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Temperature Coefficient in D₂O Moderated Reactor (Wolsung Unit 1)

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Abstract

The temperature coefficient has been investigated on the Wolsung nuclear power reactor, in which fuel is natural uranium dioxide and moderator heavy water. The numerical computations are carried out in terms of changes of the effective neutron multiplication factor with respect to fuel, moderator, and coolant temperatures. Those results are compared with the computed values of temperature coefficient based on the LATREP computer code.

요 약

천연 산화 우라늄 핵연료와 중수 감속재를 사용하는 월성 1호기에 관한 온도 계수를 조사하였다. 핵연료, 감속재 및 냉각재 온도변화에 따른 중성자의 유효 증배 인자의 변화를 계산하였다. 계산된 결과를 LATREP 전자계산 code에 의한 온도 계수 값들과 비교하였다.

1. Introduction

Of the most important parameters of a nuclear reactor, temperature coefficient of reactivity is the effect bearing on the operation of a reactor since the reactivity change introduced per degree of temperature gives either positive or negative temperature coefficient of the reactor and ultimately the safety of the system. If an increase of temperature occurs in normal reactor operation, this leads to an increase in heat release beyond normal demand. In the case of posi-

tive temperature coefficient, the increase of temperature induces positive reactivity insertion and may result in a runaway accident. The temperature coefficient should therefore be required to be negative so that an increase in temperature causes negative reactivity addition, that is, a decrease in the rate of fission and consequently power and a return to safe level. Thus for any nuclear reactor, typically for power reactor, NRC requires a temperature coefficient of reactor to be negative so as to be inherently self-regulating and safe.

It has been, however, reported¹⁾ that

the heavy water reactors have positive temperature and power coefficients so that the reactor is not stable inherently. However, reported are a few dealing with temperature coefficient of the system by experiments or by theoretical computations. Girard et.al.²⁾ made an investigation of temperature coefficient in a square lattice cell of various natural uranium fuel types in heavy water moderator. They measured the coefficient as a function of buckling and showed that the coefficient due to those types was not significantly changed but negative temperature coefficient was observed. Experiments were also conducted³⁾ in the exponential tank to measure the change in buckling as a function of temperature when the tank was heated uniformly by means of electrical heating, and negative values of temperature coefficient for buckling were also observed for the range between 20°C and 100°C.

Since CANDU type reactors fueled with natural uranium dioxide and moderated by heavy water are planned to be introduced into Korea, it is of importance to know whether the system is inherently safe associated with the temperature change of various reactor parameters. Numerical calculations on respective parameter of reactivity are carried out in order to investigate what parameter mainly contributes to determining the temperature coefficient and to obtaining overall effect of temperature to reactivity in the system.

The following section describes general formulas to compute temperature effects of each reactivity parameter. The associated cross section evaluations are presented together with the descriptions of reactivity parameters. The last section contains the

computed results and discusses these with the other results based on LATREP code.

II. Temperature Coefficient

A temperature coefficient of reactivity, α_T , is generally defined by

$$\alpha_T = \frac{d\rho}{dT} = \frac{1}{k_{eff}} \frac{d k_{eff}}{dT}, \quad (1)$$

where T stands for temperature. As the case may be, it denotes the average temperature of either the individual cell components or the reactor core. From the effective neutron multiplication factor k_{eff} , the temperature coefficient can be rewritten into six factors:

$$\alpha_T = \alpha(\eta) + \alpha(f) + \alpha(p) + \alpha(\epsilon) + \alpha(P_T) + \alpha(P_F), \quad (2)$$

that is, α_T can be obtained from the sum of individual effects to temperature change.

III. Numerical Computations

1. Cross Sections

In order to evaluate the temperature coefficient of reactivity, it is important to know the temperature-dependent cross sections which are put into each of multiplication factor estimates. The effective cross sections are generally defined by

$$\bar{\sigma} = \frac{\iint \sigma(E) \phi(\vec{r}, E) d\vec{r} dE}{\iint \phi(\vec{r}, E) d\vec{r} dE}, \quad (3)$$

where $\phi(\vec{r}, E)$ is neutron flux with energy E at \vec{r} , which gives the energy-dependent flux as

$$\phi(E) = \int_{\text{all space}} \phi(\vec{r}, E) d\vec{r}. \quad (4)$$

Together with Eq. (4), the effective cross sections can be rewritten by

$$\bar{\sigma} = \frac{\int_0^{\infty} \sigma(E) \phi(E) dE}{\int_0^{\infty} \phi(E) dE}. \quad (5)$$

In computing cross sections from the analytical form, the expressions for energy-dependent cross sections must be obtained in closed forms. Margulis⁴⁾ suggests the following relations for the energy-dependent cross sections of nuclei of interest, U-235 and U-238.

$$\sigma_f(25) = 608(1 - 0.0543E) E^{-0.5} \text{ barns}, \quad (6)$$

$$\sigma_a(28) = 2.73(1 + 0.00506E) E^{-0.5} \text{ barns}. \quad (7)$$

The energies here are measured in units of 0.0253eV over the range from 0.0253 to 0.1 eV and the subscripts have the usual meanings. As given in BNL-325⁵⁾, the capture-to-fission ratio is taken to be 0.19 for the nucleus U-235 at 0.0253eV.

