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Development of a Simplified Statistical Methodology for Nuclear Fuel Rod Internal Pressure Calculation

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Abstract

A simplified statistical methodology is developed in order to both reduce over-conservatism of deterministic methodologies employed for PWR fuel rod internal pressure (RIP) calculation and simplify the complicated calculation procedure of the widely used statistical methodology which employs the response surface method and Monte Carlo simulation. The simplified statistical methodology employs the system moment method with a deterministic approach in determining the maximum variance of RIP. The maximum RIP variance is determined with the square sum of each maximum value of a mean RIP value times a RIP sensitivity factor for all input variables considered. This approach makes this simplified statistical methodology much more efficient in the routine reload core design analysis since it eliminates the numerous calculations required for the power history-dependent RIP variance determination. This simplified statistical methodology is shown to be more conservative in generating RIP distribution than the widely used statistical methodology. Comparison of the significances of each input variable to RIP indicates that fission gas release model is the most significant input variable.

Key Words : rod internal pressure, system moment method, sensitivity factor

1. Introduction

One of the objectives of the fuel rod design is to guarantee the fuel rod integrity under the ANS conditions I and II [1]. In order to assure the fuel rod integrity, the performance of the limiting rods with appropriate allowance for the uncertainties should be within the limits specified by the rod design criteria. A deterministic methodology has been employed for the PWR fuel performance analysis according to the current KNFC's fuel rod design methodology for the KOFA(KOrean Fuel

Assembly) fuel. It is well known that the deterministic methodology would give too much conservatism in the fuel rod design calculations since the upper or the lower bound of the fuel fabrication and fuel performance model parameters are combined as a single input data set. In addition, it is very difficult to estimate the magnitude of over-conservatism of this methodology. However, fuel management and demand of fuel design for extended cycle length and high burnup make it necessary to reduce the over-conservatism contained in the deterministic methodology.

A statistical methodology has been utilized to reduce the conservatism of the fuel thermal and mechanical performance parameters [2,3,4]. For the calculation of the rod internal pressure(RIP), Kim et al.[2] have proposed the widely used sophisticated statistical methodology which employs the response surface method and Monte Carlo simulation [5]. This methodology predicts quite reliably a statistical distribution of RIP. However, it requires the same number of regression equations as all limiting rod power histories to be considered for the fuel rod performance analysis. This makes it impractical to apply the sophisticated statistical methodology to the reload fuel rod performance analysis.

In this study, a simplified statistical methodology for the RIP calculation of the PWR fuel rod is developed in order to both reduce over-conservatism of the deterministic methodology and simplify the complicated procedure of the widely used sophisticated statistical methodology.

2. Development of a Simplified Statistical Methodology

The widely used sophisticated statistical methodology proposed by Kim et al.[2] is briefly summarized and then the simplified statistical methodology developed in this work is described in detail.

2.1. Sophisticated Statistical Methodology

2.1.1. Response Surface Method

When the performance parameter, Y, is a complex function of all input variables, $X_1, \cdots, X_n$, Y can be written as below :

$$Y = f(X_1, X_2, X_3, \cdots, X_n) \quad (1)$$

Then, Y can be approximated in the form of a polynomial regression equation as follows :

$$Y = B_o + \sum_i B_i X_i + \sum_i \sum_j B_{ij} X_i X_j \quad \text{--(2)}$$

where B_O = constant

B_i = regression coefficient of input variable X_i

B_{ij} = regression coefficient of input variable $X_i X_j$

The equation (2) may be represented by a matrix form in a following manner :

$$Y = X B + E \quad (3)$$

where Y = $(m \times 1)$ observed data vector

X = $(m \times n)$ experimental design matrix

B = $(n \times 1)$ regression coefficient vector

E = $(m \times 1)$ error vector

m = number of input data set

n = number of input variable

The $(m \times n)$ experimental design matrix is usually obtained by the LHS(Latin Hypercube Sampling) method [4]. Then, B can be estimated by using the least square method that the sum of the errors becomes minimum [6]. It is noteworthy that the same number of regression equations as the limiting rod power histories should be derived since the regression coefficients, Bi, are dependent on rod power history. In addition, since limiting rod power histories are cycle-dependent, the regression equations will be repeatedly derived for every reload cycle.

