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**<Technical Note>**

# Stress Analysis of Top Hat Type Structure for Random Loading

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## Abstract

To resolve several arguments raised for the current analysis of a structure like top hat, which is composed of flange, cylinder and plate, the dynamic response analysis is performed for the full and half models. The dynamic characteristics are investigated for full and half models and the results are compared between them. The responses such as bolt reactions and stresses due to random loading are also obtained using the analysis capabilities between commercial programs which have the routine for the random vibration analysis. Several general purpose structural analysis programs are used to get the response due to the random loadings. Also the application of the random loading and the effect of correlations such as fully correlated, partially correlated and fully uncorrelated cases are studied and the general directions for the generation of design loads due to random loading are suggested.

## 1. Introduction

The top hat assembly which is also called guide structure support system is one of very important structure of the reactor vessel internals. It is located on the upper part of the reactor vessel internals and is attached to the upper flange of upper guide structure by bolts. It provides the guidance for the control element assembly (CEA) extension shafts into the closure head nozzles when the closure head is being lowered onto the reactor vessel. It also provides lateral support for the CEAs when the CEAs are lowered to the upper guide structure laydown area floor.

The current analysis used half symmetric model to represent the structure like top hat for the response evaluation. For this point several arguments were raised about this model which may not show all modes. Especially if the asymmetric loadings are imposed on the structure the response between full model and half model will be different. To resolve above problem for the future nuclear power plant design, the full and half models are developed and the dynamic characteristics are compared between them. The responses such as bolt reactions and stresses are also compared. Another issue is the difference of analysis capabilities between commercial programs with random analysis capabilities and between different types of correlations such as fully correlated, partially correlated and fully uncorrelated cases. Several general purpose structural analysis programs like STARDYNE [1], ANSYS [2] and NISA II [3] are used to get the response due to the random loadings. The responses between them are compared and the general directions for the future nuclear power plant design are suggested.

## 2. Random Vibration Method

### 2.1. Description of the Method

The theory implemented in the ANSYS code [2] is described here to show the general methodology for random analysis. For partially correlated nodal and base excitations, the complete equations of motions are segregated into the free and the restrained degree of freedom (DOF) as :

$$\begin{bmatrix} [M_{ff}] & [M_{fr}] \\ [M_{rf}] & [M_{rr}] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}_f\} \\ \{\ddot{u}_r\} \end{Bmatrix} + \begin{bmatrix} [C_{ff}] & [C_{fr}] \\ [C_{rf}] & [C_{rr}] \end{bmatrix} \begin{Bmatrix} \{\dot{u}_f\} \\ \{\dot{u}_r\} \end{Bmatrix} + \begin{bmatrix} [K_{ff}] & [K_{fr}] \\ [K_{rf}] & [K_{rr}] \end{bmatrix} \begin{Bmatrix} \{u_f\} \\ \{u_r\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{0\} \end{Bmatrix} \quad (1)$$

where $\{u_f\}$ are the free DOF and $\{u_r\}$ are the restrained DOF (excited by random loading, i.e., those with unit value of displacement). Note that the restrained DOF that are not excited are not included in equation (1). $\{F\}$ is the nodal force excitation activated by a value of force. The value of force can be other than unity, allowing for scaling of the unit PSD (power spectral density). The free displacements can be decomposed into pseudo-static and dynamic parts as :

$$\{u_f\} = \{u_s\} + \{u_d\} \quad (2)$$

The pseudo-static displacements may be obtained from equation (1) by excluding the first two terms on the left-hand side of the equation and by replacing $\{u_f\}$ by $\{u_s\}$ :

$$\{u_s\} = -[K_{ff}]^{-1}[K_{fr}]\{u_r\} = [A]\{u_r\} \quad (3)$$

in which $[A] = -[K_{ff}]^{-1}[K_{fr}]$. Physically, the elements along the ith column of $[A]$ are the pseudo-static displacements due to a unit displacement of the support DOFs excited by the ith base PSD. Substituting equations (3) and (2) into (1) and assuming light damping yields :

$$[M_{ff}]\{\ddot{u}_d\} + [C_{ff}]\{\dot{u}_d\} + [K_{ff}]\{u_d\} \cong \{F\} - ([M_{ff}][A] + [M_{fr}])\{\ddot{u}_r\}) \quad (4)$$

The second term on the right-hand side of the above equation represents the equivalent forces due to support excitations.

