DEFECT DETECTION WITHIN A PIPE USING ULTRASOUND EXCITED THERMOGRAPHY

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An UET (ultrasound excited thermography) has been used for several years for a remote non-destructive testing in the automotive and aircraft industry. It provides a thermo sonic image for a defect detection. A thermograhy is based on a propagation and a reflection of a thermal wave, which is launched from the surface into the inspected sample by an absorption of a modulated radiation. For an energy deposition to a sample, the UET uses an ultrasound excited vibration energy as an internal heat source. In this paper the applicability of the UET for a realtime defect detection is described. Measurements were performed on two kinds of pipes made from a copper and a CFRP material. In the interior of the CFRP pipe (70mm diameter), a groove (width - 6mm, depth - 2.7mm, and length - 70mm) was engraved by a milling. In the case of the copper pipe, a defect was made with a groove (width -2mm, depth - 1mm, and length - 110 mm) by the same method. An ultrasonic vibration energy of a pulsed type is injected into the exterior side of the pipe. A hot spot, which is a small area around the defect was considerably heated up when compared to the other intact areas, was observed. A test on a damaged copper pipe produced a thermo sonic image, which was an excellent image contrast when compared to a CFRP pipe. Test on a CFRP pipe with a subsurface defect revealed a thermo sonic image at the groove position which was a relatively weak contrast.

KEYWORDS : Non Destructive Test, Ultrasound Excited Thermography, Pipe, Defect Detection

1. INTRODUCTION

Thermal wave infrared imaging for detecting defects in a material has been carried out with high-power ultrasonic sources. An early example of this was the work by Mignogna et al, who utilized a high-power ultrasonic heating of a material, coupled with an imaging by means of an infrared (IR) camera [1]. The time scale for their IR images was several seconds after the application of an ultrasonic pulse, and they reported seeing a heating in the vicinity of both saw-cuts and fatigue cracks. More recently, Busse and his colleagues have reported the use of time-varying ultrasonic sources, together with a frequency domain IR imaging to observe a variety of defects, including cracks, in materials [2]. The time scale for those measurements was also in the order of seconds to minutes. The approach to this technique, by the research team of Wayne State University, was to work in the time domain, by utilizing a single short pulse of 15-40 kHz sound from an ultrasonic welder as the excitation source, and a focal plane array IR imager to monitor the surface temperature in the vicinity of a defect before, during and after the application of the pulse. The resulting images are astonishingly revealing for defect structures, especially cracks [3, 4].

Ultrasound excited thermography (UET) is a nondestructive testing technique that uses a low-frequency ultrasonic excitation and combines an infrared detection to image surface and subsurface cracks or defects [5, 6]. This UET is based on the heating generated at cracks or disbonds in a material due to the applied sound wave into it. When an ultrasound propagates through a solid body that contains a defect, the two faces of the defect do not ordinarily vibrate in unison, and dissipative phenomena such as a friction motion between the faces will convert some of the vibrational energy to heat. The heat is predominantly generated from this friction motion of the mating surfaces when they rub against each other during the application of an ultrasonic pulse. This local increase in temperature due to the friction heat is imaged by an IR camera, which sees it as a bright IR source (hot spot) within a dark background field. Since a heating occurs as soon as the ultrasound reaches a defect, the bright source first appears in a time scale in the order of a few ms after the ultrasound is turned on. An IR camera captures the images of the surface in the damage locations at a high speed frame rate. The IR camera observes the heat progression at the crack surfaces during the application of the ultrasound pulse. This method of an ultrasound excited thermography allows for a selective defect detection which enhances the probability of a defect detection in the presence of complicated intact structures. This technique has been used in detecting fatigue damage both in isotropic and composite materials [7]-[12].

In this paper the applicability of an UET for a real-time defect detection is described. In this technique, a short ultrasonic pulse (280 ms) is applied to a sample with small artifact defects. Then a local temperature increase on the sample surface is produced because of the extra acoustic damping in the defective or inhomogeneous areas. An IR camera is used to observe and record the temperature change. From a sequence of the thermosonic images, the defective or inhomogeneous regions can be detected easily. Measurements were performed on two kinds of pipes: a copper and a CFRP pipe.

2. PRINCIPLE OF AN ULTRASOUND EXCITED THERMOGRAPHY

When an ultrasound wave passes through a material with a defect, that is a crack, disbond, delamination or other adhesion weakness, in it, the two mating faces of a defect do not move in harmony. The resulting rubbing or clapping of the defect surfaces causes them to heat up. The simplest view of the physical principle underlying this UET is: a short ultrasonic pulse vibrates a target material with very small fatigue defects, which causes a rubbing or clapping between the surfaces of any defects which may be present, and this rubbing or clapping cause a friction, which results in a temperature rise at the defects. An IR camera is used to observe and record the temperature change (see Fig. 1) in this sample. From a sequence of the thermo sonic images, the defective or inhomogeneous regions could be detected easily. An ultrasound wave at frequencies of 20 kHz in solid materials has wavelengths in the order of tens of centimeters, and propagates with an appreciable amplitude over distances much longer than a normal wavelength. Unless the material is very loss, an ultrasound wave can propagate over many wavelengths without an appreciable damping. Since the speed of ultrasound in solids is typically in the order of a few kilometers per second, this ultrasound field fully permeates the regions of the test specimen under inspection during the time that the excitation pulse is applied. In most solids, the propagation of an injected ultrasonic energy to a defect site can be taken to be nearly instantaneous, based on the distance that sound travels during the period of a single IR camera frame (e.g. the speed of sound in steel is approximately 6mm/us, so the distance traveled in a single image frame period is nearly 100 meters for a camera at a 60Hz frame

