A measurement-based time-domain noise simulation of radiation detector-preamplifier (front-end) noise in nuclear spectroscopy is described. The time-domain noise simulation was performed by generating “noise random numbers” using Monte Carlo’s inverse method. The probability of unpredictable noise was derived from the empirical cumulative distribution function via the sampled noise, which was measured from a preamplifier output. Results of the simulated noise were investigated as functions of time, frequency, and statistical domains. Noise behavior was evaluated using the signal wave-shaping function, and was compared with the actual noise. Similarities between the response characteristics of the simulated and the actual preamplifier output noises were found. The simulated noise and the computed nuclear pulse signal were also combined to generate a simulated preamplifier output signal. Such simulated output signals could be used in nuclear spectroscopy to determine energy resolution degradation from front-end noise effect.

KEYWORDS: Time Domain Noise, Measurement-Based Noise Simulation, Preamplifier Noise, Nuclear Pulse Signal, Energy Resolution

1. INTRODUCTION

The signal simulation method is the most popular technique for studying behaviours of radiation detector, pulse pile-up and other electronic noise in a nuclear spectroscopy system. The generation of “random numbers” is often used to simulate the effect of noise in the electronic instrument. Time-domain simulation of electronic noises is a method used to study noise-induced energy resolution degradation in a spectroscopy system. Kuwata (1999) proposed a noise model using the SPICE computer program, which responded to a real system in computation of 1/f, step, and delta noises [1]. Tae-Hoon Lee and Gyuseong Cho (2003) presented a time-domain Monte Carlo-based Hspice noise simulation of a modern charge-sensitive preamplifier in conjunction with a CR-RC wave-shaping circuit to generate the random amplitude noise waveform. The amplitude distribution of thermal and 1/f noises was modeled and the noise sources were arranged in the produced noise waveform as referred to the noise from the preamplifier output [2]. Alberto Pullia and Stefano Riboldi (2004) proposed a computer simulation procedure for generating the electronic noise of ionizing-radiation spectrometers in the time domain [3]. In that case, the electronic noise was determined from fundamental electrical-physical parameters of the system including the equivalent noise source of detector capacitance, detector leakage current, and feedback resistor, as well as the 1/f-noise coefficient of the input transistor, and the temperature of the preamplifier input devices [4].

In practical nuclear spectroscopy, the radiation energy absorbed by a detector is converted into electrical pulses contaminated with unpredictable interference noises, which are outside the fundamental parameters. Those signals are amplified and shaped with an optimized shaping network to obtain the best possible energy resolution of the energy spectrum. The time profile of the detector signal and the response function of the pulse shaper are significant parameters for noise reduction in energy spectrum analysis. Noise is usually analyzed in the frequency domain, but time-domain noise simulation is an effective tool for noise reduction study. The time-domain front-end noise generation method applied in this study was conducted using the measurement-based Monte Carlo simulation technique to obtain similar physical characteristics in the time and frequency domains as may have occurred from actual preamplifier output. The simulated noise was investigated and compared with the actual noise, and it was then employed to study noise reduction in the signal pulse processing of a nuclear spectroscopy system.
2. THEORETICAL BACKGROUND

2.1 Power Spectral Density of Noise

The power spectral density of noise in a high-resolution spectrometer arises from electronic devices and the radiation detector. They are referred to as front-end stage and are always widespread as a color spectrum that can be categorized into several noise sources. These sources include the series noise of the preamplifier-input transistor, the parallel noise or shot noise of the detector leakage current, and the Lorentzian noise due to radiation damage to front-end devices.

A simplified diagram of the front-end charge-sensitive preamplifier with input-equivalent noise source \([3, 5]\) is shown in Fig. 1.

![Fig. 1. Equivalent Noise Source at Front-End Stage](image)

As may be seen in Fig. 1, noise is assumed to arise from four uncorrelated sources at the input. The first two are \(E_1^2\) and \(I_1^2\), which generate the white voltage and current noise of spectral densities \(a\) and \(b\), respectively. The second two are \(E_f^2\) and \(I_f^2\), which generate the 1/f voltage and current noise of spectral density \(a_f/|f|\) and \(b_f/|f|\), respectively. A global equivalent input current noise spectral density is \(N(\omega)\), which can be written as \([6]\):

\[
N(\omega) = aC^2 \omega^2 + b + C^2a_\omega |\omega| + b_\omega/|\omega| \quad (1)
\]

where \(2\pi a_\omega = a\) and \(2\pi b_\omega = b\).

