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Robust Controller Design of Nuclear Power Reactor by Parametric Method

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Abstract

The robust controller for the nuclear reactor power control system is designed. Since the reactor model is not exact, it is necessary to design the robust controller that can work in the real situations of perturbations. The reactor model is described in the form of transfer function and the bound of each coefficient is determined to set up the linear interval system. By the Kharitonov and the edge theorem, a frequency based design template is made and applied to the determination of the controller. The controller designed by this method is simpler than that obtained by the H_∞. Although the controller is designed with the basis of high power, it could be used even at low power.

Key Words : nuclear reactor, control, Kharitonov theorem, parametric uncertainty

1. Introduction

The control system design is very dependent on the exactness and reliability of the plant to be controlled. But since all the real plants have uncertainties, it is questionable that the designed controller based on those plants with uncertainties would work as intended in the real circumstances. The robust control method, which takes the uncertainties during the design process, could be an alternative to take account of such limitations. The robustness is defined as the performance and stability for the family of plants those are exposed to uncertainties. Hence the ultimate purpose of the control system design is to maintain the robustness rather than the stability only.

The robust control theory has been developed within the H_∞ frame. The theory provides a precise formulation and solution of the problem of synthesizing an output feedback compensator that minimizes the H_∞ norm of a prescribed system transfer function. The theory is quite efficient under the unstructured perturbations, and is regarded as a fairly complete theory for the control system synthesis subjected to perturbations. However, the performance of controller under real parameter uncertainty and mixed parametric-unstructured uncertainty is a vital issue to most control systems. But the optimal H_∞ theory is incapable of providing a direct and non-conservative answer[1].

The parametric robust theory is based on the polynomial theory that the roots of the polynomial depend on its coefficients. Although its concept is simple enough, there had been no generalized tool to address that problem. But with the advent of Kharitonov theorem[2], the parametric approach for the structured uncertainty has been developed, and proved to be an efficient control design technique. The advantage of the parametric approach is of the real applications. It gives the non-conservative synthesis methods to achieve robustness under parameter uncertainty. In addition, the classic control techniques can be applied directly, which avoids the problem recasting into the mathematically involved H_∞ frame.

The behavior of nuclear reactor is governed by many factors ranging from nuclear characteristics to material properties, and to operating conditions, so on. And it is difficult to establish the exact model. Even in the case of employing the simple point kinetics equations, many uncertainties are involved in the model. Hence, the robust approach is inevitable[3],[4]. The uncertainties of the model are reflected on the parameters of the system transfer functions, and it poses a typical problem of parametric perturbations.

The outline of this paper runs as follows: First, the reactor is modeled by the use of point kinetics and mass-energy equations, followed by the determination of the perturbation ranges for each parameter of reactor plant. Then the robust controller is designed by the Kharitonov together with the edge theorem[5]. Finally, through the simulations, the performance of the system is discussed.

2. Reactor Model and Perturbations

The reactor dynamics is described by use of the point kinetics equations with one group delayed neutrons. A singly lumped energy balance equation is incorporated to consider the moderator and fuel temperature feedback effects on the reactivity. Even this simple description yields the fifth order MIMO (multi input, multi output) system. In addition to the simplification and linearization of the governing equations, almost all of the physical properties that constitute the reactor model are subject to change depending on the operating conditions, that is, the reactor power, P. These errors in modeling and inexact properties are main causes of the system uncertainty.

With assumptions of that the coolant inlet temperature and coolant flow rate be constant, the MIMO reactor plant reduces to SISO(single input, single output) and is described in the following linear state variable equations[3].

$$\dot{x} = Ax + Bu, \quad y = Cx + Du \tag{1}$$

where $x = \left(\delta\overline{P} \;\; \delta\overline{C} \;\; \delta T_f \;\; \delta T_c \;\; \delta\rho_{ext}\right)^T$, $u = v_r$,

$$A = \begin{pmatrix} -\frac{\beta}{\Lambda} & \frac{\beta}{\Lambda} & \frac{\alpha_f}{\Lambda} & \frac{\alpha_c}{\Lambda} & \frac{1}{\Lambda} \\ \lambda & -\lambda & 0 & 0 & 0 \\ a & 0 & -b & b & 0 \\ 0 & 0 & c & -d & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \rho_w \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad D = 0, \quad a = \frac{P_0}{M_{fe} c_{fe}},$$

$$b = \frac{1}{R M_{fe} c_{fe}}, \quad c = \frac{1}{R M_c c_c}, \quad d = c + \frac{2W_0}{M_c},$$