Evaluating the absorption cross section of UO_2 , oxygen in the fuel element contributes so very small values of absorption to the fuel as to be negligible. Using the energy-dependent thermal cross sections of fuel nuclei mentioned above, the absorption cross section of fuel (UO_2) can then be given by

$$\sigma_a(UO_2) = 1.693(1 - 0.0373E) E^{-0.5}. \quad (8)$$

The energy-space dependent flux is now described in a certain moderator. It is assumed for numerical calculations of the cross sections that neutrons are completely thermalized in the moderator region and hence the flux can be separately expressed in the energy and spatial dependence, that is,

$$\phi(\vec{r}, E) = \phi(\vec{r}) F(E). \quad (9)$$

If a reactor is fueled with cylindrical rods and the diffusion theory is valid for a cell region, the space-dependent flux for an infinite cylindrical rod is readily written as

$$\phi(r) = \frac{I_0(\kappa_F r)}{I_0(\kappa_F R)}, \quad (10)$$

where κ_F is the reciprocal of fuel diffusion

length that is a function of energy and R is the radius of the fuel rod. Taking the Maxwellian distribution for the energy-dependent flux,

$$\phi(r, E) = \frac{I_0(\kappa_F r)}{I_0(\kappa_F R)} \frac{E e^{-E/T}}{T^2}, \quad (11)$$

where T is in units of energy. Plugging Eq. (11) into Eq. (4) yields the energy-dependent flux in the infinite cylindrical cell as following:

$$\phi(E) = 2\pi R^2 \left[\frac{I_1(\kappa_F R)}{\kappa_F R I_0(\kappa_F R)} \right] \frac{E}{T^2} e^{-E/T}, \quad (12)$$

and the numerical value of κ_F can be obtained from the well-known relation,

$$\kappa_F = N_F \sigma_f \sqrt{3\sigma_a/\sigma_s}, \quad (13)$$

in which N_F is the number density of fuel. The scattering cross section of fuel in Eq. (13) is treated⁶⁾ to be constant at the thermal region (=16.7 barns), while the energy-dependent absorption cross section of fuel is taken from Eq. (8).

Since κ_F is energy-dependent, the bracketed quantity in Eq. (12) is fitted to a following empirical function⁴⁾ of energy in making computations with ease:

$$\frac{I_1(\kappa_F R)}{\kappa_F R I_0(\kappa_F R)} = a - bE^{-c}, \quad (14)$$

where E is measured in eV. Fitting the numerical values computed at various energies, i.e., $E=0.0253$ through 0.1 eV, the coefficients of a , b and c in Eq. (14) were calculated by use of trial and error method:

$$\frac{I_1(\kappa_F R)}{\kappa_F R I_0(\kappa_F R)} = 0.59314 - 0.08056E^{-0.291}. \quad (15)$$

Since the energy variable in Eq. (12) is measured in units of 0.0253 eV, Eq. (12) together with Eq. (15) can be written by

$$\phi(E) = 3.7268 R^2 (1 - 0.395 E^{-0.291}) \frac{E}{T^2} e^{-E/T}. \quad (16)$$

Using the energy-dependent cross sections and flux, the effective cross sections of fuel may be obtained from Eq. (5) in

terms of gamma functions:

$$\begin{aligned} \bar{\sigma}_f(25) &= \frac{608 \int_0^\infty (1-0.0543 E) (1-0.395 E^{-0.291}) E^{0.5} e^{-E/T} dE}{\int_0^\infty (1-0.395 E^{-0.291}) E e^{-E/T} dE} \\ &= \frac{608 [\Gamma(1.5) T^{1.5} - 0.0543 \Gamma(2.5) T^{2.5} - 0.395 \Gamma(1.209) T^{1.209} + 0.0215 \Gamma(2.209) T^{2.209}]}{T^2 - 0.395 \Gamma(1.709) T^{1.709}}, \end{aligned} \quad (17)$$

$$\bar{\sigma}_a(\text{UO}_2) = \frac{1.693 [\Gamma(1.5) T^{1.5} - 0.0373 \Gamma(2.5) T^{2.5} - 0.395 \Gamma(1.209) T^{1.209} + 0.0147 \Gamma(2.209) T^{2.209}]}{T^2 - 0.395 \Gamma(1.709) T^{1.709}}. \quad (18)$$

The D₂O and H₂O are treated as 1/v-absorber. Their temperature-dependent thermal absorption cross sections are given by

$$\sigma_a(E) = \sigma_a(E_0) E^{-0.5}, \quad (19)$$

in which $\sigma_a(E_0)$ is the absorption cross section at the neutron energy of 0.0253 eV. Also, the following empirical relation is taken⁶⁾ for the scattering cross sections:

$$\sigma_s(E) = \sigma_s(E_0) E^{-m}, \quad (20)$$

where exponent m is 0.112 for D₂O and 0.470 for H₂O.

By using Eqs. (19) and (20), the effective cross sections of D₂O and H₂O have been obtained from Eq. (5) and expressed in terms of gamma functions:

$$\bar{\sigma}_a = \sigma_a(E_0) \Gamma(1.5) T^{-0.5}, \quad (21)$$

and

$$\bar{\sigma}_s = \sigma_s(E_0) \Gamma(2-m) T^{-m}. \quad (22)$$

If the heavy water purity is denoted by w , the average cross sections are described as

$$\bar{\sigma} = w \bar{\sigma}_{\text{D}_2\text{O}} + (1-w) \bar{\sigma}_{\text{H}_2\text{O}}. \quad (23)$$

The purity of the heavy water is taken to be 99.75 w/o and the density of the fuel to

$$\rho_{\text{H}_2\text{O}}(T_c) = \frac{1 + 0.1342489 t^{1/3} - 0.003946263 t}{3.1975 - 0.3151548 t^{1/3} - 0.001203374 t + 7.48908 \times 10^{-13} t^4}, \quad (25)$$

where t is the temperature difference between the physical temperature of moderator T_c and 374.11°C⁸⁾.