The noise sources are random signals from detector leakage, the detector bias resistor, the preamplifier feedback resistor, a series connected resistor between the detector and preamplifier, FET (Field effect transistor) gate current, channel resistance of the input FET, surface leakage and dielectric loss. Especially for high-energy nuclear spectroscopy, Lorentzian packet due to radiation damaged devices is usually found. Propagating Equation (1) to the preamplifier output yields:

\[
N_{out}(\omega) = A^2(a + a_\omega /|\omega| + b /\omega^2 C + b_\omega /|\omega|^2 C) \quad (2)
\]

It is worth pointing out that with low leakage-current detectors and with low-noise front-end electronics, \(N_{out}(\omega)\) becomes mostly white, or \(N_{out}(\omega) \approx A^2a\). In low energy x-ray and gamma spectroscopy, the CdTe detectors have high stopping power and good energy resolution. The outstanding feature is the ability to operate at room temperature. They have a wide energy band-gap, resulting in high resistivity \((10^9-10^{11}\ \Omega \cdot \text{cm})\), producing low leakage current and exhibiting practically white noise \([7]\).

Noise is a major contribution to energy resolution degradation in nuclear radiation spectroscopy. Therefore, in experimental setups the noise reduction techniques that yield the maximum induced signal (from the radiation detector) to noise ratio (S/N ratio) are needed. The pulse-shaping process for noise filtering must be applied, as shown in Fig. 2.

![Fig. 2. General form of pulse shaping process](image)

Alberto Pullia et al. \([8]\) conducted research aimed at defining a filter which would allow minimum noise in a real system. A prerequisite for a proper design of such a digital processor is an adequate knowledge of the overall input noise, because its spectral characteristics, along with those of the signal itself, dictate the shape of the minimum noise filter.

The present study demonstrates the possibility of learning more about global noise in real nuclear spectroscopy. The power spectral density characteristics of each preamplifier in the experiment were also studied. Two types of preamplifiers, the Canberra model 2006 and a locally developed one, were chosen for the study. A sequence of noise from the preamplifier output was sampled. The power spectral density was calculated by a discrete-time Fourier transform of the autocorrelation. The results yielded the power spectral density profiles illustrated in Fig. 3. The frequency distribution characteristics for different preamplifier models exhibited their own characteristics that arose from unpredictable sources.

2.2 Time-Domain Noise Simulation

In order to simulate the time-domain noise in a nuclear spectroscopy system, the probability distribution
function of the noise amplitude is needed for the inverse transform method [9, 10]. The favourite method is Monte Carlo simulation, which can convert the uniform probability distribution of random numbers between 0 to 1, \( U(0,1) \), into random numbers according to the desired probability distribution, \( f(x) \), where \( x \) is the signal amplitude, before carrying out a simulation of each nuclear spectroscopy system. If \( F(x) \) is the cumulative distribution function of \( f(x) \) and if \( F^{-1} \) is the inverse of the function \( F \), \( x \) can be found from Equation (3):

\[
x = F^{-1}(U)
\]  

(3)

Generally, most noise sources have a Gaussian or normal distribution of instantaneous amplitude with time [11], including noise from a preamplifier output. The sequence of noise amplitude distribution in a theoretical model, however, cannot represent a real distribution of noise sources due to unpredictable interferences in each operating environment. Therefore, the most popular method for generating truly random numbers is to simply capture the noise signals from the preamplifier output and then sample them at a constant sampling rate [12]. In this method, \( F(x) \) and a fraction of the total time at the noise amplitude of less than or equal to \( x \) are measured [1, 11].

Based on the fact of noise signal in an ergodic process, the sample’s ensemble average is equal to the time average. Ergodicity implies that all the statistical properties of the process are invariant in time and that these properties are deducible from measurements made in time [8].

The unknown distribution function of the noise amplitude is estimated by the empirical cumulative distribution function, \( F_n(x) \), which is the natural estimation of \( F(x) \) according to the Glivenko-Contelli theorem [13].

If \( X_i \) represents a sequence of noise samples drawn from a preamplifier output, \( F_n(x) \) can be expressed as:

\[
F_n(x) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}(X_i \leq x)
\]  

(4)

2.3 Nuclear Pulse Signal Simulation

The electronic design of next-generation radiation detectors for specific applications requires system simulation in the time domain to test the design and behaviour of those systems. The model of a nuclear pulse signal is defined as:

\[
s(t) = A h(t) + n(t)
\]  

(5)

where \( s(t) \) represents the pulse signal; \( A \) stands for the pulse amplitude, which is proportional to the nuclear radiation energy; and \( n(t) \) represents electronics noise, which depends on the characteristics of each system. The \( h(t) \) term represents the preamplifier output impulse response and is defined in Equation (6):

\[
h(t) = e^{-t/\tau}
\]  

(6)

where \( \tau \) is the decay time constant of the preamplifier.

