

Robust Design of Reactor Power Control System with Genetic Algorithm-Applied Weighting Functions

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Abstract

The H_∞ algorithms of the mixed weight sensitivity is used for the robust design of the reactor power control system. The mixed weight sensitivity method requires the selection of the proper weighting functions for the loop shaping in frequency domain. The complexity of the system equation and the non-convexity of the problem make it very difficult to determine the weighting functions. The genetic algorithm which is improved and hybridized with the simulated annealing is applied to determine the weighting functions. This approach permits an automatic calculation and the resultant system shows good robustness and performance.

1. Introduction

In the control system design, the most important thing is to model the plant which is to be controlled. But it is almost impossible to model the plant exactly. The plant modeling includes an inevitable linearization of the non-linearity as well as the approximations in the mathematical description of the plant. In addition the designed system is subject to change due to the operating conditions, controller set point drift, system degradation, and so on.

The actual system should work as intended under the real circumstances even though it is designed on the basis of inexact plant. Therefore, the ultimate purpose of the control system design is to maintain the robustness rather than the stability[1].

The system robustness is defined as the performance and stability of the system for the

family of the plants exposed to the uncertainties[2]. This robustness problem, particularly for MIMO (multi-input multi- output) systems has been one of main issues in recent years, and many methods are developed for the robust design. Among them the H_∞ paradigm provides the synthetic method in which the size of the uncertainty is measured quantitatively by the infinity norm of the system[3]. However, the design process is not so easy although this method, which has a solid mathematical theory, has been proved to be a useful approach. It requires that the system should be in the H_∞ space, that is, in the space of stability and properness. Further, since the H_∞ control is a non-convexing problem, it is difficult to determine the optimal controller. All these properties result in the messy mathematical processes and many iterations are required. The H_∞ control may be

regarded as an optimization process in the frequency domain. This optimization is directly related to the loop shaping which has been known even from the era of classical control[4]. The concept of loop shaping yields another variation of the H_∞ control, say, the mixed weight sensitivity method[5]. In this method, the robust design problem resolves itself to the determination of proper frequency-dependent weighting functions. But there is no canonical method in selection of the weighting functions. The controller design begins with the guess of reasonable but arbitrary weighting functions, and if the designed controller does not give a plausible result, the overall design is to be iterated with another set of weighting functions.

This motivates us to develop a new method to determine the weighting functions automatically by use of the genetic algorithm[6]. Since the genetic algorithm is intrinsically a stochastic searching method, it can be used as an efficient tool for the non-convexing problem such as the robust design. Once the cost function is given, the algorithm finds the optimal results without designer's interference, and guarantees the best solution under the frame of pre-determined cost functions.

2. Model Uncertainties and H_∞ Control

All the modeled plants and systems have uncertainties. Some of these uncertainties come from the mathematical description of the real plant. The mathematical modeling accompanies various linearizations and simplifications somewhat apart from the real ones. Even if the plant is modeled exactly, it is exposed to the varying operating conditions, and the system properties are subject to change due to the component aging.

For an example, the nuclear reactor plant dynamics are described by the reactor power

dependent 5th order linear state variable equations as

$$\dot{\mathbf{x}} = \mathbf{A}(P)\mathbf{x} + \mathbf{B}(P)u, \quad y = \mathbf{C}(P)\mathbf{x} + \mathbf{D}(P)u \quad (1)$$

where P is the reactor power, and the state vector \mathbf{x} consists of the variations of power, precursor density, fuel temperature, coolant temperature and external reactivity. The input u and output y are control rod velocity and reactor power, respectively. The details are fully described in previous studies[7],[8], and the limitations are discussed in Ref.[9]. The model of Eq.(1) has many uncertainties. It employs the simplified point kinetics equation and single lumped energy balance equation. The modeling includes the linearization with the assumption of small perturbations from the steady state. In addition, the system matrices are function of material properties, which again depend on the operating conditions.