$$\rho_{\text{D}_2\text{O}}(T_c) = \frac{0.4883926 - 0.011517 t^{1/3} - 8.12267 \times 10^{-4} t - 9.9394 \times 10^{-14} t^4}{1 - 0.12666627 t^{1/3} + 1.5775 \times 10^{-5} t} - \frac{10^{-11} T_c^4}{3} - \frac{500}{(380 - T_c)^4}, \quad (26)$$

be 10.283 g/cm³⁷⁾ for the numerical computations. The cross sections at the neutron energy of 0.0253 eV are taken from BNL-325.⁵⁾

2. Volume Expansion

The following consideration is the effects of physical temperature of moderator on the temperature coefficient of reactivity. The atom density is inversely proportional to specific volume, i.e. $N_M \sim \rho_M \sim \frac{1}{v_M}$, and hence the factor of $\alpha(N_M)$ may be expressed as follows⁶⁾:

$$\alpha(N_M) = \alpha(\rho_M) = -\alpha(v_M) = -\beta_M, \quad (24)$$

where v_M , ρ_M and β_M are the specific volume, physical density, and volume expansion coefficient of moderator, respectively. Computations of densities in a moderator are based on the following formulae⁸⁾:

i) pure light water

—density at 20°C is 0.99823

—density at temperature T_c (°C) is:

ii) pure heavy water

—density at 20°C is 1.1053

—density at temperature T_c (°C) is:

where $t=371.5-T_c$.

The atom fraction A of light water is calculated from its weight fraction $(1-w)$ as

$$A = \frac{(1-w)}{(1-w)+0.8996w}. \quad (27)$$

Assuming that the number of atoms per unit volume is the same for light and heavy water, the density of the mixture is then

$$\rho_M(T_c) = A\rho_{H_2O}(T_c) + (1-A)\rho_{D_2O}(T_c). \quad (28)$$

The volume expansion coefficient of moderator is calculated from the following relation:

$$\beta_M = \frac{1}{\rho_M(T_c)} \frac{\rho_M(T_c + \Delta T_c) - \rho_M(T_c)}{\Delta T_c}. \quad (29)$$

3. Various Factors on k_{eff}

$$\alpha(\bar{\sigma}_f(25)) = \frac{1}{T_0} \left[\frac{1.5\Gamma(1.5)T^{0.5} - 0.136\Gamma(2.5)T^{1.5} - 0.478\Gamma(1.209)T^{0.209} + 0.0474\Gamma(2.209)T^{1.209}}{\Gamma(1.5)T^{1.5} - 0.0543\Gamma(2.5)T^{2.5} - 0.395\Gamma(1.209)T^{1.209} + 0.0215\Gamma(2.209)T^{2.209}} - \frac{2T - 0.675\Gamma(1.709)T^{0.709}}{T^2 - 0.395\Gamma(1.709)T^{1.709}} \right], \quad (32)$$

$$\alpha(\bar{\sigma}_{aF}) = \frac{1}{T_0} \left[\frac{1.5\Gamma(1.5)T^{0.5} - 0.0933\Gamma(2.5)T^{1.5} - 0.478\Gamma(1.209)T^{0.209} + 0.0325\Gamma(2.209)T^{1.209}}{\Gamma(1.5)T^{1.5} - 0.0373\Gamma(2.5)T^{2.5} - 0.395\Gamma(1.209)T^{1.209} + 0.0147\Gamma(2.209)T^{2.209}} - \frac{2T - 0.675\Gamma(1.709)T^{0.709}}{T^2 - 0.395\Gamma(1.709)T^{1.709}} \right], \quad (33)$$

where T_0 is the room temperature in degrees Kelvin. Then the temperature coefficient of η is of the form:

$$\alpha(\eta) = \frac{1}{T_0} \left[\frac{1.5\Gamma(1.5)T^{0.5} - 0.136\Gamma(2.5)T^{1.5} - 0.478\Gamma(1.209)T^{0.209} + 0.0474\Gamma(2.209)T^{1.209}}{\Gamma(1.5)T^{1.5} - 0.0543\Gamma(2.5)T^{2.5} - 0.395\Gamma(1.209)T^{1.209} + 0.0215\Gamma(2.209)T^{2.209}} - \frac{1.5\Gamma(1.5)T^{0.5} - 0.0933\Gamma(2.5)T^{1.5} - 0.478\Gamma(1.209)T^{0.209} - 0.0325\Gamma(2.209)T^{1.209}}{\Gamma(1.5)T^{1.5} - 0.0373\Gamma(2.5)T^{2.5} - 0.395\Gamma(1.209)T^{1.209} + 0.0147\Gamma(2.209)T^{2.209}} \right]. \quad (34)$$

(b) Thermal Utilization

The expression of f in heterogeneous reactor is given⁶⁾ by

$$f = \frac{\bar{\Sigma}_{aF}V_F}{\bar{\Sigma}_{aF}V_F + \bar{\Sigma}_{aM}V_MFF_m}, \quad (35)$$

in which FF_m is the disadvantage factor, the ratio of the mean flux in the moderator to that in the fuel. Taking the derivative of Eq. (35) yields

$$\alpha(f) = (1-f)[\alpha(\bar{\sigma}_{aF}) - \alpha(\bar{\sigma}_{aM}) - \alpha(FF_m) + \alpha(N_F) + \alpha(N_M)]. \quad (36)$$