Introducing the mode superposition method as :

$$\{u_d(t)\} = [\phi]\{y(t)\} \quad (5)$$

equation (4) is decoupled yielding :

$$\ddot{y}_j + 2\xi_j\omega_j\dot{y}_j + \omega^2 y_j = G_j, \ (j = 1, 2, \ldots, n) \quad (6)$$

where $n$ = number of mode shapes chosen for evaluation
$y_j$ = generalized displacements
$\omega_j$ and $\xi_j$ = natural circular frequencies and modal damping ratios.

The modal loads $G_j$ are defined by

$$G_j = \{\Gamma_j\}^T\{\ddot{u}_r\} + \gamma_j \quad (7)$$

The modal participation factors corresponding to support excitation are given by

$$\{\Gamma_j\} = -([M_{ff}][A] + [M_{fr}])^T\{\phi_j\} \quad (8)$$

and for nodal excitation

$$\gamma_j = -[\phi_j]^T\{F\} \quad (9)$$

## 2.2. Response PSD and Mean Square Resp-onse

Using the theory of random vibrations, the response PSD's can be computed from the input PSD's with the help of transfer functions for single DOF systems $H(\omega)$ and mode superposition. The response PSD' s for ith DOF are given by :

*Dynamic Part :*

$$S_{d_i}(\omega)=\sum_{j=1}^{n}\sum_{k=1}^{n}\phi_{ij}\phi_{ik}\left(\sum_{l=1}^{r_1}\sum_{m=1}^{r_1}\gamma_{lj}\gamma_{mk}H_j^*(\omega)H_k(\omega)\bar{S}_{lm}(\omega)+\sum_{l=1}^{r_2}\sum_{m=1}^{r_2}\Gamma_{lj}\Gamma_{mk}H_j^*(\omega)H_k(\omega)\hat{S}_{lm}(\omega)\right) \quad (10)$$

*Pseudo-static Part :*

$$S_{s_i}(\omega)=\sum_{l=1}^{r_2}\sum_{m=1}^{r_2}A_{il}A_{im}\left(\frac{1}{\omega^4}\hat{S}_{lm}(\omega)\right) \quad (11)$$

*Covariance Part :*

$$S_{sd_i}(\omega)=\sum_{j=1}^{n}\sum_{l=1}^{r_2}\sum_{m=1}^{r_2}\phi_{ij}A_{il}\left(-\frac{1}{\omega^2}\Gamma_{mj}H_j(\omega)\hat{S}_{lm}(\omega)\right) \quad (12)$$

where n = number of mode shapes chosen for evaluation

$r_1$ and $r_2$ = number of nodal and base PSD tables, respectively.

The transfer functions for the single DOF system assume different forms depending on the type of the input PSD and the type of response desired. The forms of the transfer functions for displacements as the output are listed below for different inputs.

*For a force or acceleration input PSD :*

$$H_j(\omega)=\frac{1}{\omega_j^2-\omega^2+i(2\xi_j\omega_j\omega)} \quad (13)$$

*For a displacement input PSD :*

$$H_j(\omega)=\frac{\omega^2}{\omega_j^2-\omega^2+i(2\xi_j\omega_j\omega)} \quad (14)$$