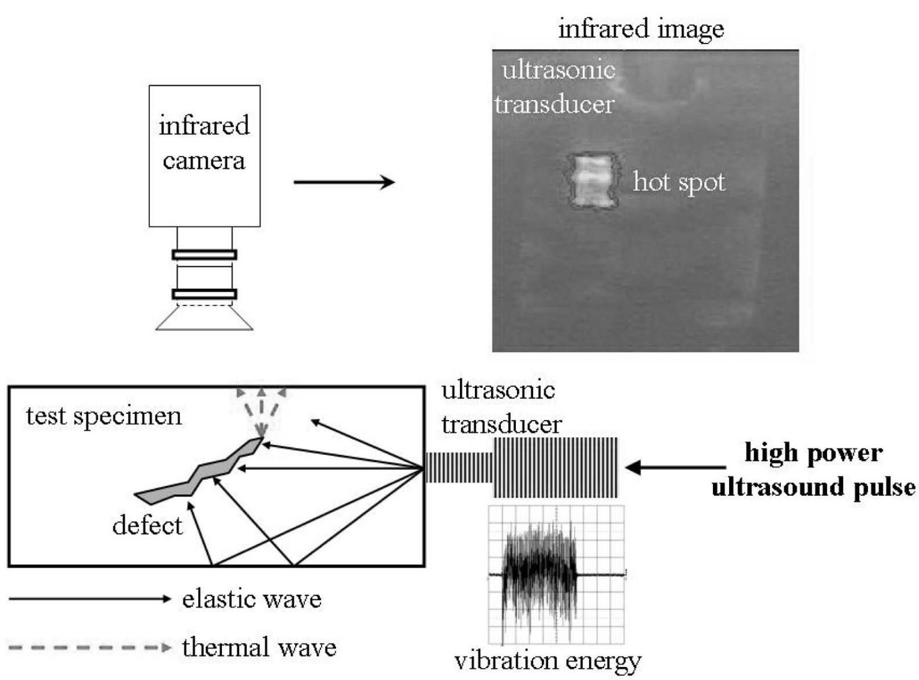


Fig. 1 Principle of an Ultrasound Excited Thermography: Ultrasound Excitation Pulse Generates Thermal Waves in the Defect Itsellf, and They are Sensed at the Surface by an Infrared Camera.

rate). On the other hand, an acoustic damping in solids is proportional to the square of the frequency of the acoustic wave [13]. Therefore it is obvious that a higher frequency requires a lesser amplitude of the ultrasonic excitation. This is the reason why we use acoustic waves in the ultrasonic frequency range (20 ~ 40 kHz) to generate heat.

3. EXPERIMENT

In this section, the UET technique used to detect a defect of pipes, made of a copper and a CFRP (carbon fiber reinforced plastic) material, is described. The UET experimental setup includes an ultrasonic transducer, an infrared camera, a high power (20 kHz / 2 kW) ultrasound generator, and an image recording system. The diagram of the experimental configuration is shown in Figure 2. A low frequency (20 kHz) ultrasonic transducer was used to infuse the pipe with a short pulse of ultrasound for 280 ms. The ultrasonic source has a maximum power of 2 kW. The acoustical energy provided by the source was, in most experiments, 1.5 kW. The outer surface temperature of the pipe was imaged by an IR camera. The IR camera is an Mitsuibishi IR-M500 camera (3~5 micron, NETD 0.15°C), which acquires images at a 60Hz field rate with a 512x512 PtSi cell array. For an IR camera, which detects a thermal radiation from objects, an important measure of its performance is its ability to detect small changes in temperature. The smallest temperature difference an IR camera can detect is called the thermal resolution. Changes which are too small to be distinguished from the background noise in the system will not be detected. Sometimes, a thermal resolution is described by the NETD, which stands for a noise-equivalent-temperature-difference. It specifies the minimum detectable temperature difference. The IR detector head is based on a Stirling-cooled PtSi mid-wave focal plane array having an image area of 0.52x0.40 inches. And it is equipped with a high brilliance IR objective. A 50mm/f1.2 lens was used for this experiment. The metal tip of the ultrasonic transducer made of SUS (stainless steel, grade 304) material is mechanically held against the specimen being tested to maintain a good mechanical contact. A photograph of the experimental setup is shown in Figure 3. A bakelite plate was used as the test stand. The test stand has a V-groove to keep the specimen from a secession when a vibration energy is injected into the pipe.

The image recording system is based on a PC. The system acquires the 60 Hz images. The acquired image stream was saved in a frame-by-frame manner. A TV monitor, which is not shown in the experimental setup diagram, was attached to an infrared camera to provide a visual reference of the camera's field-of-view. Some effort was required to tune the ultrasonic transducer and