(a) η effect

The factor η is given by

$$\eta = \frac{\nu \bar{\Sigma}_f(25)}{\bar{\Sigma}_{aF}}, \quad (30)$$

where the subscript F stands for fuel. The effective fission cross section of ^{235}U and absorption cross section of UO_2 fuel are previously described in detail. Assuming that the average number of neutrons per fission ν is constant in the thermal region and the number density of fuel does not change due to temperature, the factor η may be expressed by

$$\alpha(\eta) = \alpha(\bar{\sigma}_f(25)) - \alpha(\bar{\sigma}_{aF}). \quad (31)$$

Taking the logarithmic derivatives of Eqs. (17) and (18) yield:

As temperature varies, the density change in the fuel is small compared to that in the moderator. The factor $\alpha(\bar{\sigma}_{aM})$ can be written from Eq. (19):

$$\alpha(\bar{\sigma}_{aM}) = -\frac{1}{2TT_0}. \quad (37)$$

In the derivatives of cross sections, one can see that the gradients are proportional to the inverse of temperature. Since Eq. (33) is mainly dominated by the second term in the bracket, the difference between the

first two terms in Eq. (36) is also found to be very small. Then Eq. (36) may be written as

$$\frac{1}{f} \frac{df}{dT} \simeq -(1-f) [\alpha(F F_m) + \alpha(N_M)]. \quad (38)$$

When diffusion theory is used to calculate the thermal utilization, f is also given⁷⁾ by

$$\frac{1}{f} - 1 = \frac{\bar{\Sigma}_{aM} V_M}{\bar{\Sigma}_{aF} V_F} F + (E-1). \quad (39)$$

Here E and F are the lattice functions for a cylindrical cell in a heterogeneous reactor:

$$F = \frac{\kappa_F R}{2} \frac{I_0(\kappa_F R)}{I_1(\kappa_F R)}, \quad (40)$$

$$E-1 = \frac{(\kappa_M R_C)^2}{2} \left[\frac{R_C^2}{R_C^2 - R^2} \ln\left(\frac{R_C}{R}\right) - \frac{3}{4} + \frac{1}{4} \left(\frac{R}{R_C}\right)^2 \right], \quad (41)$$

where κ_M is the reciprocal of the diffusion length of moderator and R_C the radius of a cell.

Combining Eq. (35) and Eq. (39), the disadvantage factor may be obtained from the following relations:

$$F F_m = F + \frac{\bar{\Sigma}_{aF} V_F}{\bar{\Sigma}_{aM} V_M} (E-1). \quad (42)$$

Taking logarithm on both sides of Eq. (42) and differentiating the result with respect to temperature yields

$$\alpha(F F_m) = \frac{1}{F F_m} \left[\frac{1}{T_0} \frac{dF}{dT} + \frac{\bar{\Sigma}_{aF} V_F}{\bar{\Sigma}_{aM} V_M} (E-1) \left[\alpha(\bar{\sigma}_{aF}) - \alpha(\bar{\sigma}_{aM}) + \frac{1}{T_0(E-1)} \frac{dE}{dT} \right] \right]. \quad (43)$$

The derivative of F is

$$\frac{dF}{dT} = -\frac{(\kappa_F R)^2}{16T} \left[1 - \frac{(\kappa_F R)^2}{12} \right], \quad (44)$$

and the factor of $\frac{1}{E-1} \frac{dE}{dT}$ may be derived from Eq. (41)

$$\frac{1}{E-1} \frac{dE}{dT} = 2\alpha(\kappa_M). \quad (45)$$

Since the absorption cross section of heavy water moderator is negligible compared to the scattering cross section, its total cross

section may be replaced by the scattering cross section. Then κ_M can be expressed by

$$\kappa_M = \sqrt{3 \bar{\Sigma}_{aM} \bar{\Sigma}_{sM}}. \quad (46)$$

From Eq. (46), the factor $\alpha(\kappa_M)$ may be obtained to be

$$\alpha(\kappa_M) = \frac{1}{2} [\alpha(\bar{\sigma}_{aM}) + \alpha(\bar{\sigma}_{sM})]. \quad (47)$$

Taking logarithmic derivatives of Eqs. (19) and (20),

$$\alpha(\kappa_M) = -\frac{1}{2} \frac{m+1/2}{T}. \quad (48)$$

Then Eq. (45) can be written by

$$\frac{1}{E-1} \frac{dE}{dT} = -\frac{m+1/2}{T}. \quad (49)$$

The variation of $F F_m$ due to temperature now can be written in the form:

$$\alpha(F F_m) = -(F F_m T T_0)^{-1} \left[\frac{(\kappa_F R)^2}{16} \left\{ 1 - \frac{(\kappa_F R)^2}{12} \right\} + \left(\frac{\bar{\Sigma}_{aF}}{\bar{\Sigma}_{aM}} \right) \left(\frac{V_F}{V_M} \right) (E-1) \{ m - T T_0 \alpha(\bar{\sigma}_{aF}) \} \right]. \quad (50)$$

Next the effects of physical temperature of moderator on the reactivity of f should be considered. As was shown in Eq. (24), the number density variation of moderator due to temperature is expressed by volume expansion coefficient, which can be computed from Eq. (29).

