3D: A computational approach," *Results in Physics*, vol. 19, no. 2-3, p. 103440, 2020.
- [5] B. Schulz, "Thermal conductivity of porous and highly porous materials," *High Temperatures – High Pressures*, vol. 13, no. 6, pp. 649-660, 1981.
- [6] B. Yoon and K. Chang, "Effective Thermal Conductivity Model of Porous Polycrystalline UO₂: A computational Approach," *In Press*, 2021.
- [7] H.-S. Yang et al. "Interfacial Thermal Resistance in Nanocrystalline Yttria-Stabilized Zirconia," *Acta Materialia*, vol. 50, no. 9, pp. 2309-2317, 2002.
- [8] P. Nikolopoulos and G. Ondracek, *Journal of Nuclear Materials*, vol. 114, no. 2-3, p. 231-233, 1983.



Nuclear Materials
Modeling Lab.



Thank You
For Your Attention