$\delta\bar{P}$, $\delta\bar{C}$ = normalized power and precursor density variation, respectively, T_f, T_c = fuel and coolant temperature, respectively, v_r = rod speed, ρ_{ext} = external reactivity, β = delayed neutron fraction, P_0 = initial power in %, Λ = neutron effective lifetime, α_f, α_c = reactivity temperature coefficient of fuel and coolant, respectively, M_{fe}, M_c = total mass of fuel and coolant, respectively, c_{fe}, c_p = heat capacity of fuel element and coolant respectively, and W_0 = coolant flow rate.

In addition to the physical properties that depend on the reactor power, the fuel and moderator temperature coefficients and the fuel gap heat transfer coefficient h_g have great effects on the plant parameters. For example, the fuel gap heat transfer coefficient has a wide range of 2,500 to 11,000 W/m$^2\cdot{}^o$K [6]. And the fuel and moderator feedback temperature coefficient depend on the boron concentration, reactor lifetime, fuel temperature and rod position, so on. The FSAR of Kori Unit 2[7] reads that the temperature feedback coefficients have the values ranging over $\alpha_c \in$(-57pcm/^{o}K, 13.5pcm/^{o}K), $\alpha_f \in$(-4.7pcm/^{o}K, -2.8pcm/^{o}K), dependent on the fuel burn up and boron concentration.

In this study, three cases depending on the properties are considered. α_f = −3.7pcm/^{o}K, α_c=0pcm/^{o}K, and h_g=4,850W/m$^2\cdot{}^o$K. On the other hand, the 'optimistic plant' is of α_f=−4.7pcm/^{o}K, α_c = −57pcm/^{o}K, and h_g=10,000W/m$^2\cdot{}^o$K. And finally the 'worst plant' has the properties of α_f = −2.8pcm/^{o}K, and α_c = 13.5pcm/^{o}K,

The system dynamics of Eq.(1) is converted to the form of transfer function as h_g=2,000W/m$^2\cdot{}^o$K.

$$G(s,p) = \frac{228.5s^3 + 710.4s^2 + 229.1s + 13.7}{s^5 + 406.3s^4 + p_3 s^3 + p_2 s^2 + p_1 s} \qquad (2)$$

In obtaining this equation, it should be noted that the properties have the constant values of the normal plant. Then the system matrix is

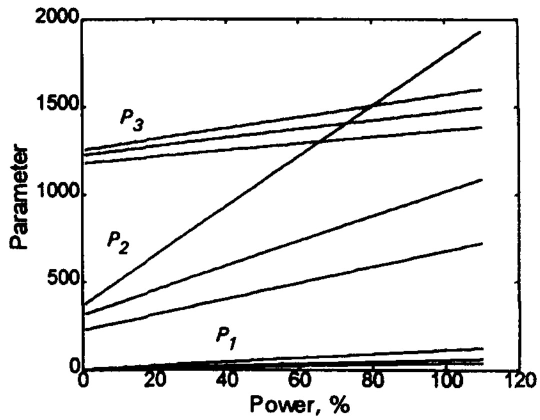

Fig. 1. The Bound of Parameter Values of the Reactor Plant

dependent only on the power, yielding three undetermined parameters. The zeros of the plant are constant regardless of the power variations. The reactor plant has one pole on the origin, which plays the role of an integrator. And as the power decreases, the poles become smaller. Particularly, the governing pole approaches to the origin. This makes the reactor plant become more unstable, accordingly, more difficult to control as the power becomes lower.

Figure 1 shows the parametric values of p_1, p_2 and p_3 for the optimistic, nominal, and worst plants. The parameters vary with the reactor power and the perturbation range of each parameter becomes larger with the increase of power.

3. Generalized Kharitonov Theorem

The characteristic equation of the reactor plant is a linear interval system of which each parameter has bounded values. There are numerous polynomial theories regarding the stability problems related to families of polynomials. The theories trace back to 1920s, and the typical ones are zero exclusion principle and phase bounded theorem[8]. But these theories can be used only in the analysis of the stability rather than the synthesis problem. But with the advent of Kharitonov theorem, the parametric approach can be used as a reliable tool for the control synthesis. However, although the Kharitonov theorem is simple and convenient to use, it provides only a sufficient condition for the system stability and yields a conservative result. On the other end, the edge theorem gives a more exact solution. But since the number of exposed edges depends exponentially on the number of uncertain parameters, the computation amount is very large, and there are redundant calculations. Therefore, it is natural to merge two theorems together, bringing about the Generalized Kharitonov Theorem (GKT)[1].