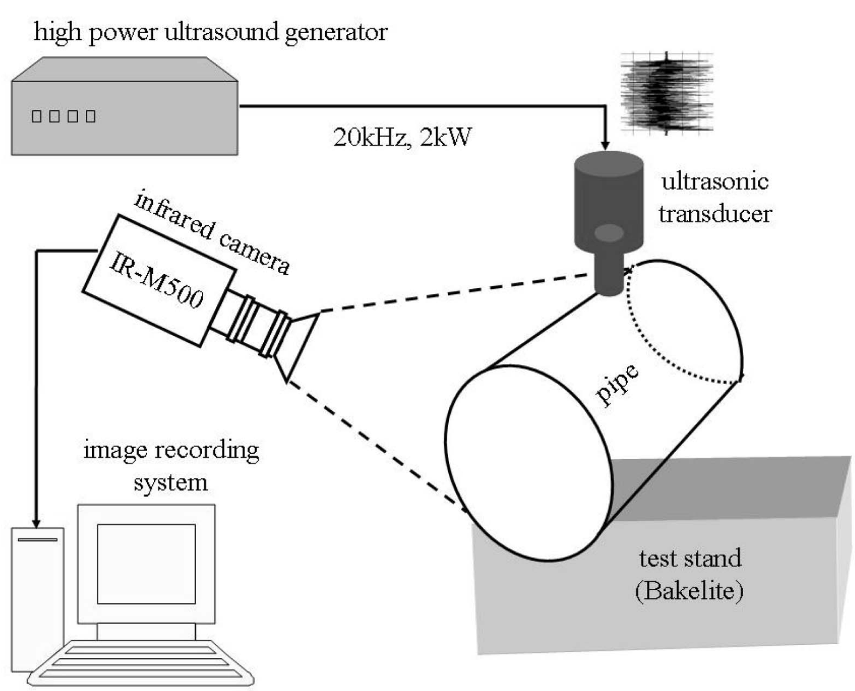


Fig. 2 Experimental Configuration of an Ultrasound Excited Thermography

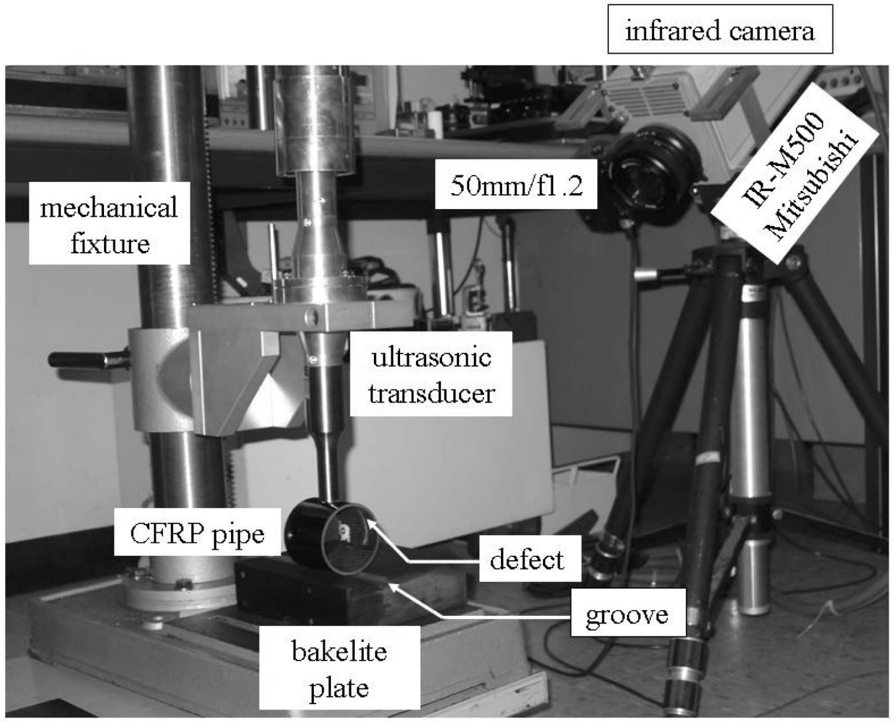


Fig. 3. A photograph of the Experimental Setup of the Ultrasound Excited Thermography

the specimen. The vibration energy injected into the ultrasonic transducer and the contact force between the ultrasonic transducer and the specimen are the most critical parameters to be tuned. It is well known among those who use an ultrasound excited thermography that the amount of the contact force between the ultrasonic transducer and the test specimen is critical to obtain a screech (loudest chirp) and hence a high quality infrared image. This loudest chirp sound is a good indicator of a mechanical coupling between the specimen and the ultrasonic transducer. If the contact force is too little, the system is quiet, and very little ultrasound is coupled into the sample. If the contact force is too great, the result is the same. The exact amount of force necessary to produce a screech depends on the particular ultrasonic transducer used to inject the ultrasound, presumably because different transducers have different vibration amplitudes. These parameters were empirically set to produce the loudest chirp. These parameter tunings were a subjective process, and the optimal settings were found by an empirical rule. Fastening specimens required some deliberation to avoid a slippage or a breakage. The specimens tended to move both from the steady mechanical force holding the ultrasonic transducer to the specimen, and from the dynamic load of the ultrasonic excitation. Both of these forces needed to be tuned to avoid a specimen slippage during a test. The impact of the ultrasonic transducer as it contacts with the specimen may cause a motion. The dynamic excitation as the high voltage was applied to the ultrasonic transducer caused the specimens to squirm. Therefore, several configurations were attempted in order to minimize the slippage of a specimen.

3.1 CFRP Pipe

The defect dimensions of the CFRP pipe are shown in Figure 4. As shown in Figure 4, a defect was etched into the inside surface of the CFRP pipe that was 6 x 2.7 x 70 mm by using a milling. An ultrasonic vibration energy was applied to the CFRP pipe with a nominal 1.5kW ultrasonic signal at 20kHz over a 280 ms duration.

A vibration wave caused by the ultrasonic excitation meets with the outside surface of the CFRP pipe specimen. The contact surface between the ultrasonic transducer and the exterior face of the pipe specimen creates a line as shown in Figure 5, because the cross-section of an ultrasonic transducer is a flat surface (diameter 20mm) as indicated in Figure 5. And the contact line covers the whole diameter of the transducer.