Usually the control system design is based on the models of plants or processes which are to be controlled. Therefore, if the modeled plant is different from the actual plant, the designed system may not have the intended stability and performance. The Wiener-Hopf-Kalman (WHK) LQR control is a typical example of this discrepancy between the theory and the reality. Since the WHK optimal control theory itself is perfect and the design procedure is once-through and simple, it was expected to be a reliable design tool. But it turned out to be almost useless in the industrial process plant. The WHK has the presumptions that the process plant be exactly described with no uncertainty and the stochastic properties of the noises be known. This is impossible or uneconomic except for the some special cases such as spaceship and small scale electro mechanical system.

The control system should take account of the

uncertainties in advance, and the designed system should work as intended under the real situation. Therefore it can be said that the purpose of the control system design is the robustness rather than the stability. The H_∞ optimal control technique provides an efficient method which can deal with the modeling errors and external disturbances. The major target of the H_∞ design is to determine the transfer function of the overall system including the controller. That is, the H_∞ control can be regarded as an optimization in the space of transfer function with the performance function of H_∞ norm of the transfer function.

The robust problem can be divided into two categories of the robust analysis and the robust synthesis. The analysis problem is to find out the MSM (multivariable stability margin) of the system seen by the uncertainties. In other words, it is to find the maximum permissible uncertainties for the system stability. The MSM, $K_m(T_{y1u1})$ is defined as

$$K_m(T_{y1u1}(j\omega)) = \frac{1}{\mu_\Delta(T_{y1u1}(j\omega))} \quad (2)$$

$$= \inf_\Delta \{ \bar{\sigma}(\Delta(j\omega)) \mid \det(I - T_{y1u1}(j\omega)\Delta(j\omega)) = 0 \}$$

where μ_Δ = structured singular value (SSV) with the uncertainty, $\bar{\sigma}$ = singular value, $\Delta(j\omega)$ = frequency dependent uncertainty, $T_{y1u1}(j\omega)$ = frequency dependent system transfer function with input and output vector of u_1 and y_1 , respectively.

The robust analysis can be thought as the MIMO version of the Nyquist small gain theorem. It says that the perturbed system $\frac{1}{(I - T\Delta)}$ is stable if and only if $\|\Delta(j\omega)\|_\infty \leq 1$ and $K_m(T(j\omega)) > 1$ for all frequencies where $T(j\omega)$ is the transfer function of an unperturbed nominal plant.

On the other hand, the robust synthesis problem is to find out the controller $K(s)$ which makes the structured singular value, $\mu_\Delta(T(j\omega))$, conform to the desirable loop shaping. The concept of loop

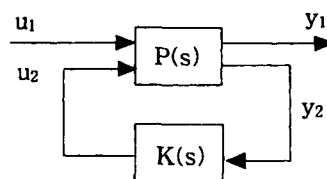


Fig. 1. Canonical Two-Port Model

shaping has been known from the classical PID, and all the modern algorithms such as H_2 , LQG/LTR or H_∞ can be regarded as related to the loop shaping. Among these, the H_∞ is the most direct and reliable method for the controller design which satisfies the design specification given in terms of the singular value loop shaping.

The robust problem could be configured into the canonical two-port model as in Fig. 1. The system equation poses as

$$\Phi = P(s) \Psi \quad (3)$$

where $\Phi = (\dot{x} \ y_1 \ y_2)^T$, $\Psi = (x \ u_1 \ u_2)^T$, x = state variable vector, u and y = system input and output vector, respectively. And

$$P(s) = \begin{pmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}.$$

Then the synthesis is to find the stable feedback control law of $u_2(s) = K(s)y_2(s)$ which minimizes the infinite norm of T_{y1u1} . The mathematical algorithm includes the relations between the input vector and output vector by use of an observer, which yields two Riccati equations of the control law and the observer law. One of important characteristics of the H_∞ control is that the cost function T_{y1u1} is of all pass, which means that the singular value is unity for all frequencies. This implies that it is always possible to control the loop shape with some proper frequency-dependent weighting functions in the H_∞ paradigm.