2.1.2. Monte Carlo Simulation

Monte Carlo simulation with the derived regression equation has been performed to generate a statistical distribution of rod internal pressure under the given range of all input variables. From the distribution of the internal pressure, the rod internal pressure of one-sided 95% probability(95% RIP) can be easily estimated.

2.2. Simplified Statistical Methodology

Mathematical equations employed in the simplified statistical methodology are derived by using the system moment method [7]. With the assumption of no intercorrelation among input variables and the consideration of only the first order term for each input variable, the equation (1) may be rewritten in the form of a differential equation as follows :

$$\frac{\delta Y}{Y} = S_1\frac{\delta X_1}{X_1} + S_2\frac{\delta X_2}{X_2} + S_3\frac{\delta X_3}{X_3} + \cdots + S_n\frac{\delta X_n}{X_n} \quad (4)$$

where δY = differential change in Y resulting from differential changes of each input variable, X_i

δX_i = differential change of each input variable, X_i

S_i = sensitivity factor of each input variable, X_i

The sensitivity factor, S_i represents significance of each input variable with respect to the performance parameter, Y. Each sensitivity factor can be easily calculated with all the input variables held constant except one input variable considered :

$$S_i = \frac{[\delta Y/Y]}{[\delta X_i/X_i]} = \frac{\delta(\ln Y)}{\delta(\ln X_i)} \quad (5)$$

From the equation (5), the value of S_i can be interpreted as the percent change of the performance parameter resulting from one percent change of input variable, X_i

Integration of the equation (4) gives

$$Y = C\,X_1^{S1}\,X_2^{S2}\,X_3^{S3}\cdots X_n^{Sn} \quad (6)$$

where C = integration constant

If the mean value of the input variable, X_i, is μ_i, Y can be expanded in a Taylor's series about μ_i as follows :

$$Y - \mu_Y = \frac{\partial Y}{\partial X_1}(X_1-\mu_1)+\frac{\partial Y}{\partial X_2}(X_2-\mu_2)+\cdots+\frac{\partial Y}{\partial X_n}(X_n-\mu_n) \quad (7)$$

where μ_Y = the mean value of the performance parameter

If the perturbations from the mean values are small, the higher order terms in the equation (7) can be neglected. Then, the variance of $(Y - \mu_Y)$ is represented in the following way :

$$\sigma_Y^2 = \left(\frac{\partial Y}{\partial X_1}\right)^2\sigma_1^2+\left(\frac{\partial Y}{\partial X_2}\right)^2\sigma_2^2+\cdots+\left(\frac{\partial Y}{\partial X_n}\right)^2\sigma_n^2 \quad (8)$$

From the equation (6), we have

$$\mu_Y = C\mu_1^{S1}\;\mu_2^{S2}\;\mu_3^{S3}\cdots\mu_n^{Sn} \quad (9)$$

Substitution of the equations (6) and (9) into the equation (8) leads to

$$\left(\frac{\sigma_Y}{\mu_Y}\right)^2 = \sum_i S_i^2\left(\frac{\sigma_i}{\mu_i}\right)^2 \quad (10)$$

The equation (10) indicates that the variance of the performance parameter, σ_Y^2, can be calculated by using μ_i and σ_i of each input variable, the

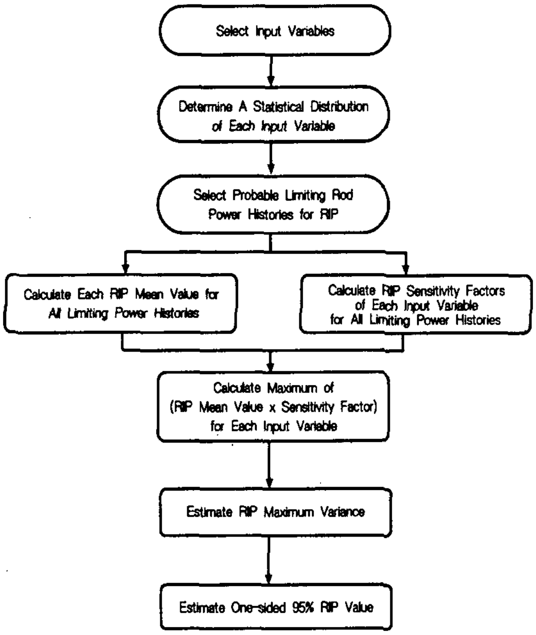

Fig. 1. Flowchart of Simplified Statistical Methodology for RIP Calculation.