2.4 Noise Evaluation

The evaluation of the noise efficiency of a wave-shaping network in a spectroscopy amplifier was first performed by F.S. Goulding (1972) [14]. He showed a simple approach to the problem of comparing the noise behavior of various pulse-shaping networks used in a nuclear pulse-amplifier. His method is based on an elementary physical view of noise in the time domain. Noise sources were categorized into at least two types, e.g., Step noise and Delta noise. Only step and delta noises were assumed to be present, although there were many well-known noise sources including flicker-noise, excess-noise and Lorentzian package, not all of which could be presented by step and delta functions at amplifier input. The analysis focused on the cumulative effect of the noise energy occurring before signal measuring time, presented by step-noise and delta-noise indices. The indices provided a reasonably detailed view on the subject of detector processing signals and indicated factors which should be the most favorable choices for optimizing the system.

Noise indices were determined by the rational factor of the mean square noise to the square of the signal amplitude that passed through the wave-shaping network. An
alternative index of noise evaluation was Equivalent Noise Charge (ENC) [15]. This parameter represented the contribution of the noise charge to the total charge of the system. The value of ENC was usually expressed in a unit of electrons, so it could be readily compared to a signal charge generated in the detector. In the frequency domain, the ENC of particular noise sources could be calculated if the noise power spectral density function and the wave-shaping network transfer function were known. In the time domain, the ENC value of any noise component could be evaluated from output noise waveforms by dividing the root-mean-square of the noise voltage by the amplitude of the signal pulse generated from a single energy at the shaped output. A justifiable test aimed to use the ENC to compare the behaviour of actual noise to that of simulated noise. The most general wave-shaping network or filter in a nuclear spectroscopy amplifier unit consisted of a combination set of a differentiator and an integrator with the same corner frequency (usually called the reciprocal function) as the shaping time [14].

3. MATERIALS AND METHODS

In this experiment, the equipment set for preparing Monte Carlo’s input probability consisted of an AMPTEK XTR100 CdTe low energy x-ray detector, a preamplifier, a high voltage bias power supply and a Tektronix digital oscilloscope model TDS3000. A personal computer was used for controlling data acquisition and for data computation. The software for calculation was written in MATLAB. Fig. 4 shows the setup of the experimental system.

3.1 Noise Sampling

The front-end noise measuring system was set up in an environment with low background noise. A warm-up time of at least half an hour was needed to ensure system stability. For data acquisition, a signal containing only noise generated from the preamplifier’s devices and the CdTe radiation detectors was digitized by the digital oscilloscope at the sampling rate of 5 Ms/s. A set of ten thousand noise samples was sent to the computer via a serial port on the digital oscilloscope to evaluate their statistical parameters and to perform the noise simulation process.

3.2 Noise Simulation

The empirical cumulative distribution function of noise amplitude, which was related in Equation (4), is illustrated in Fig. 5. The signal simulation was generated by the inverse transformation method, which was related in Equation (3). Then, the simulated noise signals of the front-end output were regenerated. The mean, standard
deviation, root mean square, amplitude distribution and power spectral density of both the actual and simulated front-end noise were calculated. The power spectral densities were evaluated from the sequence of noise under the selected Gaussian window for smoothing the power spectral density curve. The average of a hundred simulations was taken from ten measurements. For statistical analysis, a statistical non-parametric test with two independent samples was used.

3.3 Noise Efficiency Evaluation

The ENC was evaluated using the MATLAB program. The command ‘filter’ was used for high-pass and low-pass filtering functions. The corner frequency of the high-pass was represented by the reciprocal function of $2\pi RC$, and this parameter was used for the ‘filter’ command function. The same corner frequency was used in the ‘butter’ command for generating the 3rd order low-pass filter numerator and denominator parameters.

The frequency response in the time domain of an equivalent wave-shaping network was configured as a band-pass filter. In this study, the network function of a cascade stage of a single CR high-pass filter (differentiator) and a third order low-pass Butterworth filter (integrator) was used as the wave-shaping function and as the numerical calculation. An experiment was setup to calculate the ENC of both actual and simulated noises at shaping time ranging from 0.25 to 12 ms. The equivalent-function schematic of the wave-shaping network is shown in Fig. 6.

Fig. 6. The Schematic of the Numerical Experiment

(a) Wave shaping network

(b) Equivalent function

Fig. 6. The Schematic of the Numerical Experiment

The noise behavior of the pulse-shaping network was evaluated by applying the time-domain simulated noise through the MATLAB function of the wave-shaping network. The root-mean-square voltage output was obtained for ENC calculation at each shaping time.