As shown in Figure 5, when subject to ultrasonic excitations, the CFRP pipe specimen was observed to generate a localized heating (hot spot) at the defect. Figure 6 shows a selection from a sequence of the thermo sonic images of this CFRP pipe which were acquired prior to, during, and immediately after an application of the ultrasonic excitation pulse. The left frame shown in Fig.

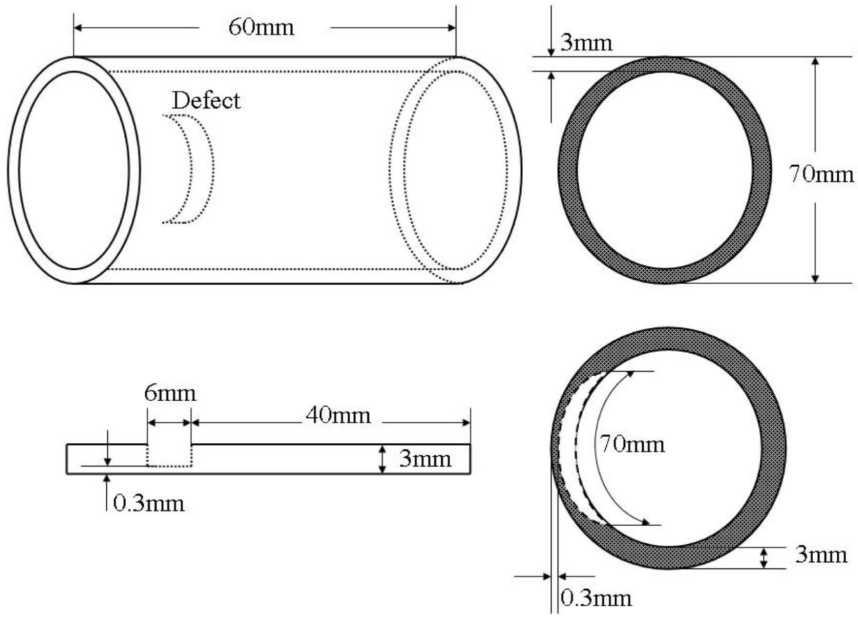


Fig. 4. Defect Dimensions of the CFRP Pipe

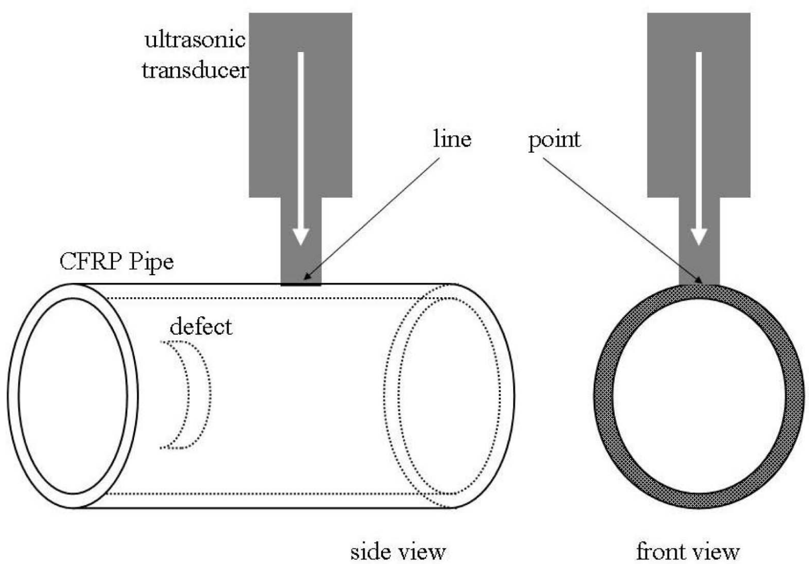
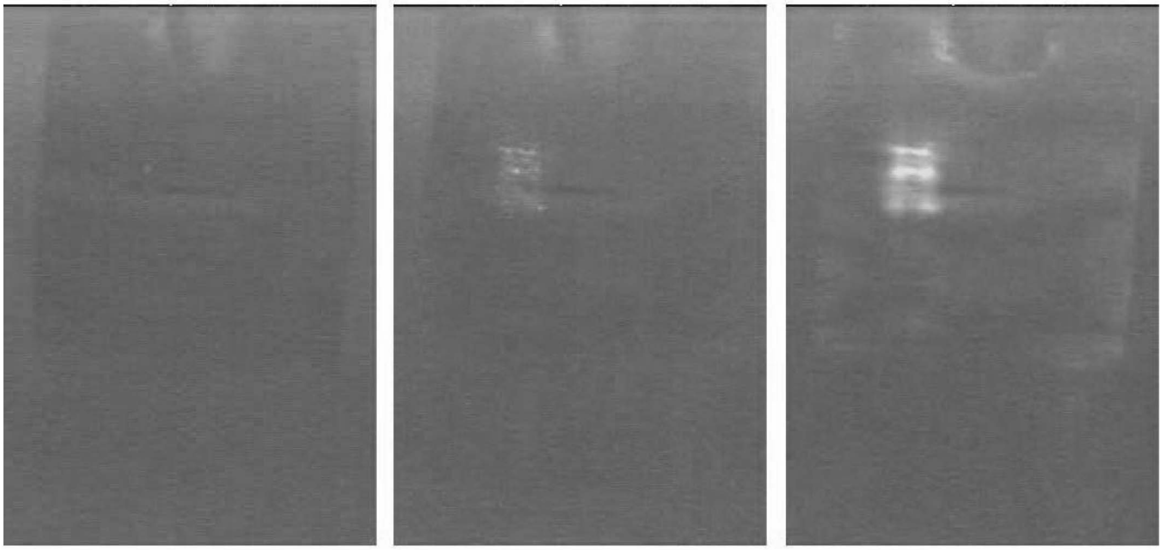