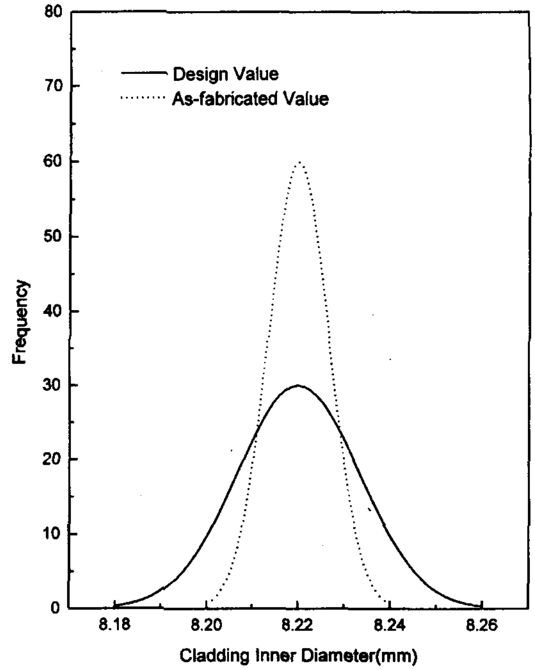

Fig. 2. Statistical Distribution of Cladding Inner Diameter.

sensitivity factor of each input variable, S_i, and the mean value of the performance parameter, μ_Y. It is noted that the μ_Y is dependent on rod power histories of each reload cycle. Since the variance of the performance parameter, σ_Y^2, is to be calculated considering all limiting rod power histories of each reload cycle, the application of the equation (10) to the routine reload cycle design is not practical. In this study, therefore, a deterministic approach is developed to obtain a power history-independent μ_Y from the equation (10). In detail, the maximum value of $\mu_Y \cdot S_i$ for each input variable(X_i) is selected from all values of $\mu_Y \cdot S_i$ calculated for all limiting rod power histories considered. Then, all the maximum values of μ_Y time S_i for all input variables are square summed up to calculate the maximum variance of the performance parameter considered, $(\sigma_{Y,max})^2$, from the equation (10). This deterministic approach will eliminate the calculation of the rod power history-dependent variance of the performance parameter for every reload cycle design. With the introduction of the deterministic approach, the equation (10) can be modified as follows :

$$(\sigma_{Y,\,max})^2 = \sum_i [\max \sum_j (\mu_{Yj} \cdot S_{ij})]^2 \left(\frac{\sigma_i}{\mu_i}\right)^2 \quad (11)$$

where μ_{Yj} = mean value of the performance parameter generated with j-th rod power history

S_{ij} = sensitivity factor of i-th input variable generated with j-th rod power history

$\max\Sigma(\mu_{Yj} \cdot S_{ij})$ = maximum value of $\mu_Y \cdot S_i$ selected from the data set of $(\mu_{Y1} \cdot S_{i1}, \mu_{Y2} \cdot S_{i2}, \cdots, \mu_{Ym} \cdot S_{im})$ where the subscript i represents the input variable of X_i and the subscript j represents all probable limiting rod power histories (j = 1, 2, ···, m)

The maximum variance of the performance parameter obtained from the equation (11) is used for the performance parameter of the one-sided 95% probability in the conservative direction as follows :

$$Y(95\%) = \mu_Y + 1.645\ \sigma_{Y,\max} \quad (12)$$

3. Application of the Simplified Statistical Methodology to Rod Internal Pressure Calculation

The calculational procedure of RIP with the aid of the simplified statistical methodology is shown in Fig. 1. Each step in this figure can be described in the followings.

3.1. Step 1 : Selection of Input Variables

It is necessary to select the RIP-related key input variables from all input variables used in the fuel performance analysis code [8]. The key input variables selected for the RIP calculation are divided into two groups of fabrication-related variables and model-related ones. The fabrication-related variables are pellet porosity, pellet outer diameter, clad inner and outer diameters, dishing volume, plenum volume, and initial fill gas pressure, whereas the model-related variables are fission gas release, radial relocation, swelling/densification, and clad creep models.