To utilize the GKT, the extremal set of line segment, $\Delta_E(s)$, should be determined first. With a perturbed plant and a fixed controller of

$$G(s)=\frac{P_1(s)}{P_2(s)},\quad C(s)=\frac{F_1(s)}{F_2(s)} \tag{3}$$

the characteristic equation of the closed loop system is

$$\Delta(s)=P_1(s)F_1(s)+P_2(s)F_2(s) \tag{4}$$

From the segment polynomials of $P_1(s)$ and $P_2(s)$, eight Kharitonov vertex equations are obtained and they are

$$K_1^j(s),\ j=1,2,3,4\ \ \text{for}\ P_1(s),$$
$$K_2^k(s),\ k=1,2,3,4\ \ \text{for}\ P_2(s)$$

$$P_m(s)=\sum_0^n[q_i^-,\ q_i^+]s^i,$$

$$K_m^1(s)=q_0^-s^0+q_1^+s^1+q_2^+s^2+q_3^-s^3+q_0^-s^4+\ldots$$
$$K_m^2(s)=q_0^-s^0+q_1^-s^1+q_2^+s^2+q_3^+s^3+q_0^-s^4+\ldots$$
$$K_m^3(s)=q_0^+s^0+q_1^+s^1+q_2^-s^2+q_3^-s^3+q_0^+s^4+\ldots$$
$$K_m^1(s)=q_0^+s^0+q_1^-s^1+q_2^-s^2+q_3^+s^3+q_0^+s^4+\ldots$$

The extremal subset, $P_E^l(s),\ l=1,2,$ consists of

$$P_E^1(s)=\frac{\lambda_l K_1^j+(1-\lambda_l)K_1^k}{K_2^i},$$
$$P_E^2(s)=\frac{K_1^i}{\lambda_m K_2^j+(1-\lambda_m)K_2^k} \tag{6}$$

where $\lambda\in(0,\ 1)$, l = 1, 2, 3, 4, m = 1, 2, 3, 4, i = 1, 2 and $[j,\ k]$, = [1, 2], [1, 3], [2, 4], [3,4].

In the above equation, the number of extremal equations is $32(m\cdot 4^m)$, where m is the number of perturbed polynomials. And $[j,\ k]$ indicates connection points to make the Kharitonov polytope.

Some of the subset equations may be the same, hence the extremal subset is described as

$$P_E(s)=\cup_{l=1}^m P_E^l(s) \tag{7}$$

With this extremal subset, the extremal subset of line segment (or, generalized Kharitonov segment polynomials) is

$$\begin{aligned}\Delta_E(s)&=\cup_{l=1}^m\Delta_E^l(s)\\&=\{<F(s),P(s)>\ :\ P(s)\in P_E(s)\}\end{aligned} \tag{8}$$

Then, since $\Delta_E(s)\subset\Delta(s)$, if all the polynomials of linear interval system were stable, the system with the perturbed parameters is stable.

In the controller design, or in the synthesis problem, the GKT permits the use of classic control techniques. Although the classic methods are heavily dependent on the designer's discretion, they have the merit of that the design can be made directly within the frequency domain, which provide a familiar visual method.