However the H_∞ has some disadvantages. Since the problem is to solve a set of coupled Riccati equations, the system equation itself should satisfy several mathematical conditions. Some of these conditions are : 1) both of the control law Riccati solution, P , and the observer Riccati solution, S , should be positive-definite, 2) spectral radius of (PS) should be positive, 3) the rank condition between the input/output and packed matrix order should be satisfied, 4) D_{11} of the packed matrix should be sufficiently small for the system roll-off at high frequencies. The most difficult one is the rank condition. If the system does not meet this condition, dummy variables should be added or the overall system should be reconfigured. Another drawback of the H_∞ is the non-convexity. Hence it is very difficult to obtain the globally optimal solution. This implies physically that there might be another controller which is better than the designed one.

3. Mixed Weight Sensitivity Method

Figure 2 shows the unity feedback system with plant disturbance and sensor noise. By defining the sensitivity as $S = 1/(1+GK)$ and the complementary sensitivity as $T = GK/(1+GK)$, the system output and the control input are put into

$$y_p(s) = T(r - n) + Sd, \quad u(s) = \frac{T}{G}(r - d - n) \quad (4)$$

Usually the noise has high frequencies and the disturbance has low frequencies. Therefore, it is desirable to decrease the magnitude of T in the higher frequency region to eliminate the noise effects. Similarly, the magnitude of S should be small in the lower frequency region to remove the disturbance effects. This is the concept of the loop shaping and the robust control design problem is to achieve the desirable loop shaping with the predetermined ideal loop shapes of W_1 and W_3 .

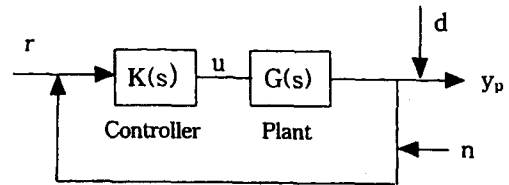


Fig. 2. Feedback System with Noise and Disturbance

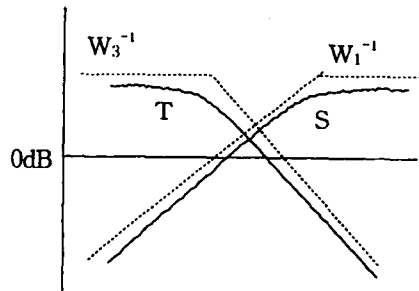


Fig. 3. Loop Shaping with Weighting Functions

This is described in Fig. 3. The target of the design is to find such S and T which satisfy the conditions of $\bar{\sigma}(S(j\omega)) < |W_1^{-1}|$ and $\bar{\sigma}(T(j\omega)) < |W_3^{-1}|$.

Figure 4 shows the feedback system perturbed by disturbance and noise. In this configuration, all the disturbances acting on the plant are treated as one multiplicative uncertainty. The weighting functions of W_1 and W_3 are applied to the error signal and plant output signal, respectively.

The overall system is a MIMO system and the transfer function between the input vector u and output vector y is

$$\begin{aligned} T_{yu} = \frac{y}{u} &= \begin{pmatrix} W_1 S & -W_1 S \\ W_3 T & -W_3 T \end{pmatrix} \\ &= \begin{pmatrix} W_1 S \\ W_3 T \end{pmatrix} (I, -I) \end{aligned} \quad (5)$$

where $u = (u_n \ u_d)^T$, $y = (e_w \ y_w)^T$. The problem is to decrease the infinite norm of T_{yu} to minimize the

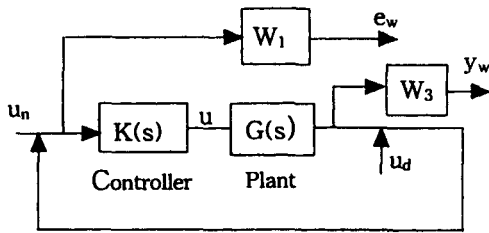


Fig. 4. Weighting Augmented System

system responses to the noise and disturbance inputs, and the mixed weight sensitivity method boils down to the selection of weighting functions. The calculational procedure of the mixed sensitivity method is simpler than that of the μ analysis method which uses the heavy calculation of the structured singular value. The result of the mixed sensitivity gives a good approximation to the μ analysis with the maximum deviation of 3dB.