Fig. 5. Schematic Diagram of Measurement I

6 is from just before the ultrasound pulse was turned on, and the center one in Fig. 6 is from the 2nd frame (~ 60ms) after it was turned on, and the right one in Fig. 6 is from the 9th frame (~270 ms) after it was turned on. The hot spots as shown in Figs. 6b and 6c indicate a defect of the inside surface of the CFRP pipe. It is evident from this sequence of thermo sonic images that such a defect is easily detected by the UET technique. A heat is also visible at the ultrasonic transducer contact as shown in the upper side of Figure 6, which was common with all the tests.

Figure 7 shows a schematic diagram of another



(a) before an ultrasonic excitation pulse (b) 2^{nd} frame after the pulse (c) 9^{th} frame after the pulse

Fig. 6 Thermal Response of the Defect Area in the CFRP Pipe to an Ultrasound Excited Vibration

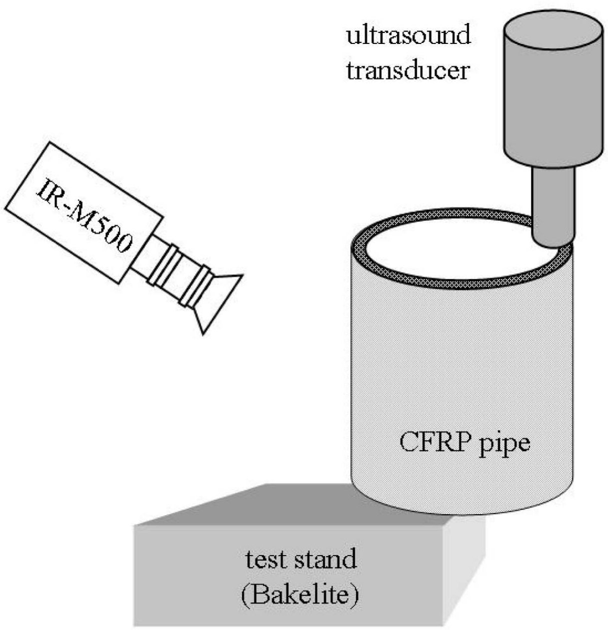
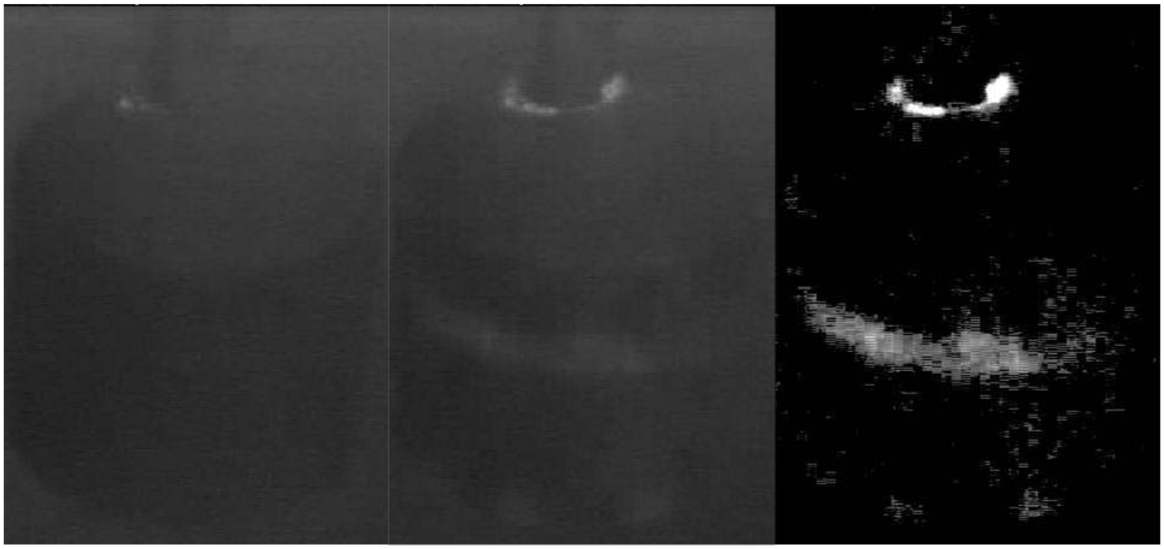