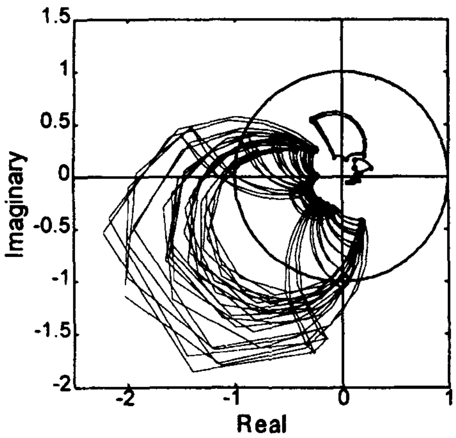

Fig. 2. Nyquist Diagram of the Perturbed Plant

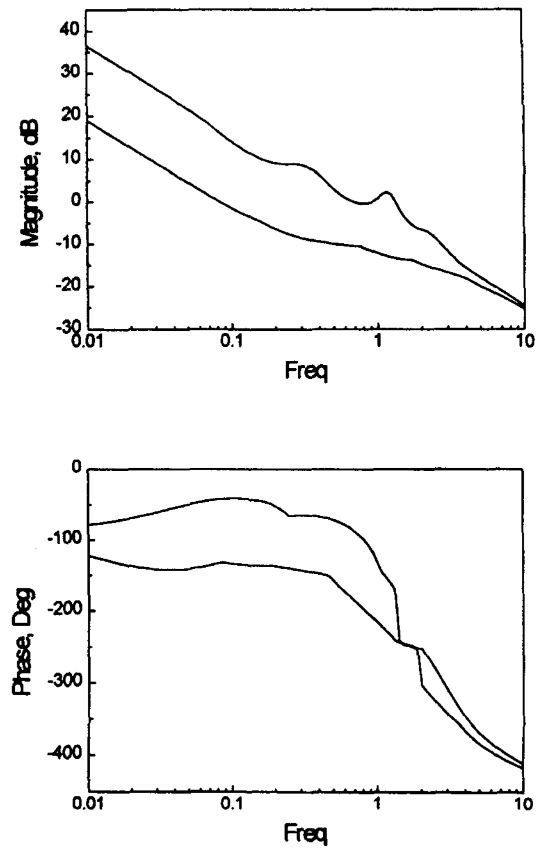

Fig. 3. Bode Diagram of the Perturbed Plant

4. Determination of the Controller

Figure 2 shows the Nyquist diagram of $G(s, \boldsymbol{p})$.

The coefficients of the plant cover all the bounds of heat transfer coefficient, fuel and moderator temperature feedback coefficients, and power ranges from 0% to 100%. And to make the situation more conservative by considering the lag of control rod drive movement, a delay of 2 seconds is included in the plant[2]. As shown in the figure, there are some characteristics strings which are unstable.

The Bode diagram of the perturbed system is shown in Fig. 3. As a design basis, 90% of power is used. But considering the possible measurement error, the bound values of $P \in (70\%, 100\%)$ are considered. The rationale of determining the 90% power as a design basis is the increase in performance. That is, if the design is made at low power, a larger stability could be obtained, but at the expense of the performance degradation.

The Bode diagram of Fig. 3 has been developed using the GKT, but it has some conservatism. It means that the envelope of the plot is, in general, not of the specific member of the polynomial family. In other words, there is no system in the family which generates the entire boundary of the envelope itself[1].

With the aid of Bode diagrams that are constructed from the set of extremal edges of $G(s, \boldsymbol{p})$, the controller is designed by the routine classic design procedures. In determining the controller, the lower boundary of the phase envelope is matched with the upper boundary of the gain envelope. Considering the desirable margins, a lead controller with the gain of 0.27 is determined with the additional pole of [-1, 0] to take account of the curve shift. The controller designed by this procedure is

$$C(s) = \frac{0.27(1+8.7s)}{(1+10s)(1+5s)} \qquad (9)$$

This controller is simpler than the one designed by the H_∞ methods[3],[4]. Also it gives sufficient margins even under the perturbations, say, it gives robustness. Figure 4(a) and (b) show the Nyquist envelope diagrams of $G(s, \mathbf{p})$ and $G(s, \mathbf{p})C(s)$, respectively. Figure 4(a) is the Nyquist of $G(s, \mathbf{p})$ only, and there are some strings in the unstable region. On the other hand, the controller of Eq.(10) makes the system stable as shown in Fig. 4(b). Further, since the Nyquist envelope has the conservatism, the actual margins are expected to be larger than those described in the figure.

5. Numerical Simulations

With the parameter perturbed plant of $G(s, \mathbf{p})$ and fixed controller of $C(s)$, the system is configured into the unity feedback system.

Two cases are considered for simulation. The first case is the power increase by step change from the initial steady power of 90% to 100%, and the second one is from the initial steady power of 5% to 15%. In the simulation, all the perturbations are considered.