The feedback system with the weighting functions of Fig. 4 is recast into the two-port model as described in Fig. 5.

In this figure, the noise signal is replaced by the command signal u_1 to make the tracking system as is the reactor power control system. An additional weighting function W_2 are applied to the plant input signal to regulate the control effort. Even if it is not necessary to regulate the control effort, this weighting function is necessary for the rank condition. Then the system is expressed in the following SIMO (Single Input Multi-Output) transfer function vector.

$$\mathbf{T} = \frac{(y_1 \ y_2 \ y_3)^T}{u_1} = (W_1 S \ W_2 R \ W_3 T)^T, \quad (6)$$

$$R = \frac{K}{1 + KG}$$

The calculational procedure is outlined in Figure 6. The most important and the most difficult step is the selection of weighting functions. It is a common practice to use a first order lead or lag as a weighting function. A higher order function can

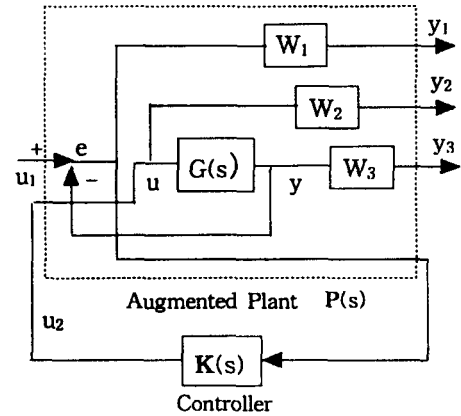


Fig. 5. Weight Augmented Two Port Model

be used for the more exact result, of course, but it increases the order of the packed matrix, and may breach the conditions of the H_∞ controller existence. But even for the case of a first order lag or lead, it is very difficult to determine its parameters, particularly when the frequency characteristics of the system such as bandwidth, breakpoint frequency and saturated limits are not known. Although the selected weighting functions satisfy the mathematical conditions, the designed system may differ from the intended system with respect to performance or stability. Then the whole calculation starts again from the first step with new weight functions.

The real problem lies in the fact that the problem is non-convexing. Because of this non-convexity, it is difficult to relate the performance improvement with the gradients, or rate of changes, of weighting function parameters. Hence the weighting function selection heavily depends on the designer's discretion. The calculation should be iterated until the 'plausible' results are obtained. But it can not be assured that the designed controller is optimized globally. The controller thus obtained is optimized locally.

Figure 7 shows the results of this conventional

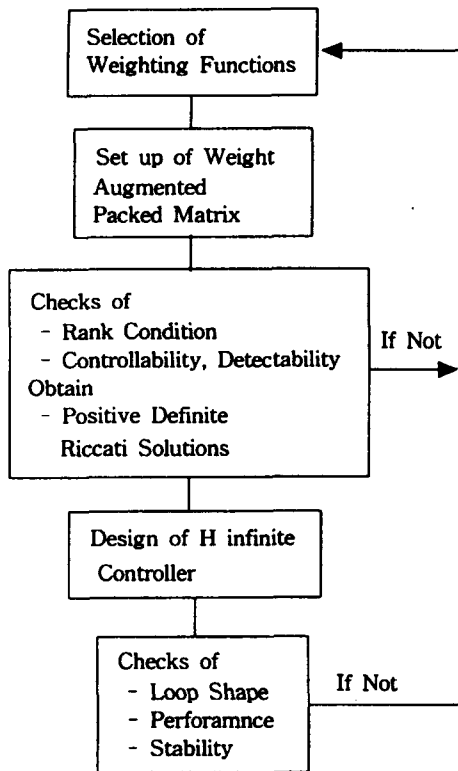


Fig. 6. Controller Design by Mixed Weight Sensitivity

approach. The reactor power control system is configured as in Fig. 5. The system is in the steady state of 90% rated power. The situation is such that a step command input signal acts on the system to increase the power from 90% to 100%. It is required that the maximum power during the transient should not exceed 103% and the rod speed be less than 2cm/sec. The purpose is to design the robust controller $K(s)$ which satisfies these constraints. The reactor plant $G(s)$ of 90% is obtained from Eq.(1). With numerous trial and errors, the weighting functions are determined as