(c) Resonance Escape Probability

The temperature coefficient of p will be obtained by considering the temperature effects of neutrons absorbed in the medium. The following expression⁶⁾ will be used to compute p :

$$p = \exp \left[-\frac{N_F V_F I}{\xi_M \bar{\Sigma}_{sM} V_M} \right], \quad (51)$$

where I is the resonance integral. Throughout the numerical computations, the fuel and moderator volumes are assumed to be fixed and the slowing down power $\xi_M \bar{\Sigma}_{sM}$ is taken¹⁾ to be 0.177 cm^{-1} . Therefore the fuel temperature coefficient of p is mainly dependent on the factor I . Now the effect of

fuel temperature on ρ , sometimes known to be the prompt temperature coefficient, can be calculated by assuming that the temperature of the moderator remains constant while the temperature of the fuel changes. The resonance integral is expressed by the empirical formula⁹⁾

$$I(T) = I(300) [1 + \beta_I (\sqrt{T} - \sqrt{300})], \quad (52)$$

where T is the effective temperature of fuel in degrees Kelvin, and β_I is a function of the properties of the fuel. The value of resonance integral I at room temperature for cylindrical fuel rods of UO₂ is computed from the relation⁷⁾

$$I(300) = 3.0 + 39.6 / \sqrt{\rho_F R}, \quad (53)$$

where ρ_F and R are the density and the radius of fuel, respectively. The parameter β_I is given¹⁾ approximately by

$$\beta_I = A' + C' / \rho_F R, \quad (54)$$

in which A' and C' are 6.1×10^{-3} and 9.4×10^{-3} for UO₂ fuel, respectively.

Then the prompt temperature coefficient of ρ is obtained by differentiating $\ln \rho$, that is,

$$\alpha_{\text{prompt}}(\rho) = - \frac{N_F V_F}{\xi_M \sum_{sM} V_M} \frac{dI}{dT}. \quad (54)$$

Differential of $I(T)$ from Eq. (52) is:

$$\frac{dI}{dT} = \frac{I(300^\circ) \beta_I}{2\sqrt{T}}. \quad (55)$$

Then the factor $\alpha_{\text{prompt}}(\rho)$ can be expressed by

$$\alpha_{\text{prompt}}(\rho) = - \frac{\beta_I}{2\sqrt{T}} \left[\ln \frac{1}{\rho(300)} \right]. \quad (56)$$

Next consideration follows the effects of moderator temperature on ρ . Assuming that only the moderator temperature changes while the fuel temperature remains constant, the moderator temperature coefficient of ρ is expressed by

$$\begin{aligned} \alpha(\rho) &= - \frac{N_F V_F I}{\xi_M \sum_{sM} V_M} \frac{d}{dT} \left(\frac{1}{N_M} \right) \\ &= \alpha(N_M) \ln \left[\frac{1}{\rho(300)} \right] = -\beta_M \ln \left[\frac{1}{\rho(300)} \right]. \end{aligned} \quad (57)$$

(d) Thermal Non-Leakage Probability

According to the diffusion theory, thermal non-leakage probability P_T is given by

$$P_T = 1 / (1 + B_g^2 L_T^2). \quad (58)$$

The temperature coefficient of P_T can then be

$$\alpha(P_T) = - \frac{B_g^2 L_T^2}{1 + B_g^2 L_T^2} [\alpha(L_T^2) + \alpha(B_g^2)]. \quad (59)$$

Because the amount of moderator is assumed to be remained constant, the geometrical buckling does not change due to temperature. The value of $B_g^2 = 0.7618 \times 10^{-4} \text{ cm}^2$ was used⁹⁾ under the full power condition.

The definition of the diffusion area L_{T0}^2 without considering the intermediate medium such as air gap is

$$L_{T0}^2 = \bar{\lambda}_{tr} / 3 \bar{\Sigma}_a, \quad (60)$$

where the mean of the absorption cross section $\bar{\Sigma}_a$, and transport mean free path $\bar{\lambda}_{tr}$ in the cell are defined by weighting with volumes and flux,

$$\bar{\lambda}_{tr} = \frac{1}{\bar{\Sigma}_{tF}} \frac{V_F}{V_F + T_F F F_m} + \frac{1}{\bar{\Sigma}_{tM}} \frac{V_M F F_m}{V_F + V_M F F_m} \quad (61)$$

$$\text{and } \bar{\Sigma}_a = \frac{\bar{\Sigma}_F V_F + \bar{\Sigma}_{aM} V_M F F_m}{V_F + V_M F F_m}. \quad (62)$$

The diffusion area L_{T0}^2 obtained may be shortened because the gaps are assumed to be filled with moderator for computation of L_{T0}^2 . So that it should be corrected for the air gap by following relation⁷⁾:

$$L_T^2 = \left(1 + \frac{2V_i}{V_c} \right) L_{T0}^2, \quad (63)$$

where V_c and V_i are the volumes of the cell and intermediate medium except coolant region, respectively. The diffusion area is also written in the following form⁶⁾:

$$L_T^2 = (1-f) L_{TM}^2 \frac{(1 + V_F / V_M F F_m)^2}{1 + D_M V_F / D_F V_M F F_m}, \quad (64)$$

where L_{TM}^2 is the diffusion area of moderator. As the volume ratio of moderator to fuel in the considered cell is very large,

$$L_T^2 \sim (1-f)L_{TM}^2, \quad (65)$$

the derivative of which is

$$\alpha(L_T^2) = -\frac{f}{1-f}\alpha(f) + \alpha(L_{TM}^2). \quad (66)$$

The diffusion area of moderator is then obtained by

$$L_{TM}^2 = 1/(3N_M^2 \bar{\sigma}_{aM} \bar{\sigma}_{sM}). \quad (67)$$

Taking the logarithmic derivative of Eq. (67) yields

$$\alpha(L_{TM}^2) = -2\alpha(N_M) - \alpha(\bar{\sigma}_{aM}) - \alpha(\bar{\sigma}_{sM}). \quad (68)$$

The temperature coefficient of L_T^2 then may be calculated by

$$\alpha(L_T^2) = -\frac{f}{1-f}\alpha(f) + 2\beta_M + \frac{m+1/2}{T}, \quad (69)$$

where T is measured in degrees Kelvin.