Fig. 7. Schematic Diagram of Measurement II

measurement. In Figure 7, the vibration wave caused by the ultrasonic excitation meets with the circumference surface of the CFRP pipe specimen. A ultrasonic vibration energy was applied to the circumference surface with a nominal 1.5kW ultrasonic signal at 20kHz over a 280 ms duration. The thermal response of the defect area which lies at the inside of the CFRP pipe is shown in Figure 8. As compared to Figures 5 and 6 (measurement I), the thermal response of measurement II is weak. From the hot spot image as shown in Fig. 8b, we know that an elaborate image processing algorithm is necessary for the extraction of a defect area from this weak thermosonic image. When the hot spot is faint and the gray level of it is hidden by a background noise, we are not able to segment the hot spot area from the background noise by using the proper threshold method. After careful consideration, regarding various aspects of this thermosonic image, an image processing algorithm such as background subtraction was chosen to eliminate the background noise. From a sequence of the thermosonic images of the CFRP pipe which were acquired prior to, during and after the expiration of an application of the ultrasonic excitation pulse, we found that the intensity and the position of the background noise level overwhelming the gray level of the hot spot was almost invariable if the ultrasonic transducer had not slipped from the contact surface of the test specimen during the injection of an ultrasonic excitation pulse. For the background noise, a background subtraction was found to be a good way to make the thermosonic images clearer. By subtracting a background noise image taken just prior to an ultrasonic excitation, a change in the temperature as a result of the ultrasonic excitation is observed in the thermosonic signal. Background subtraction is a basic arithmetic digital image operation. This operation



(a) before an ultrasonic excitation pulse　(b) 9th frame after the pulse　(c) difference frame after background subtraction

Fig. 8. Thermal Response of the Defect Area to an Ultrasound Excited Vibration (Before, Injection, and Difference).

is performed on a pixel-by-pixel basis between two or more images. The two images under a background subtraction operation must be aligned to avoid the introduction of a blurring and other artifacts in the resultant difference image. It means that the ultrasonic transducer must not slip from the contact surface of the test specimen when an ultrasonic excitation pulse is applied to the sample. From the result of the background subtraction method, it is established that the defect area of the CFRP pipe is clearly shown in the right image of Figure 8.

From the experimental results of measurement I and measurement II, it is estimated that the sensitivity of the thermal response of a defect area is not related to the contact cross-section between the ultrasonic transducer and the test specimen. The contact area of measurement I (Figure 5) is thought of as a line (one dimension), on the other hand, the contact area of the measurement II is a two dimensional area. Therefore, the contact area of measurement I is much less than that of measurement II. But the thermal response of measurement I is much more sensitive than that of measurement II. From this fact, it is assumed that the factor determining the thermal responsivity of the defect area when exposed to the ultrasound excited vibration energy is not the size of the contact area between an ultrasonic transducer and a test specimen but the mechanical condition bringing about a considerable vibration.

3.2 Copper Pipe

The measurement setup of the copper pipe (diameter-55mm, length-228mm) is shown in Figure 9. A defect was made with a groove (width -2mm, depth - 1mm, and length - 110 mm) by a milling as shown in Figure 9.

The copper pipes were measured in the same modes as the CFRP pipe measurements as shown in Figures 5 and 7. The ultrasonic transducer tip should be parallel to the surface of the test specimen, and held in place with a sufficient mechanical pressure to insure a constant contact throughout the insonification process. And the ultrasonic transducer tip should be a hard metal alloy, e.g. SUS material. The action of the transducer tip is essentially a hammering motion, i.e. the tip lifts off of the test specimen surface and then makes contact with each ultrasonic excitation period. The effect of tens of thousands of such impacts during a brief period (ultrasonic excitation time) is often some scuffing of the specimen surface. In order to efficiently transfer a mechanical vibration energy from the excitation device (ultrasonic transducer) to the surface of the test specimen (copper pipe) and in order to protect the test specimen surface, an aluminum adhesive tape as a suitable couplant was used. Figure 10 shows the heating patterns (hot spots), generated by the ultrasonic excitation (measurement I), of the copper pipe. As the ultrasonic vibration energy was injected into the copper pipe, multi-heating patterns were observed along the defect line engraved on the outer surface of the pipe specimen. In the test using the copper pipe specimen, the highest brightness level of the hot spot at the defect was observed at the 4^{th}~7^{th} (120~210ms) image frames. The bright diagonal line of Fig. 10a results from the fact that an ambient heat