$$W_1(s) = \frac{1}{0.2} \frac{s+0.06238}{s+0.0006267}, \quad W_2(s) = 5, \quad (7)$$

$$W_3(s) = \frac{1}{2.4} \frac{s+0.06238}{0.06175}$$

and the controller is

$$K(s) = \frac{0.1119s^6 + 6953.7s^5 + 2.8072 \times 10^6 s^4}{(s^7 + 6.2261 \times 10^4 s^6 + 3.1577 \times 10^7 s^5 + 1.0189 \times 10^7 s^3 + 7.0329 \times 10^6 s^2 + 1.3033 \times 10^8 s^4 + 1.3213 \times 10^8 s^3 + 3.7512 \times 10^5 s + 3.224 \times 10^7 s^2 + 1.7921 \times 10^6 s + 1110.5)} \quad (8)$$

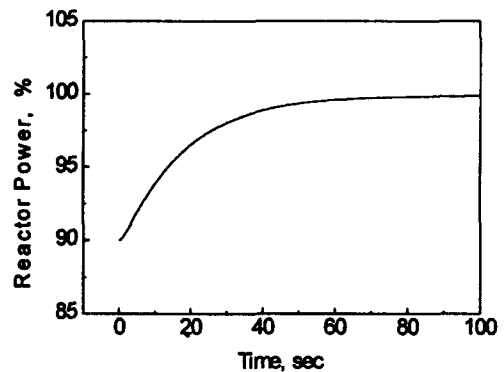


Fig. 7-1. Reactor Power - Conventional Method

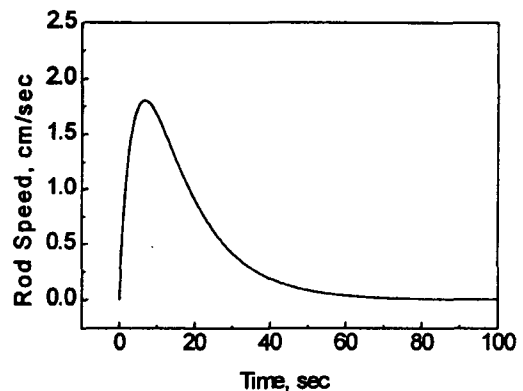


Fig. 7-2. Rod Velocity - Conventional Method

Figure 7 shows the reasonable results. There is no overshooting during the transient and the maximum rod speed is less than 2cm/sec. The maximum rod acceleration is 0.76cm/sec². Although no formal constraint is set forth on the rod acceleration, a large acceleration is not

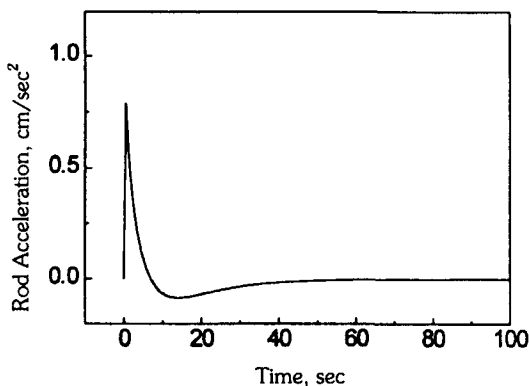


Fig. 7-3. Rod Acceleration - Conventional Method

desirable because of the possible problems in mechanical components.

It is found that the calculational algorithm is very sensitive to the parameters of the weighting functions. A small change of the weighting function parameter may result in a quite different result, and it is almost impossible to find out the best solution.

4. Application of GA to Weighting Functions

The difficulties of the optimization arising from the non-convexity can be eliminated by use of the genetic algorithm (GA). The GA is based on the survival-of-the fittest principle in nature. In terms of calculation, the GA maps a problem onto a subset of candidate solution which is called an individual. Each solution is associated with a fitness value to measure how good it is. By using three major operators of reproduction, crossover and mutation, it searches the optimal solution of the given problem. Since the GA does not depend on the coupling between the parameters, it provides more flexibility in dealing with the concerned system. Throughout the repeated searches, the attributes of the candidate solutions will improve toward an unknown optimal one, and

some of these solutions will converge to the globally best one. However, the GA has several shortcomings. For example, its convergence rate is slow at the final stage and may wander around the true solution. And the detail algorithm may be inefficient depending on the problem characteristics.