Since the amount of moderator is kept to be constant, the factor $\alpha(P_T)$ can be written in the form:

$$\alpha(P_T) = -\left(2\beta_M + \frac{m+1/2}{T}\right)(B_g^2 L_T^2)^{-1} \quad (70)$$

(e) Fast Non-Leakage Probability

The factor of P_F is defined for a finite reactor by

$$P_F = 1/(1+B_g^2 \tau). \quad (71)$$

The temperature coefficient of P_F is

$$\alpha(P_F) = -\frac{B_g^2 \tau}{1+B_g^2 \tau} [\alpha(\tau) + \alpha(B_g^2)]. \quad (72)$$

The slowing down area at room temperature τ_0 is taken to be 106 cm^2 in the D_2O purity of 99.8% obtained experimentally by Wade⁷⁾. The temperature correction is introduced¹⁰⁾ as a change in density

$$\tau_M = \tau_{0M} \left[\frac{\rho_M(20^\circ\text{C})}{\rho_M} \right]^2. \quad (73)$$

The τ_M is the moderator slowing down area at temperature $T^\circ\text{K}$. Using the simple formula¹⁰⁾

$$\tau = \frac{1}{3\bar{\Sigma}_{tr}\bar{\xi}\bar{\Sigma}_s} \ln \frac{E_1}{E_2}, \quad (74)$$

where E_1 and E_2 are the energy limits of slowing down, leads finally to

$$\tau_c = \tau_M \frac{\bar{\xi}\bar{\Sigma}_{trM}(\bar{\xi}\bar{\Sigma}_s)_M}{\bar{\Sigma}_{trC}(\bar{\xi}\bar{\Sigma}_s)_C} \left[\frac{V_c}{V_M} \right]^2. \quad (75)$$

Here $\bar{\Sigma}_{tr}$ and $\bar{\xi}\bar{\Sigma}_s$ are conventionally the transport cross section and slowing down power, and the subscripts M and C means moderator and cell, respectively. Cross section data for $\bar{\Sigma}_{tr}$ was obtained from ANL-5800⁷⁾.

In view of Eq. (73), the temperature effect of reactivity on τ is mainly affected by physical temperature of moderator, *i. e.*,

$$\tau \sim \frac{1}{\rho^2} \sim \frac{1}{N^2}. \quad (76)$$

The factor $\alpha(\tau)$ is derived by the above relation:

$$\alpha(\tau) = 2\beta_M. \quad (77)$$

Then the temperature coefficient of P_F may be calculated from the form:

$$\alpha(P_F) = -2\beta_M B_g^2 \tau (1+B_g^2 \tau)^{-1}. \quad (78)$$

IV. Results and Discussions

Along with the fuel bundle of the Wolsung unit 1, shown in Fig. 1 is the configuration of the equivalent unit cell assumed for the single rod approximation in order to make use of LATREP code. It is, however, assumed that the intermediate media, such as air gap, are filled with heavy water for the numerical computations by using the analytical forms.

In order to make comparisons, four factors of neutron multiplication are computed through utilizing the lattice cell code LATREP which is a multigroup, collision probability, fuel burn-up code using 32 epithermal and one thermal neutron groups. The thermal group, up to 0.625 eV, uses a modified Westcott formulism. Table 1 shows the results from the code for the cases of the single rod and the 37-rod fuel bundle at two

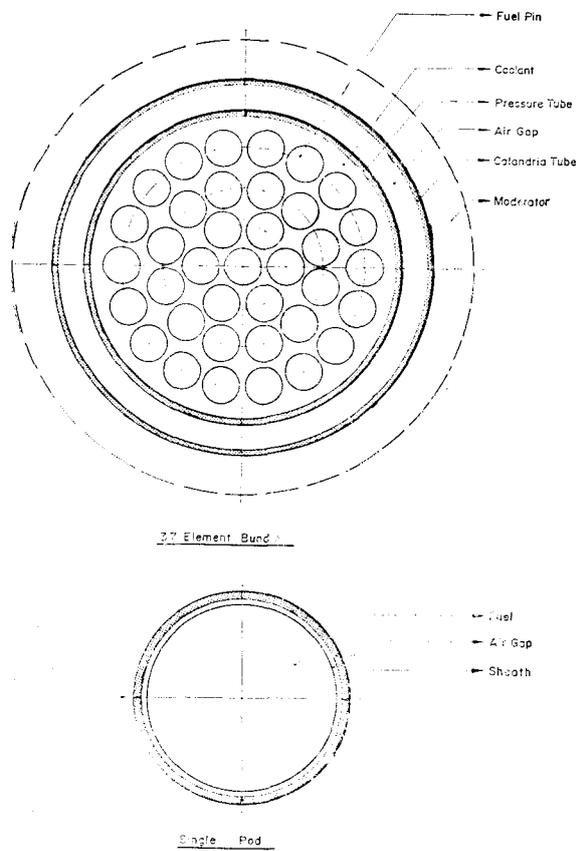


Fig. 1. Cell Configuration

different conditions of boron concentration in moderator. Also given are the numerical values of PSR of the Wolsung unit.

Using the code for the fuel bundle, calculations are made by the rubber band approximation in which respective ring region is uniformly distributed by the composite materials.

The reactivity effects of moderator, coolant, and fuel temperatures are computed and presented through Fig. 2 to 4 for the single rod approximation and the fuel bundle.