3.2. Step 2 : Determination of Statistical Input Variable

Statistical distributions of the input variables are to be determined in order to calculate their standard deviations which are needed in the equation (11). Based on the as-fabricated data and the data base used for the fuel performance models, the statistical distributions of the fabrication-related variables and the model-related ones are considered to be normal and uniform, respectively [2]. As shown in Fig. 2, the assumption of normal distribution for the fabrication-related variables is conservative since it covers conservatively the as-fabricated data of clad inner diameter with setting the tolerance limits as $\pm 3\sigma$. For the model-related variables, on the other hand, the assumption of uniform distribution between the maximum and the minimum data is very conservative since most of data are naturally concentrated on a best-estimated value. The statistical values employed in the rod internal pressure calculations are given in Table 1.

3.3. Step 3 : Determination of Sensitivity Factors

The sensitivity factors of each input variable and the RIP mean values are calculated by using sixteen limiting rod power histories(see Table 2), which are generally employed in the routine fuel performance analyses to generate rod power history-dependent fuel performance. The sensitivity factors of each input variable are found to be constant. As examples, the monotonic behaviors of clad inner diameter and fission gas release model are shown in Figs. 3 and 4. The maximum value of the RIP mean value(μ_{RIP}) times the sensitivity factor(S_i) for each input variable is selected from the data set of $\mu_{RIP} \cdot S_i$ generated for all limiting rod power histories considered. The data set of $\mu_{RIP} \cdot S_i$ which is dependent on rod power history is given in Table 2.

3.4. Step 4 : Determination of 95% RIP

The maximum variance of RIP is calculated by

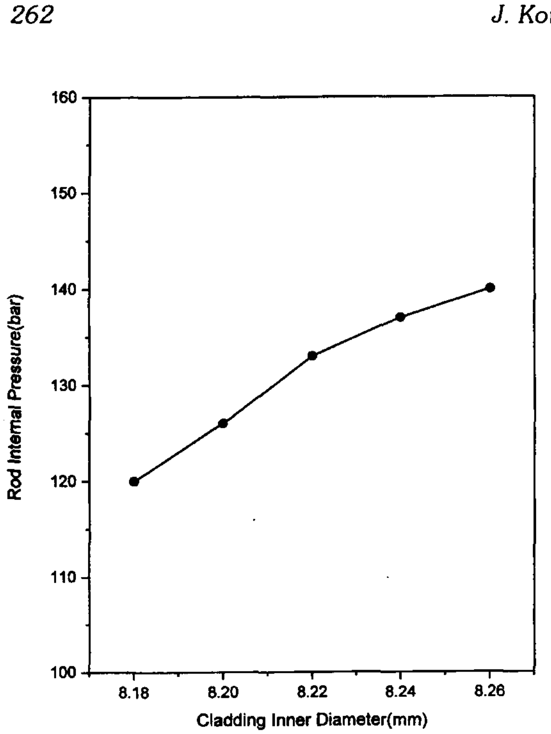

Fig. 3. Statistical Distribution of Cladding Inner Diameter.

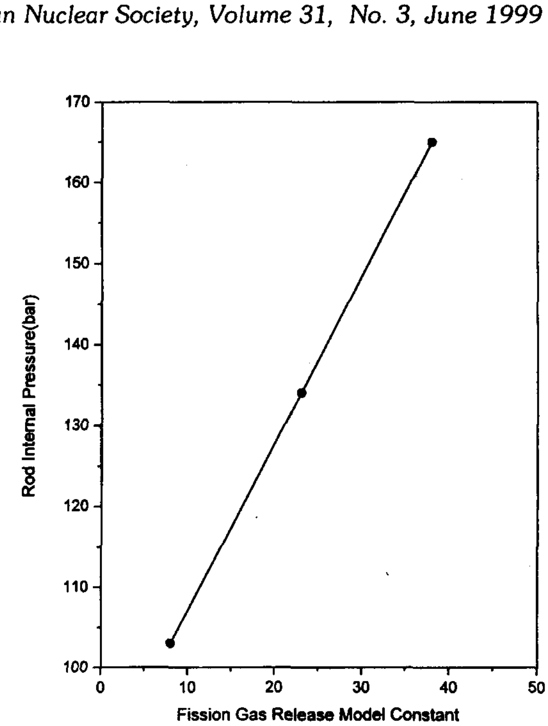

Fig. 4. Effect of Fission Gas Release on Rod Internal Pressure.

substituting into the equation (11) the maximum value of $\mu_{RIP} \cdot S_i$ for each input variable and the statistical data of each input variable such as standard deviation and mean value. Then, by using the mean value(μ_{RIP}) and the maximum standard standard deviation of RIP($\sigma_{RIP,max}$), the one-sided 95% RIP is calculated from the equation (12).