To eliminate these generic demerits of the GA, the simulated annealing (SA) is employed into the GA along with the improvements of the GA algorithms. And this scheme is named the hybrid Modified GA Simulated Annealing (MGA-SA)[10]. The major features of the MGA are as below.

- 1) *exponential-wise representation of the parameters* : This permits the efficient search when there exists little information on the parameter range, or when it ranges widely.
- 2) *modified crossover scheme* : A linear inter/extrapolation is used instead of the bit-wise cross over to avoid the hamming cliff effects.
- 3) *ancestor pool and periodic reinitialization* : To reduce the run time, fitness values of the visited solutions are saved in a storage named ancestor pool, and is reused when the same candidates are generated in the future generation. And by the periodic reinitialization from this pool, the searching process becomes more stable and less divergent.

The MGA is a multi point searching scheme and is efficient for the global search in the earlier searching stage. But it tends to wander around the true solution at the final stage. The SA, on the other hand, is a one point searching scheme and suitable to locate the final solution[11]. Therefore the schemes are made in such a way that the MGA is switched to the SA to locate the final solution after a solution of pre-specified quality is obtained (hybrid MGA-SA).

The MGA-SA is applied to determine the weighting functions of Fig 5. The reactor plant of 90% is used as before. The weighting functions W_1 and W_3 are assumed as a first order lead/lag

circuit, respectively, and W_2 as a constant.

$$\begin{aligned} W_1(s) &= \gamma_1 \frac{1+b_1s}{1+a_1s}, \quad W_2(s) = \gamma_2, \\ W_3(s) &= \gamma_3 \frac{1+b_3s}{1+a_3s} \end{aligned} \quad (9)$$

Then the robust problem is to determine seven parameters of $[\gamma_1, \gamma_2, \gamma_3, a_1, b_1, a_3, b_3]$ which yield the best fitness. The multiplication of the reactor power deviation, rod speed and acceleration are referred for the fitness. The system fitness, or cost function, can be defined arbitrarily at designer's convenience, which permits the flexibility and practicability. This is due to the independency of the GA cost from the governing equations of the system.

In terms of the GA, the problem is set forth as

Find $C = [c_1, d_1, c_2, d_2, \dots, c_7, d_7]$

To Maximize $\text{fitness}(X) = 1/\text{Cost}(X)$

Subject to

$$\begin{aligned} \text{Cost}(X) &= \sum_{k=0}^N |y(k) - y_o| * u(k) * a(k) \\ X &= [\gamma_1, \gamma_2, \gamma_3, a_1, b_1, a_3, b_3] = X(x_i) \\ N &= \frac{(\text{final time}, T_f)}{(\text{sampling period}, \Delta T)} \\ x_i &= c_i \cdot e^{d_i}, \quad i=1, 2, \dots, 7 \\ 1.00 \leq c_i \leq 9.99, \quad i=1, 2, \dots, 7 \\ -4 \leq d_i \leq 5, \quad i=1, 2, \dots, 7 \end{aligned} \quad (10)$$

All the searching module of the hybrid MGA-SA are implemented in the C language. The H_o algorithms written in MATLAB[12] are called to calculate the cost through the interfacing programs. The limits of the overshooting and the rod speed of the FSAR are applied to evaluate the cost function. And the maximum rod acceleration is limited to 0.5cm/sec². After 50 generations in MGA, the SA finally locates the weighting functions as follows.

$$\begin{aligned} W_1(s) &= 14 \frac{1+0.34s}{1+45000s}, \quad W_2(s) = 0.078, \\ W_3(s) &= 0.0036 \frac{1+4.7s}{1+0.071s} \end{aligned} \quad (11)$$