3.4 Noise Effect in Energy Resolution

In this experiment, the simulated preamplifier output signal was applied to the MATLAB wave-shaping function for noise effect in the energy resolution study. The composite preamplifier output, also known as the simulated preamplifier output signal, was generated by mixing the simulated noise containing the specific noise characteristics of the CdTe semiconductor detector-preamplifier with a computed exponential pulse signal, as referred to in Equation (5) and as shown in Fig. 7.

Fig. 7. The Composite Preamplifier Output Generation

Fig. 8 shows a series of nuclear pulse signals at the preamplifier output simulated by the addition of a computed exponential pulse signal with a simulated noise. The pulse height amplitude, the pulse rate and the decay time constant of signals can be adjusted by the setting parameters. The signal amplitude in volt
equivalent to the imparted energy of a 60-keV gamma ray in the CdTe detector was simulated from Equation (6). Then, the simulated preamplifier output was applied to the same MATLAB wave-shaping function. The amplitude distribution permits calculation of the FWHM (full width at half maximum) at each shaping time.

4. RESULTS

4.1 Simulated Noise Investigation

The results representing the front-end noise were investigated and then compared with the actual noise as a function of time, frequency, statistical domains. The simulated noise waveform at the preamplifier output, generated from the empirical distribution function of the sample noise sequence, is shown in Fig. 9(a). The means, standard deviation, and the root mean-square are 0.0036, 0.0012, and 0.0021 volts, respectively. Fig. 9(b) shows the actual noise in the time domain having the means, standard deviation, and the root mean-square of 0.0015, 0.0012, and 0.0019 volts, respectively. Fig. 10 plots the power spectral densities of the simulated noise compared with the actual noise. The statistical test of two independent samples revealed the dissimilarity between their statistical parameters. This is due to the difference in the high frequency region over $2 \times 10^6$ MHz found in the power spectral density of the simulated noise. As shown in Fig.11, the ENC responses of the simulated and actual noise sources through the wave-shaping function are very similar.

4.2 Simulated Noise Application

The simulated preamplifier output, which was a combination of noise characteristics from the CdTe semiconductor detector front-end and computed nuclear pulses, was applied to the same noise investigation wave-shaping function to determine the energy resolution in terms of FWHM. A pulse height equivalent to 60 keV was assumed in order to calculate the peak spectrum with
and without noise at the shaping time of 0.5 μs, when the pulse rate was 10 kcps and when the preamplifier decay time constant was 50 μs. Fig. 12(a) shows the line peak spectrum when the nuclear pulse signal was free from noise interference. Fig. 12(b) illustrates the spectrum when the natural nuclear-like pulse signal was interfered by noise. In Fig. 13, the better energy peak resolution was obtained when the longer shaping time (at 4.0 and 8.0 μs) of the filter function was applied to reduce the noise interference. The results of the energy peak resolution in this experiment illustrate that this noise simulation technique could be employed for determining a noise reduction feature for any wave-shaping function.

5. CONCLUSION

Random noise generated through the inverse transform method using the empirical distribution function of a noise sequence sampled from preamplifier output with the measurement-based technique could be used to simulate the time-domain noise representing the front-end noise characteristics of a nuclear spectroscopy system. A method for the generation of random noise with characteristics comparable to those of the front-end noise in a nuclear spectroscopy system was presented. Simulated noise produced using the presented method exhibited characteristics that were almost the same in all respects as the real-time noise (so-called “actual noise”) with regard to waveforms, spectral density, and equivalent noise charge. This enabled the investigation of noise bandwidth and the characteristics of the wave-shaping network in a spectroscopy amplifier. A simulated preamplifier output could be simply generated by the combination of simulated noises and computed exponential pulses to study the behaviour of wave-shaping function and the degradation in energy resolution due to the unpredictable interference noise from the front-end stage.

However, nuclear spectroscopy noise simulation intended to improve energy resolution requires two important conditions. One is a power spectrum with the same noise characteristics, referred to as spectroscopy amplifier input, which is important for evaluating frequency response of the filter function. The other is the...
probability distribution function, which is important for time-domain analysis. In the measurement-based time-domain noise simulation, various cases and types of front-end characteristics similar to actual noise can be collected as a noise profiles database. These noise profiles could be used for digital filter design and development to obtain the best energy resolution for a nuclear spectroscopy system. The simulated noise can also be modified to investigate the optimum filter selection for the best energy resolution in a nuclear spectroscopy setup. Furthermore, the above simulation technique could be applied to study the impact of other statistical nuclear radiation phenomena such as pulse pile up and ballistic deficit effects on nuclear spectroscopy energy resolution.

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REFERENCES