One can see from the figures that there are some discrepancies between the two approximations. However, the differences, which are negligibly small except for the coolant temperature effect, are not really concerned in this calculations but the behaviour of the respective temperature effect to the reactivity. Among the results, the reactivity effect of fuel temperature can only be available from the PSR of the Wolsung unit for a comparison with the results. It should be pointed out from results of com-

Table 1. Comparison of Four Factor Parameters at HFP with Equilibrium Poisons

Moderator Condition	Case	η	ϵ	p	f	k_{∞}
Fresh Fuel, No Boron in Moderator	PSR	1.241	1.026	0.9047	0.9358	1.078
	Bundle	1.2466	1.0308	0.8985	0.9343	1.0786
	Single Rod	1.2455	1.0433	0.9296	0.9223	1.1140
Fresh Fuel, 3.4 ppm Boron in Moderator	PSR	1.241	1.026	0.9047	0.9085	1.047
	Bundle	1.2466	1.0308	0.8985	0.9059	1.0459
	Single Rod	1.2455	1.0433	0.9296	0.8910	1.0762

*HFP Condition: Fuel Temperature=936°C
 Moderator // =68°C
 Coolant // =290°C
 Sheath // =339°C

*Material Properties: UO₂ density=10.283 (g/cm³)
 D₂O purity (Coolant and Moderator)=99.722 atom %
 Zr-4 density=6.55 (g/cm³)

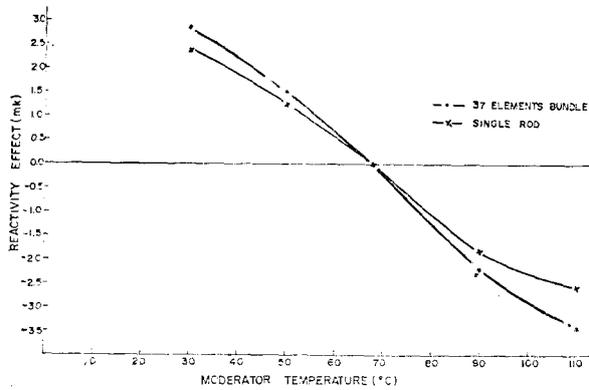


Fig. 2. Reactivity Effect of Moderator Temperature

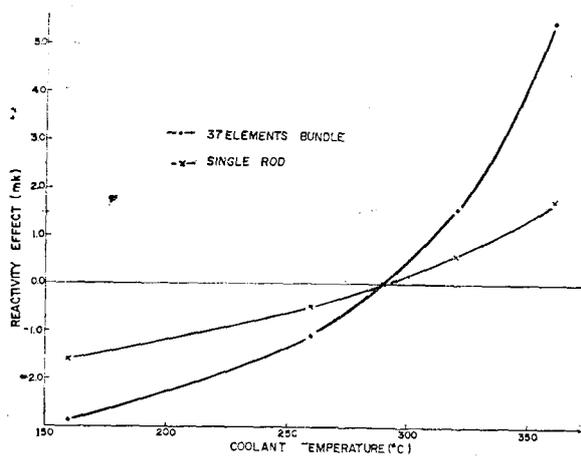


Fig. 3. Reactivity Effect of Coolant Temperature

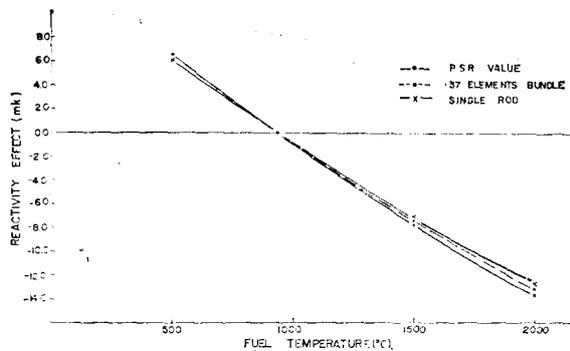


Fig. 4. Reactivity Effect of Fuel Temperature

putations that the heavy water coolant temperature coefficient is positive in the heavy water moderated reactor. Each parameter change of the effective multiplication factor is computed with respect to coolant temperature variations in the unit cell with the 37-rod fuel bundle, and the results are provided in Table 2.

Table 2. Coolant Temperature Coefficients

Factors	Temp.	(37 element bundle) ($\times 10^{-6}/^{\circ}\text{C}$)			
		160°	260°	290°	320°
$\alpha(\eta)$		-12.4	-8.5	-6.2	-2.6
$\alpha(\rho)$		16.4	27.2	33.4	44.0
$\alpha(f)$		0.4	4.0	5.9	9.2
$\alpha(\epsilon)$		6.3	10.5	12.8	17.0
$\alpha(P_T)$		-1.8	-1.2	-0.82	-0.23
$\alpha(P_F)$		-0.91	-1.5	-1.9	-2.6
$\alpha(k_{eff})$		8.1	30.5	43.3	65.0

Of the contributions to temperature coefficient, it has been taken to be equal to the prompt temperature coefficient for the fuel coefficient since this varies significantly along with the effects of Doppler broadening of ^{238}U resonance. And it was found⁽¹⁾ that the contribution of the changes of fast fission factor due to temperature variations was very small compared to other factors and hence negligible in the lattice. Thus calculations are only made of other parameters except ϵ .

The factor η is determined by the energy spectrum of thermal neutron, and neutrons in a unit cell are primarily thermalized in the moderator. The gradients of absorption rate in the fuel is larger than those of fission rate with respect to increasing temperature. One can thus expect to get negative temperature coefficient of η . And the results shows that the magnitude with ne-

gative sign represents the most important contribution to the overall temperature coefficient.