4. Results and Discussion

Table 1 summarizes the statistical values for the RIP calculations with the help of the simplified statistical methodology developed in this work. In this table, the maximum standard deviation of RIP for the 17×17 KOFA fuel was calculated to be 17.9 bar. This standard deviation was obtained by the system moment method combined with the deterministic approach described in the previous section 2.2. Based on the deterministic approach, the maximum variance of RIP is obtained by taking into account the maximum values of $\mu_{RIP} \cdot S_i$ for each input variable selected from the data set of $\mu_{RIP} \cdot S_i$ generated for all limiting rod power histories considered. In this work, sixteen limiting rod power histories are considered. The central limit theorem [9] indicates that the statistical distribution of RIP for the 17×17 KOFA fuel is considered to be normal with a standard deviation of 17.9 bar and a RIP mean value. Using the maximum standard deviation of 17.9 bar, the one-sided 95% RIP value for the 17×17 KOFA fuel can be given by the following formula :

$$\mathrm{RIP}(95\%) = \mathrm{RIP} + 1.645 \times 17.9 \quad (13)$$

where RIP = rod internal pressure, bar

μ_{RIP} = mean value of rod internal pressure, bar

Table 1. Statistical Distributions and Parameters for Each Input Variable

Input Variables	μ_i	σ_i	$\alpha_i^{(1)}$	$\beta_i^{2(2)}$	$f_i^{(3)}$
Porosity (%) [4]	5.10	0.4667	39.78	13.25	0.041
Clad I.D. (mm) [5]	8.22	0.0133	3295.87	28.44	0.088
Dish Volume (mm^3) [6]	11.00	1.0000	22.77	4.28	0.013
Plenum Volume (cm^3) [7]	6.40	0.2000	78.72	6.05	0.019
Fission Gas Release [8] Model	23.00	8.6603	34.50	168.75	0.523
Relocation Model [9]	0.66	0.1013	14.22	4.76	0.015
Swelling Model [10]	0.40	0.0346	97.60	71.27	0.221
Creep Model [11]	1.13	0.21	26.77	26.12	0.081

(1) $\alpha_i = \max[\sum_j \mu_{RIP,j} \times S_{ij}]$, bar : see Equation (11)

(2) $\beta_i^2 = \alpha_i^2 \times (\sigma_i/\mu_i)^2$, $(\sigma_{RIP})_{max} = (\Sigma \beta_i^2)^{1/2} = 17.9$ bar

(3) $f_i = \beta_i^2 / \Sigma \beta_i^2$

(4) Porosity (%) : Pore Volume Fraction in the Pellet

(5) Clad I.D. (mm) : Cladding Inner Diameter

(6) Dish Volume (mm^3) : Dishing Volume in the Pellet

(7) Plenum Volume (cm^3) : Plenum Volume in the Fuel Rod

(8) Fission Gas Release Model : Fission Gas Release Model Constant to Simulate Fission Gas Release through the Pellet Grain Boundaries to the Void in the Fuel Rod

(9) Relocation Model : Pellet Relocation Model Constant to Simulate Pellet Movement in the Radial Direction Caused by Cracks Formation in the Pellet

(10) Swelling Model : Pellet Swelling Model Constant to Simulate Pellet Volume Increase Caused by Formation of Gas and Solid Fission Products in the Pellet

(11) Creep Model : Cladding Creep Model Constant to Simulate Cladding Diameter Decrease Caused by Combination of Overpressure Acting on the Cladding and Neutron Fluence

The above formula indicates that the higher μ_{RIP} generates the higher 95% RIP. To determine μ_{RIP}, one limiting rod power history is to be selected, which generates the maximum μ_{RIP} for a certain reload cycle. In Fig. 5, it can be seen that the statistical distribution with a standard deviation of 17.9 bar and a maximum RIP mean value bounds conservatively actual one-sided 95% RIP values calculated by using sixteen limiting rod power histories. This is because the maximum standard deviation of 17.9 bar was determined by using the maximum values of μ_{RIP} times S_i for each input variable. On the other hand, the over-conservatism of the deterministic methodology employed for the KOFA fuel can be estimated with the equation (13). Using one limiting rod power history for a certain reload cycle which generates μ_{RIP} of 140 bar, the deterministic methodology generates the maximum RIP value of 211 bar, while the equation (13) gives the one-sided 95% RIP value of 169 bar. This indicates that the simplified statistical methodology can reduce over-conservatism of the deterministic methodology as much as 42 bar in predicting RIP