*For a velocity input PSD :*

$$H_j(\omega)=\frac{i\omega}{\omega_j^2-\omega^2+i(2\xi_j\omega_j\omega)} \quad (15)$$

where $\omega$= forcing frequency

$\omega_j$ = natural frequency for jth mode

$i$ = $-1$

Now, random vibration analysis can be used to show that the absolute value of the mean square response of the ith free displacement is

$$\sigma_{f_i}^2=\int_0^\infty S_{d_i}(\omega)d\omega+\int_0^\infty S_{s_i}(\omega)d\omega+2\left|\int_0^\infty S_{sd_i}(\omega)d\omega\right|_{Re}$$

$$=\sigma_{d_i}^2+\sigma_{s_i}^2+2C_v(u_{s_i},u_{d_i}) \quad (16)$$

where $| \ |_{Re}$ = denotes the real part of the argument

$\sigma_{d_i}^2$ = variance of the ith relative free displacements

$\sigma_{s_i}^2$ = variance of the ith pseudo-static displacements

$C_v(u_{s_i}, u_{d_i}^2)$ = covariance between the static and dynamic displacements.

The general formulation described above gives simplified equations for several situations commonly encountered in practice. For fully correlated nodal excitations and identical support motions, the subscripts $l$ and $m$ would drop out from the equations (10) through (12). When only nodal excitations exist, the last two terms in equation (16) do not apply, and only the first term within the large parentheses in equation (10) needs to be evaluated. For uncorrelated nodal force and base excitations, the cross PSD' s (for which $l$ = $m$) are zero, and only the terms for which $l$ = $m$ in equations (10) through (12) need to be considered, which can be written

$$S_{d_i}(\omega)=\sum_{j=1}^{n}\sum_{k=1}^{n}\phi_{ij}\phi_{ik}R_{jk}(\omega) \quad (17)$$

$$S_{s_i}(\omega)=\sum_{l=1}^{r_2}\sum_{m=1}^{r_2}A_{il}A_{im}\bar{R}_{lm}(\omega) \quad (18)$$

$$S_{sd_i}(\omega)=\sum_{j=1}^{n}\sum_{l=1}^{r_2}\phi_{ij}A_{il}\hat{R}_{jl}(\omega) \qquad (19)$$

where $R_{jk}(\omega)$, $\bar{R}_{lm}(\omega)$, $\hat{R}_{jl}(\omega)$, = modal PSD's terms within large parentheses of equations (10) through (12). After integration, the variances become

$$\sigma_{d_i}^2=\sum_{j=1}^{n}\sum_{k=1}^{n}\phi_{ij}\phi_{ik}Q_{jk} \qquad (20)$$

$$\sigma_{s_i}^2=\sum_{l=1}^{r_2}\sum_{m=1}^{r_2}A_{il}A_{im}\bar{Q}_{lm} \qquad (21)$$

$$\sigma_{sd_i}^2=\sum_{j=1}^{n}\sum_{l=1}^{r_2}\phi_{ij}A_{il}\hat{Q}_{jl} \qquad (22)$$

where $Q_{jk}$, $\bar{Q}_{lm}$, $\hat{Q}_{jl}$ = modal covariance matrices. The variance for stresses, nodal forces or reactions can be computed from equations similar to (20) through (22). If the stress variance is desired, replace the mode shapes ($\phi_{ij}$)and static displace-ments ($A_{il}$) with mode stresses ($\phi_{ij}$) and static stress-es ($\bar{A}_{il}$). Similarly, if the node force ($\hat{\phi}_{ij}$)variance is desired, replace the mode shapes and static displacements with mode nodal forces ($\hat{\phi}_{ij}$) and static nodal forces ($\hat{A}_{ij}$). Finally, if reaction variances are desired, replace mode shapes and static displacements with mode reaction ($\tilde{\phi}_{ij}$) and static reactions ($\tilde{A}_{ij}$)

## 3. Analysis

### 3.1. Finite Element Model

The finite element models of the top hat structure are made for the full and half models. The model configuration is shown in Figure 1 for full model. The bolt locations are fixed for all six degrees of freedom and the symmetric boundary conditions are imposed for the half symmetric model.