And the controller is calculated as

$$\begin{aligned} K(s) &= \frac{0.06146s^6 + 25.84s^5 + 440.9s^4}{(s^7 + 426.2s^6 + 9601s^5 + 3.01 \times 10^4 s^4} \\ &\quad + \frac{1313s^3 + 822.9s^2 + 42.31s}{+ 2.01 \times 10^4 s^3 + 3323s^2 + 154s + 0.003421} \end{aligned} \quad (12)$$

Figures 8-1 through 8-3 show the time responses of the reactor power, rod speed and rod acceleration, respectively, of the resultant system. The reactor power converges to the full power more rapidly than that of Fig. 7-1. However, the rapid system requires a larger control effort, which is confirmed by comparison with Fig. 7. The rod speed is larger than the previous result. But the acceleration, whose maximum value is 0.44 cm/sec², is much smaller, which means the milder actuator operation.

The convergence of the parameters of $[\gamma_1, \gamma_2, \gamma_3, a_1, b_1, a_3, b_3]$ as the generation proceeds is described in Fig. 9. As shown in the figure, there are three notable changes. The parameters change rapidly at the initial stage which reflects the efficiency of the GA in multi-point searching. Over the several generations, they show small variation. Around the 12th generation, they change to new values owing to mutation algorithm. At the 36th generation they are converged to the final global solution. To describe the betterment qualitatively, the relation between the improvement and generation is presented in Fig. 10. The improvement is defined as the ratio of fitness to the reference. And the fitness of Fig. 7 is used as the reference for the comparison. The overall trend matches the parameter