Variations of parameter f depending on temperatures mainly come from the depression of the thermal flux distribution across the cell. And the density changes of moderator due to its thermal expansion will contribute to the positive temperature coefficient of reactivity. There are some discrepancies between analytical formulas and LATREP in calculating f because the flux depression in the vicinity of fuel and moderator boundary is found to be small according to diffusion theory. Thus it follows from Eq. (38) that the value of $\alpha(f)$ may be underestimated. Since the two terms in the bracket of Eq. (38) are quantities with negative signs, it is evident that the temperature coefficient is positive but small com-

pared with other parameters.

As fuel temperature increases, which always gives rise to broadening of Doppler effect, the absorption rate is found to be largely increased in resonance region. This effect leads to the negative temperature coefficient of fuel. The atom density of heavy water is decreasing as a function of moderator temperatures. This results that the concentration of fuel in the cell becomes large compared to the moderator concentration. As was noted that values of p decrease with increasing fuel concentration, the changes of moderator temperatures affect to negative temperature coefficient of p .

Changes in the neutron leakage due to temperature are determined by the density of heavy water and the cross sections. The calculated results show that temperature effect to reactivity of neutron leakage is

Table 3. Fuel Temperature Coefficients ($\times 10^{-6}/^{\circ}\text{C}$)

	Temp $^{\circ}\text{C}$	500 $^{\circ}$	936 $^{\circ}$	1500 $^{\circ}$	2000 $^{\circ}$
$(\alpha)\eta$	Single Rod	-5.1	-5.3	-4.9	-4.6
	Bundle	-3.6	-3.7	-3.5	-3.3
$\alpha(p)$	Single Rod	-5.8	-4.5	-3.8	-3.4
	Bundle	-10.2	-8.1	-6.7	-5.8
	Calculation	-7.7	-6.2	-5.1	-4.5
$\alpha(f)$	Single Rod	-1.7	-1.7	-1.5	-1.4
	Bundle	-1.0	-1.0	-1.0	-0.9
$\alpha(\epsilon)$	Single Rod	0	0	0	0
	Bundle	0	0	0	0
$\alpha(P_T)$	Single Rod	-0.81	-0.85	-0.82	-0.79
	Bundle	-0.60	-0.63	-0.61	-0.60
$\alpha(P_F)$	Single Rod	0	0	0	0
	Bundle	0	0	0	0
$\alpha(k_{eff})$	Single Rod	-13.4	-12.5	-11.2	-10.2
	Bundle	-15.3	-13.4	-11.7	-10.7
	Calculation	-7.7	-6.2	-5.1	-4.5

Table 4. Moderator Temperature Coefficient ($\times 10^{-6}/^{\circ}\text{C}$)

Factors	Temp $^{\circ}\text{C}$	30 $^{\circ}$	50 $^{\circ}$	68 $^{\circ}$	90 $^{\circ}$	110 $^{\circ}$
$\alpha(\eta)$	Calculation	-81.8	-83.4	-84.9	-86.6	-88.1
	LATREP(A)	-72.3	-70.7	-69.0	-66.9	-64.9
	LATREP(B)	-57.7	-56.9	-56.0	-54.9	-53.7
$\alpha(\rho)$	Calculation	-16.5	-28.9	-37.9	-47.2	-54.9
	LATREP(A)	-13.6	-21.6	-27.9	-35.0	-42.7
	LATREP(B)	-21.8	-35.0	-45.0	-56.5	-68.3
$\alpha(f)$	Calculation	34.3	36.2	37.3	38.2	39.0
	LATREP(A)	68.1	66.3	64.2	61.7	58.2
	LATREP(B)	48.0	47.5	46.6	45.2	42.8
$\alpha(\epsilon)$	Calculation	—	—	—	—	—
	LATREP(A)	0.3	0.5	0.5	0.7	0.9
	LATREP(B)	0.5	0.7	0.8	1.0	1.2
$\alpha(P_T)$	Calculation	-25.4	-28.9	-31.6	-34.4	-36.8
	LATREP(A)	-25.3	-28.1	-30.3	-32.8	-35.3
	LATREP(B)	-21.7	-24.2	-26.2	-28.4	-31.3
$\alpha(P_F)$	Calculation	-4.9	-8.7	-11.5	-14.8	-17.7
	LATREP(A)	-5.4	-8.6	-11.1	-13.19	-16.6
	LATREP(B)	-5.4	-8.7	-11.3	-14.2	-16.8
$\alpha(k_{eff})$	Calculation	-94.4	-113.8	-128.6	-144.7	-158.5
	LATREP(A)	-48.1	-62.3	-73.5	-86.2	-100.2
	LATREP(B)	-58.1	-76.5	-90.9	-107.5	-125.3

(A): Single Rod

(B): 37 Elements Bundle

small for a large power reactor.

The temperature coefficients of fuel and moderator are given in Table 3 and 4, respectively.

As was noted before, Table 3 includes only the computations of the prompt temperature coefficient which lie in between the two approximations and show that a good prediction can be made by using the analytical formulas. However, comparing the results of calculations using the formulas to those of LATREP, observed in Table 4 are some discrepancies in the calculations of the parameters η , ρ , and f .

Although the numerical values among the computations differ in an individual moderator temperature, the variations associated with the moderator temperature coefficients are appeared to be along the same signs of quantities.

Although calculations have been carried out for a unit cell, it can be concluded that the overall temperature coefficient is negative at operating range of Wolsung Unit 1.

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