### 3.2. Modal Analysis

The finite element models of the top hat

**Table 1. Comparison of Frequencies Between Full and Half Models**

| Mode no. | Full model | | | Half model | | | comments |
|---|---|---|---|---|---|---|---|
| ($\theta$, r) | ANSYS | STARDYNE | Mode | ANSYS | STARDYNE | Mode | |
| (0,0) | 12.6 | 12.7 | (1) | 12.6 | 12.7 | (1) | |
| (0,1) | 26.2 | 26.4 | (2,3) | 26.2 | 26.4 | (2) | |
| (0,2) | 43.2-43.5 | 43.6-43.8 | (4,5) | 43.5 | 43.6 | (3) | |
| (1,0) | 47.9 | 48.3 | (6) | 47.9 | 48.3 | (4) | |
| (1,1) | 55.1 | 56.8 | (7,8) | 55.1 | 56.8 | (5) | 1) |
| (0,3) | 64.2 | 64.4 | (9,10) | 64.3 | 64.4 | (6) | |
| (2,0) | 74.0 | 75.6 | (11) | 74.1 | 75.6 | (7) | |
| (1,1) | 75.9-76.0 | 75.6-76.2 | (12,13) | 75.8 | 76.2 | (8) | |
| (0,4) | 86.2 | 85.7 | (14) | N/A* | N/A | | 1) |
| | 88.5 | 88.5 | (15) | 88.3 | 88.4 | (9) | |
| | 92.9 | 92.3 | (16) | N/A | N/A | | 1) |
| (0,5) | 99.3 | 98.4 | (18) | 99.2 | 98.3 | (10) | |
| (1,2) | 99.2-99.3 | 98.3-99.2 | (17,19) | N/A | N/A | | 1) |
| | 100.2 | 99.5 | (20) | 99.9 | 99.2 | (11) | |
| (2,1) | 102.3-102.4 | 102.5 | (21,22) | 102.2 | 102.5 | (12) | |
| (1,5) | 116.1-116.5 | 112.9 | (23,24) | 115.6 | 112.9 | (13) | |

1) flange coupling mode. * N/A : not appeared.

structure are made using the ANSYS code and a frequency analysis of the top hat structure is performed to get the eigenvalues and eigenvectors for the full and half models. Several different codes are used and the results are compared with each other.

A modal analysis of the top hat structure is carried out to calculate the natural frequencies and modeshapes. For ANSYS run, the reduced method is used for the mode extraction where the HBI algorithm (Householder - Bisection - Inverse iteration) is applied for the calculation of the eigenvalues and eigenvectors. The master degrees of freedom were selected enough not to have missing modes. For STARDYNE run, Lanczos extraction method is used. This method is an iterative process which extracts the requested number of modes from the full size mathematical model and associated degrees of freedom.