For the simulation conditions, a sufficient deal of conservatism is considered to reflect the uncertainties during the operation. The reactor plant is assumed to be subject to random combinations of the perturbations of :

1) $P \in [P_0 \pm 5]$, 2) $h_g \in [h_{g0} \pm 1000]$,

3) $\alpha_f \in [\alpha_{f0} \pm 1.85]$, 4) $\alpha_c \in [\alpha_{c0} \pm 10]$ and

5) *delay* (sec) $\in [0, 2]$.

For the case of power increase from 90 to 100%, the nominal values are $[P_0, h_{g0}, \alpha_{f0}, \alpha_{c0}]$ $=[90\%, 4850 \text{W/m}^2 \,^\circ\text{K}, -3.7 \text{pcm}/^\circ\text{K}, 0 \text{pcm}/^\circ\text{K}]$.

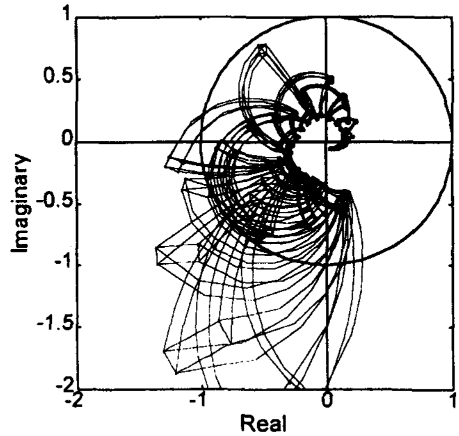

Fig. 4(a). Nyquist Diagram of $G(s, \mathbf{p})$

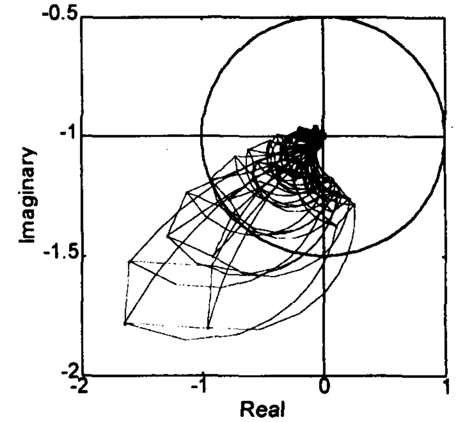

Fig. 4(b). Nyquist Diagram of $G(s, \mathbf{p})C(s)$

Figure 5(a) shows the results of power transient.

The initial powers, heat transfer coefficient, reactivity temperature feedback coefficients and the delay are randomly determined within the bounds described above, and found to be :

Case	*Power*	h_g	α_f	α_c	*delay*
A	92.3	3958	−3.2	−2.3	1.8
B	91.2	4928	−4.1	−1.7	2.0
C	91.7	5215	−2.6	+0.4	1.7
D	88.7	5723	−2.5	−6.6	0

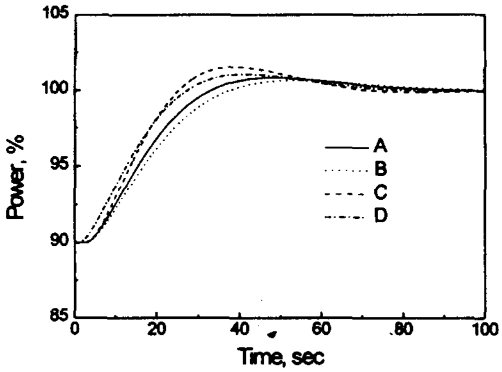

Fig. 5(a). Power Transients for Power Increase from 90 to 100%

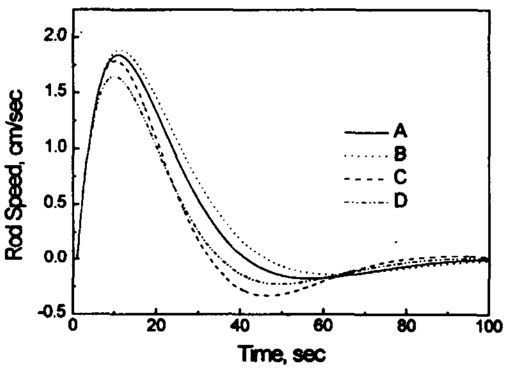

Fig. 5(b). Control Rod Speeds for Power Increase from 90 to 100%

where the units of the properties of above table are [%, $W/m^{2}\cdot{}^{o}K$, $pcm/{}^{o}K$, $pcm/{}^{o}K$, sec].