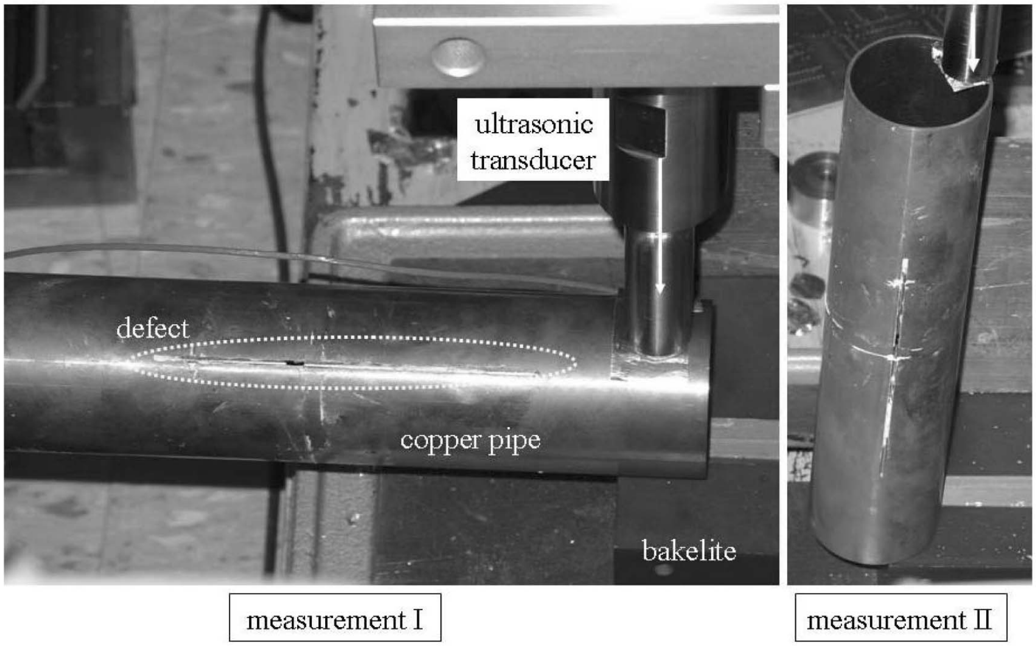


Fig. 9. A Photograph of the Measurement Setup of the Copper Pipes

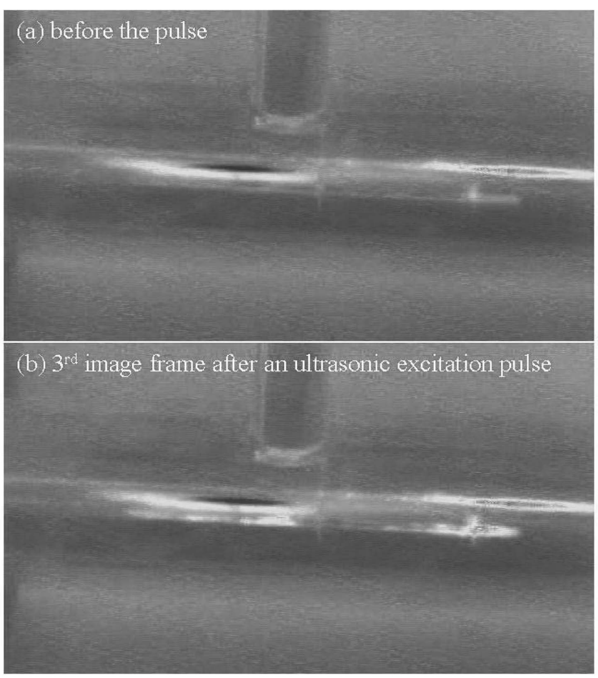


Fig. 10. Thermosonic Images of the Copper Pipe

of the atmosphere in the room is reflected on the outer surface of the copper pipe and it is imaged by an infrared camera as the reflectivity coefficient of the surface of the copper pipe is high. Generally the reflectivity of a metal (Cu) is influenced by the polishing status of the surface of this metal. The polishing status of the surface of a copper pipe is impaired by an oxidation, staining, and roughness. The reflectivity of the outer surface of a copper pipe ranges from 0.98 to 0.4 with the polishing status. In order to achieve a low and homogenous reflectivity, the surface of the test specimen is generally sprayed with black paint for the infrared thermography. It is a shortcoming of an infrared thermography and is an obstacle to a wide field application in the industry. From the hot spot noise images as shown in Fig. 10, we know that an elaborate image processing algorithm is necessary for the segmentation of a defect area from these high background noise images. In a typical image from an infrared camera (Fig. 11, 1st difference image) in which the pre-excitation image has been subtracted, several weak indications of an anomalous heating appear as a result of friction at a defect. After the elapse time of the 7th image frame (Fig. 11, 7th difference image), it is known that several hot spot patterns generated by the ultrasonic excitation apparently bloomed along the defect line. And after the 11th image frame, the hot patterns gradually disappeared from the defect.

Another thermal response of the copper pipe was measured. As shown in the schematic diagram of Figure 7 (measurement II), the vibration wave caused by the ultrasonic excitation meets with the circumference surface of the copper pipe specimen. The thermal response of the

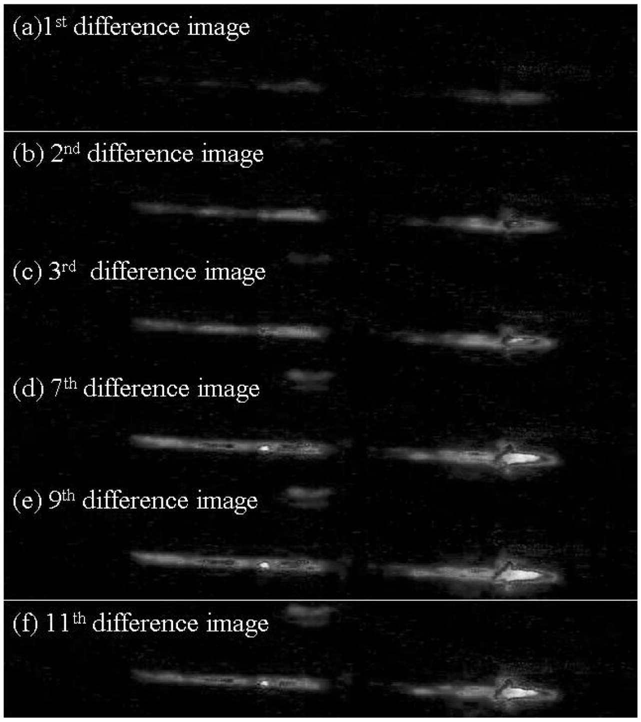
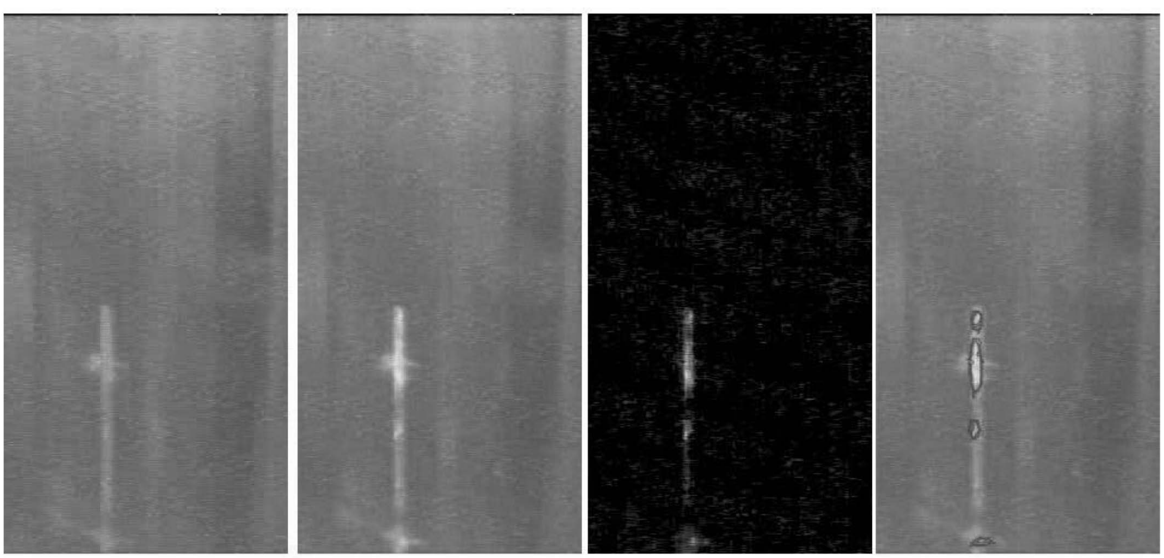