The frequencies obtained are shown in Table 1

**Table 2. Comparison of Bolt Reactions for Base Excitation.**

| Halfmodel | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Node | ANSYS | | | | | STARDYNE | | | | |
| no | Fx | Fy | Fz | Mx | My | Fx | Fy | Fz | Mx | My |
| 501 | 99.4 | 55.0 | 236.7 | 242.1 | 659.0 | 104.0 | 55.1 | 260.9 | 487.5 | 681.2 |
| 507 | 471.6 | 286.1 | 335.3 | 765.2 | 726.3 | 484.6 | 290.9 | 368.7 | 750.8 | 611.1 |
| 513 | 760.0 | 257.6 | 15.4 | 18.9 | 105.9 | 771.6 | 0.0 | 0.0 | 0.0 | 145.6 |
| 519 | 471.3 | 285.1 | 335.7 | 766.2 | 727.4 | 484.7 | 291.0 | 368.7 | 750.9 | 611.0 |
| 525 | 99.1 | 54.8 | 237.1 | 242.6 | 660.1 | 104.1 | 55.1 | 261.0 | 487.6 | 681.3 |
| Total | 1901.4 | 938.7 | 1160.3 | 2035.0 | 2878.7 | 1949.0 | 692.1 | 1259.3 | 2476.8 | 2730.2 |
| Full model | | | | | | | | | | |
| Node | NSYS | | | | | STARDYNE | | | | |
| no | Fx | Fy | Fz | Mx | My | Fx | Fy | Fz | Mx | My |
| 501 | 198.7 | 122.8 | 472.0 | 476.2 | 1320.1 | 210.0 | 0.0 | 522.0 | 0.0 | 1375.8 |
| 507 | 474.6 | 287.7 | 335.2 | 764.8 | 726.6 | 478.4 | 282.3 | 368.9 | 755.4 | 619.9 |
| 513 | 755.0 | 255.9 | 20.9 | 68.2 | 98.3 | 757.0 | 0.0 | 0.0 | 0.0 | 140.0 |
| 519 | 476.7 | 292.8 | 336.1 | 767.0 | 728.7 | 478.3 | 282.2 | 368.9 | 755.4 | 619.8 |
| 525 | 199.1 | 130.9 | 473.3 | 477.9 | 1324.2 | 210.0 | 0.0 | 522.1 | 0.0 | 1375.9 |
| 531 | 466.4 | 281.8 | 335.1 | 764.6 | 727.4 | 478.5 | 282.4 | 368.8 | 755.5 | 619.7 |
| 537 | 762.1 | 257.5 | 25.2 | 67.2 | 103.7 | 756.8 | 0.0 | 0.0 | 0.0 | 139.8 |
| 543 | 469.1 | 285.0 | 333.2 | 759.6 | 722.6 | 478.5 | 282.3 | 368.8 | 755.4 | 619.7 |
| Total | 3801.7 | 1914.4 | 2330.9 | 4145.6 | 5751.7 | 3847.5 | 1129.2 | 2519.5 | 3021.7 | 5510.6 |
| Node | NISA | | | | | | | | | |
| no | Fx | Fy | Fz | Mx | My | | | | | |
| 501 | 240.7 | 1.5 | 569.3 | 0.3 | 1470.7 | | | | | |
| 507 | 558.8 | 340.3 | 406.3 | 808.2 | 668.9 | | | | | |
| 513 | 883.3 | 0.5 | 1.0 | 2.2 | 150.7 | | | | | |
| 519 | 559.5 | 340.9 | 406.4 | 808.4 | 668.8 | | | | | |
| 525 | 240.7 | 1.2 | 569.6 | 0.2 | 1471.1 | | | | | |
| 531 | 559.2 | 340.7 | 406.4 | 808.5 | 668.9 | | | | | |
| 537 | 883.0 | 0.4 | 0.7 | 1.4 | 150.5 | | | | | |
| 543 | 559.6 | 341.0 | 406.4 | 808.4 | 668.9 | | | | | |
| Total | 4484.8 | 1366.5 | 2519.5 | 3237.6 | 5918.5 | | | | | |

**Table 3. Comparison of Stress Intensities for Base Excitation (unit = psi).**

| Component | Location | ANSYS | | STARDYNE | | NISA |
|---|---|---|---|---|---|---|
| | | Half | Full | Half | Full | Full |
| Assembly | Top | 134.0(302.1) | 134.5(303.0) | 36.9 | 37.0 | 50.2 |
| | Bot. | 135.0(290.9) | 135.6(291.9) | 58.9 | 59.3 | 43.0 |
| Plate | Top | 23.7(36.6) | 24.3(35.9) | 23.1 | 23.1 | 33.3 |
| | Bot. | 22.9(35.9) | 23.9(36.3) | 23.5 | 23.4 | 33.9 |
| Cylinder | Top | 31.6(62.0) | 31.6(61.9) | 25.7 | 26.2 | 43.2 |
| | Bot. | 51.2 (115.1) | 51.3(115.1) | 58.9 | 59.3 | 51.5 |

( ) obtained from post-processing plot of the ANSYS run using commands as follows ;

```
/post1
  :
esel,elem,···
top
plesol,s,int
```