As in the figure, all the transients do not exceed the overshoot of 2% which is specified in the FSAR[8]. And they settle to the target value around 50 seconds into the transient.

The control input is another important factor to be considered. And in the reactor power control system, control input energy is the rod speed which is described in Fig. 5(b). It shows that although there are large perturbations, the rod speeds are less than the FSAR specified value of 2cm/sec.

The second simulation is for the power increase from 5 to 15%. The nominal values are the same as the first case except the power of 5%. The perturbed initial conditions are determined by the same way as the first case and they are :

Case	*Power*	h_g	α_f	α_c	*delay*
A	3.1	5287	−3.4	+5.3	0.8
B	5.6	4224	−1.9	−2.3	1.3
C	5.0	4967	−2.1	+0.3	1.1
D	5.7	5739	−4.2	−1.6	0.7

Since the power is low, the plant becomes less stable. It should be noted that the controller of Eq.(9) is determined with the design basis power of 90%. But this controller yields still tolerable results even at low power. Figure 6(a) describes the power transients and Fig. 6(b) shows the rod speeds. Even with the large perturbations with a delay, the maximum power during the transient is about 23%, and the rod speed does not exceed the FSAR value.

Comparing Fig. 5 and Fig. 6 each other, the designed controller of Eq.(10) at low power does not secure as much stability margins as at high power. This results from the fact that the design is made in the high power range. If the design is made in the lower power range, the controller gives a sufficient margins, but at the expense of performance degradation in high power ranges.

6. Conclusions

In the control system design, the mathematical model of the plant to be controlled has always uncertainties. These uncertainties arise from the limitations of the governing physical equations, linearizations, and aging, so on. One of the important problems in the control system is the

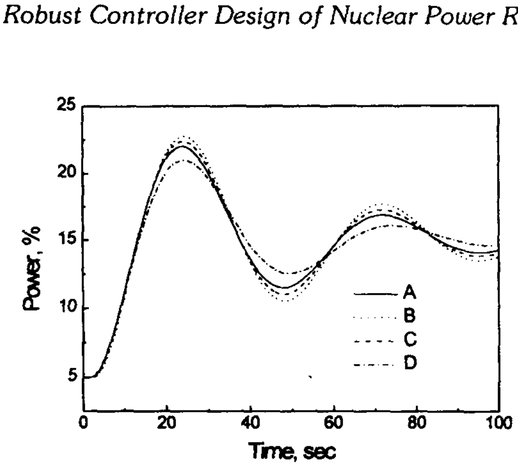

Fig. 6(a). Power Transients for Power Increase from 5 to 15%

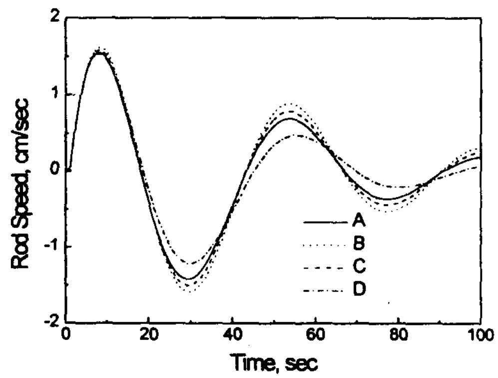

Fig. 6(b). Control Rod Speeds for Power Increase from 5 to 15%

performance and stability of the system under the real world parametric uncertainty or mixed parametric-unstructured uncertainty.

The H_∞ optimal and its offspring do not provide a direct and non-conservative solution on those problems. The nuclear reactor power control system poses a typical parametric problem with structured uncertainty. Even for the case of simple point kinetics equations model, each parameter of the transfer function has wide bounded values due to the operating conditions and material properties. And it is reasonable to deal with the family of plants rather than a specific plant.

By use of the Kharitonov and the edge theorem, the robust controller is designed. Since the design can be made with the frequency response envelopes, the classical methods can be used directly. The designed controller is of the second order one, which is much simpler than the controller designed by H_∞ method. The controller is designed on the basis of high power, but it can be used at low powers although the stability is somewhat degraded.

The main factor that influences the stability of the reactor plant is the power level. As the power level becomes lower, the plant itself becomes less stable. So in the future works, it is suggested that several controllers classified by power regions be designed, e.g., low, middle, and high power. Then by switching the controller each other with the power transients, the overall system is expected to maintain proper stability and performance.

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