Fig. 11. Thermosonic image frames sequence of a defect in the copper pipe specimen

defect line which was engraved on the outside of the copper pipe is shown in Figure 12. Fig. 12 (a) reveals the status of the outer surface of a copper pipe before the application of an ultrasonic excitation pulse. The bright vertical line as displayed in an approximate mid-center position of Fig.12 (a) represents a defect. The reason that the brightness level of a defect line is greater than that of the surrounding surface of the copper pipe results from the fact that an ambient heat of the atmosphere in the room is reflected more on the defective surface of the copper pipe. Because the engraved surface of a defect is more polished than the surrounding surface of the copper pipe, the reflectivity of the etched surface of a defect is higher than that of the other oxidized or stained surfaces of this metal. Figure 12 (b) reveals the status of the copper pipe after the application of an ultrasonic excitation pulse. When compared to Figure 12 (a), the brightness level in the defect position of the copper pipe is slightly more brilliant than that of the left image. When compared to Figure 10 (measurement I), the thermal response of measurement II is relatively weak. In order to discriminate between the defect line and the intact area, an image processing technology such as the background subtraction method was used, as with the CFRP pipe. From the results of the background subtraction method, it is observed that the defect line of the copper pipe is clearly shown in Figure 12 (c).

Also the contour of the brightest white spot region was extracted from the difference image as shown in Fig. 12 (c). A contour description of an image has been shown to assist in many fields of an image processing; for example feature recognition, segmentation or manipulation. The contour extracted was superimposed onto the 6th image frame shown in Fig. 12(b). The red ellipses shown in Figure 12(d) show the result of a superimposition. From



(a) before (b) 6th frame after the pulse (c) difference (d) superimposition

Fig. 12. Thermal Response of the Defect Area to an Ultrasound Excitation Pulse (Before, After, Difference and Superimposition)

Fig. 12 (d), we can clearly see the location of the defects of the copper pipe. As with the experimental results of the CFRP pipe, it is also estimated that the sensitivity of the thermal response of a defect line is not related to the contact cross-section between the ultrasonic transducer and the copper pipe. From the results of the copper pipe, it is established that the ultrasound excited thermography is an efficient technique for detecting any type of surface-breaking crack.

4. CONCLUSION

Ultrasound excited thermography based inspection methods have the potential to facilitate in an extremely sensitive and fast detection of small, tightly-closed defects that are undetectable by using other thermographic methods. As the vibration energy generated at the ultrasonic transducer is transferred to a test sample, it is converted into a thermal energy by friction at a defect. The advantage of a selective heating, generated at a defect, is that the vibration energy generated by an ultrasonic excitation is used in a very efficient way since it is not wasted for a heating of the intact areas.

In this paper the applicability of the UET for a realtime defect detection of a pipe is described. A 280ms ultrasonic pulse from a 2 kilo joule transducer operating at a 75% amplitude was applied to the outer surface of a CFRP pipe and a copper pipe respectively. Test results indicated that the thermal signal response of a defect in the pipes is not related to the size of the contact cross-section between the ultrasonic transducer and the test specimen. In the case where the ultrasonic transducer tip crosses perpendicular to the outer surface of the pipe diameter, the contact area is thought of as a line (measurement I). In the other case where the ultrasonic transducer tip meets with the circumference surface of the pipe, the contact area is a two dimensional area (measurement II). Therefore, the contact area of measurement I is much less than that of measurement II. But the thermal response of measurement I is much more sensitive than that of measurement II. It is tentatively concluded that that the factor determining the thermal responsivity of the defect area, when exposed to an ultrasound excited vibration energy, is not the size of the contact area between an ultrasonic transducer tip and a test specimen but the mechanical condition bringing about a considerable vibration. In both cases (measurement I and II), the detection is intuitively interpretable, rapid, and sensitive, and it can be utilized to detect subsurface defects over wide areas. From the results of this study, it is concluded that the ultrasound excited thermography is a versatile and potential NDT technique that has a real-time detection capability of a defect in a pipe. Also, the method is applicable to disbonds and delaminations, as well as to fatigue cracks.

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