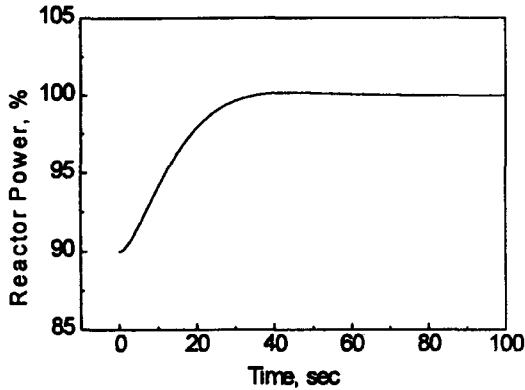


Fig. 8-1. Reactor Power - by GA Method

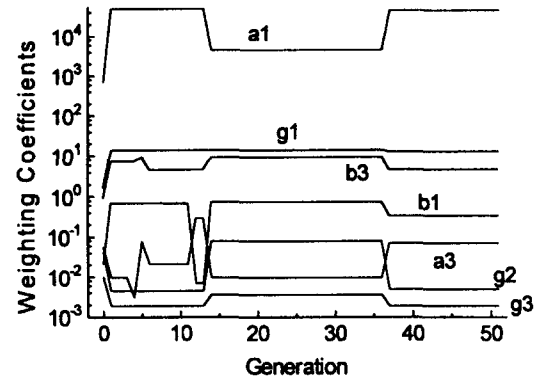


Fig. 9. Convergence of Parameters

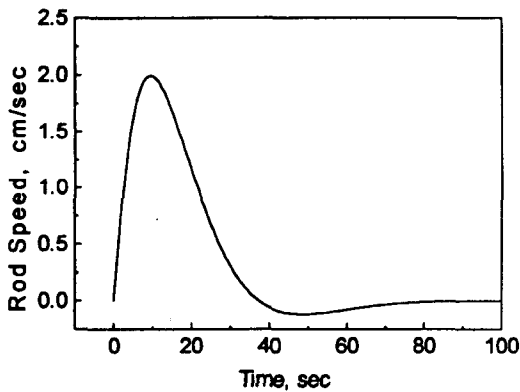


Fig. 8-2. Rod Velocity - by GA Method

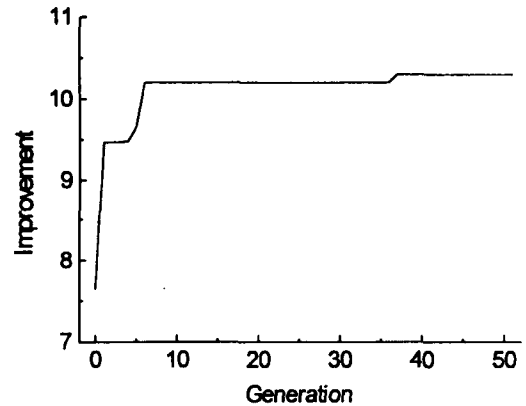


Fig. 10. Improvement vs. Generation

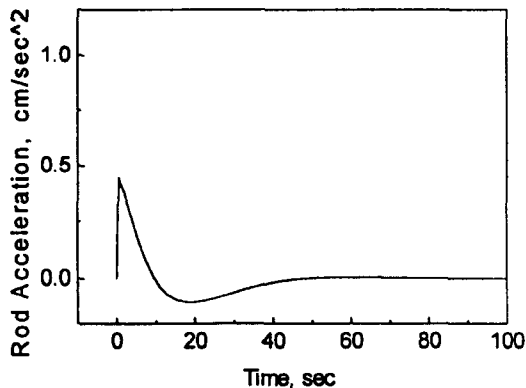


Fig. 8-3. Rod Acceleration - by GA Method

convergence of Fig. 9. The improvement increases rapidly at the initial stage, and increases further with the parameter changes. This illustrates the fact the GA search proceeds in the optimal direction owing to the heredity of good attributes.

The controller of Eq.(12) is of 7th order, and the coefficients have wide ranges. This means that the controller is impractical in implementation, and the setting problem may arise. Hence the controller order is reduced to the 4th order by truncation of the insignificant Hankel singular values. The reduced controller is

$$K(s) = \frac{0.062s^3 + 0.92s^2 + 0.73s + 0.04}{s^4 + 20.9s^3 + 17.1s^2 + 2.96s + 0.14} \quad (13)$$

Table 1. System Margins

Power Level, (%)	Gain Margin, (dB)	Phase Margin (degree)
90	97.0	71.3
80	96.9	68.5
70	96.9	68.5
60	96.8	65.2
50	96.5	57.1
40	96.4	52.0
30	96.3	45.9
20	96.2	38.7
10	96.1	29.9

The frequency responses of Eq.(12) and (13) are exactly the same each other. The designed controller has the gain margin of 97 dB, and phase margin of 71.3°.

The reactor plant is the function of the power. As the power decreases, the dominant pole approaches to the origin, and the reactor becomes more difficult to control. To estimate the robustness, the margins of the system which consists of the power dependent reactor plant and the controller are calculated as in Table 1. The controller is designed with the 90% reactor plant. Hence the reactor plants of other than 90% can be regarded as perturbed plants from the controller's view point and the larger the power differs from the 90%, the larger the degree of perturbation or uncertainty becomes. Table 1 shows that the designed controller provides the system with a sufficient robustness. The gain margins are almost constant and even the largely perturbed plant of 20% rated reactor has the phase margin of about 30°. Also the simulations show that all the perturbed plants up to 20% satisfy the requirements on the overshooting, rod speed and acceleration.

5. Conclusions

The control system design starts from the plant

modeling. But the modeled plant is different from the real one due to the simplification, operating conditions and so on. These uncertainties should be considered to keep the robustness so that the designed system works as intended in the actual situation. The H_∞ control is an useful method, and one of its variation, the mixed weight sensitivity, is employed for the robust design of the reactor power control system. In the frame of mixed weight sensitivity, the design problem is to determine the proper weighting functions which give the desirable system loop shaping. But because of the complexity of the system equations and the non-convexity of the problem, it is very difficult to determine the proper weighting functions, and the design may be finished without knowing the existence of the better solution.

The GA algorithm which is modified for improvement in algorithms and hybridized with the SA is employed to eliminate this problem. The GA searches the best solution throughout the global solution space without trapping in the local solutions. The GA also provide the automatic design procedure, which removes the laborious and blind-wise iteration. The designed system has sufficient margins for the robustness, and the simulation shows good performances.

The design parameter of the GA-applied approach is a cost function. Depending on how to define the cost function, the results may be different. Once the problem-specific cost function is defined, the GA applied mixed weight sensitivity method results in the best solution in an automatic procedure.

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