Table 2. RIP Mean Value ($\mu_{RIP,j}$) Times Sensitivity Factor (S_{ij}) for Each Input Variable Versus Each Rod Power History

Rod Power History Type	RIP Mean Value ($\mu_{RIP,j}$) × Sensitivity Factor (S_{ij}), bar							
	Fuel Porosity	Clad I.D	Dish Volume	Plenum Volume	Fission Gas Release Model	Relocation Model	Swelling Model	Creep Model
1	28.56	1002.84	18.59	51.84	28.29	11.08	74.00	14.17
2	29.58	1175.47	18.37	56.96	28.28	12.65	66.60	4.27
3	22.44	904.20	13.65	39.04	21.62	7.67	79.40	25.20
4	39.27	3295.87	22.22	77.45	34.50	14.22	44.59	8.55
5	32.13	995.67	19.00	59.10	28.32	11.06	59.96	3.12
6	36.21	1471.38	20.90	72.95	29.45	11.74	62.42	4.96
7	24.99	1132.83	16.50	51.83	23.91	8.98	68.62	9.67
8	20.40	1019.25	14.08	40.96	20.70	5.83	84.00	26.77
9	32.13	813.77	22.77	71.04	26.46	9.83	79.00	10.12
10	39.78	1597.65	21.01	78.72	31.51	10.81	37.19	10.80
11	25.50	509.78	16.83	47.37	22.55	5.77	66.81	16.00
12	24.22	1198.03	15.65	49.56	22.53	5.04	87.22	24.34
13	20.40	961.74	12.87	39.04	18.40	4.06	69.41	25.65
14	30.09	690.59	21.23	65.28	23.23	7.54	91.20	13.28
15	22.44	945.30	12.54	36.48	18.64	3.41	76.40	26.10
16	24.98	1159.02	16.27	54.40	19.77	5.50	97.60	25.66

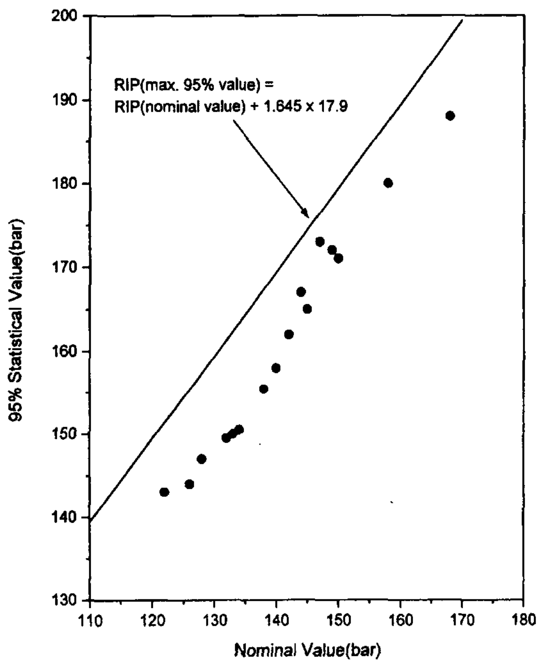

Fig. 5. The 95% RIP Value Predicted by the Simplified Statistical Methodology.

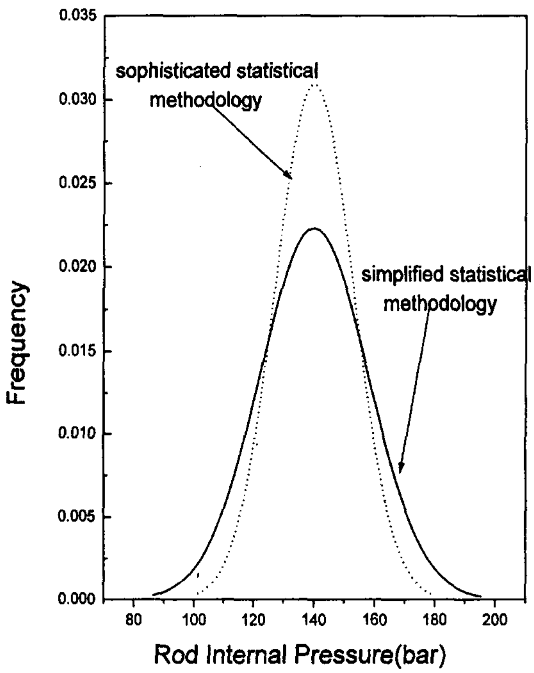

Fig. 6. Rod Internal Pressure Distribution Predicted by the Statistical Methodologies.

with the limiting rod power history considered.

In order to check the reliability of the simplified statistical methodology, statistical distributions of the internal pressure predicted by the simplified methodology and the sophisticated one are compared in Fig. 6. It can be seen that the statistical distribution of the internal pressure with the simplified methodology bounds that with the sophisticated methodology. This is because a standard deviation of RIP predicted by the simplified methodology is 17.9 bar while that predicted by the sophisticated methodology is 12.9 bar with the use of the same rod power history. This indicates that the simplified methodology generates the statistical distribution of RIP more conservatively than the sophisticated methodology. In addition, the simplified methodology is very efficient in estimating the 95% RIP since it eliminates the time-consuming process on the derivation of the rod power history-dependent standard deviation which is needed for the sophisticated statistical methodology.

As a criterion of significance of each input variable with respect to RIP, a fractional variance of each input variable, f_i, may be defined as follows :

$$f_i = \frac{S_i^2 (\sigma_i/\mu_i)^2}{\sum_i S_i^2 (\sigma_i/\mu_i)^2} \qquad (14)$$

Comparison of the f_i values of each input variable given in Table 1 indicates that fission gas release model is the most significant variable to RIP and clad inner diameter is the second most significant variable, while pellet dish volume is the least significant variable. This shows that the optimization of pellet microstructure is the most important factor in developing high burnup fuel since fission gas release depends strongly on pellet microstructure and rod internal pressure is one of the limiting performance parameter for high burnup fuel [10].

The one-sided 95% RIP value determined can be utilized to check the clad non-lift-off criterion which limits a clad creepout rate less than a pellet swelling rate. According to Kim et al, [11], RIP is shown to be larger than the system pressure(P_{sys}) when the clad non-lift-off criterion is employed. In addition, the amount of RIP overpressure(Δp = RIP - P_{sys}) is dependent on linear hear generation rate, as given in the following :

$$\Delta P = RIP - P_{sys} = -0.517\, q' + 153.7$$

where ΔP = RIP overpressure, bar

P_{sys} = coolant pressure, bar

q' = local linear heat generation rate, W/cm

Then, the allowable RIP mean value(μ_{RIP}) against the clad lift-off may be given as follows :

$$\mu_{RIP,max} = P_{sys} + \Delta P - 1.645 \times 17.9$$

where $\mu_{RIP,max}$ = allowable RIP mean value against the clad lift-off, bar

In order to check whether the clad non-lift-off criterion is met or not, one should determine μ_{RIP} for a limiting rod power history. If μ_{RIP} is less than $\mu_{RIP,max}$, the clad non-lift-off criterion is met.

5. Conclusions

A simplified statistical methodology for the rod internal pressure calculation is developed in order to both reduce over-conservatism of the currently employed deterministic methodology and simplify the existing sophisticated statistical methodology. The conclusions are summarized as follows :

(1) For the probability distribution function of RIP, the simplified statistical methodology employs the system moment method combined with a deterministic approach in determining a maximum variance of RIP, while the

sophisticated statistical methodology employs the LHS method, the response surface method and Monte Carlo simulation. The simplified methodology is easy and quick to use since it eliminates the time-consuming process on the regression equation derivations or the RIP variance derivations which are dependent on rod power history. The simplified methodology is very reliable since the statistical distribution of the internal pressure with the simplified methodology is more conservative than that with the sophisticated statistical methodology.

(2) By comparing the deterministically obtained RIP with the one-sided 95% RIP predicted by the simplified statistical methodology, the simplified methodology reduces RIP by 42 bar.

(3) Comparison of the significance of each input variable to RIP indicates that fission gas release model is the most significant variable and clad inner diameter is the second most significant variable, while pellet dish volume is the least significant variable. For the high burnup fuel development the most important parameter is the optimization of pellet microstructure.

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