**Table 4. Comparison of Bolt Reactions for Pressure Excitation in Axial Direction.**

| Halfmodel | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Node | Full Correlated | | | | | Fully Uncorrelated | | | | |
| no | Fx | Fy | Fz | Mx | My | Fx | Fy | Fz | Mx | My |
| 501 | 35.7 | 15.3 | 531.6 | 978.6 | 1387.1 | 10.4 | 4.6 | 86.7 | 159.7 | 225.1 |
| 507 | 50.9 | 51.0 | 1063.2 | 1961.2 | 1961.3 | 22.1 | 22.3 | 160.1 | 299.6 | 292.4 |
| 513 | 0.0 | 71.4 | 1063.4 | 2774.4 | 0.0 | 31.1 | 13.1 | 150.4 | 391.4 | 25.6 |
| 519 | 50.9 | 50.9 | 1063.4 | 1961.5 | 1961.4 | 22.0 | 22.3 | 160.2 | 299.7 | 292.4 |
| 525 | 35.7 | 15.3 | 531.7 | 978.8 | 1387.3 | 10.5 | 4.7 | 86.7 | 159.8 | 225.2 |
| Total | 173.2 | 203.9 | 4253.3 | 8654.5 | 6697.1 | 96.1 | 67.0 | 644.1 | 1310.2 | 1060.7 |
| Full model | | | | | | | | | | |
| Node | Full Correlated | | | | | Fully Uncorrelated | | | | |
| no | Fx | Fy | Fz | Mx | My | Fx | Fy | Fz | Mx | My |
| 501 | 71.1 | 0.0 | 1065.5 | 0.0 | 2779.4 | 12.9 | 31.5 | 129.2 | 23.1 | 335.4 |
| 507 | 50.6 | 50.6 | 1065.4 | 1964.9 | 1964.9 | 24.5 | 24.5 | 128.6 | 237.0 | 237.0 |
| 513 | 0.0 | 71.0 | 1065.5 | 2779.5 | 0.0 | 31.5 | 12.9 | 129.3 | 335.5 | 23.0 |
| 519 | 50.6 | 50.6 | 1065.4 | 1965.0 | 1965.0 | 24.5 | 24.5 | 128.6 | 237.0 | 237.0 |
| 525 | 71.1 | 0.0 | 1065.6 | 0.0 | 2779.7 | 12.9 | 31.5 | 129.3 | 23.1 | 335.4 |
| 531 | 50.6 | 50.6 | 1065.5 | 1965.1 | 1965.1 | 24.5 | 24.5 | 128.7 | 237.0 | 237.1 |
| 537 | 0.0 | 71.1 | 1065.6 | 2779.7 | 0.0 | 31.5 | 12.9 | 129.3 | 335.5 | 23.1 |
| 543 | 50.6 | 50.6 | 1065.4 | 1965.0 | 1965.0 | 24.5 | 24.5 | 128.7 | 237.0 | 237.1 |
| Total | 344.6 | 344.5 | 8523.9 | 13419.2 | 13419.1 | 186.8 | 186.8 | 1031.7 | 1665.2 | 1665.1 |

for full and half models. There is a good agreement between full and half model even though several flange modes do not appear in the half symmetric model for the high frequency range. But it is not important because the mode participation factors for those modes are very low and is negligible from the response point of view. Modeshape contour plots show a good agreement between full and half models.

### 3.3. Random Analysis

Using the modal analysis information, response analysis of the top hat structure for random excitations is performed to get the bolt reactions and stresses for the full and half models. Several types of forcing functions and different codes are used and the results are compared with each other.

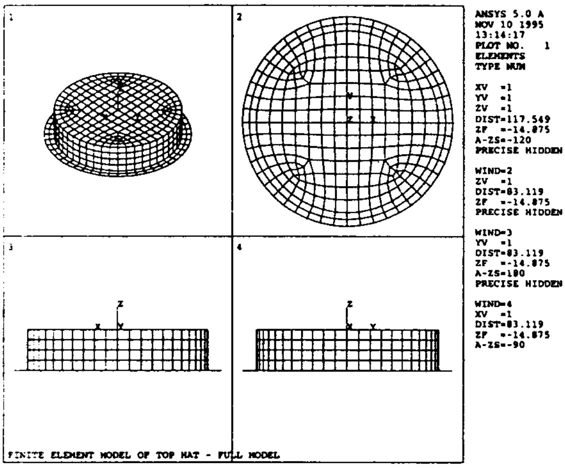

**Fig. 1. Finite Element Model of Top Hat Structure**

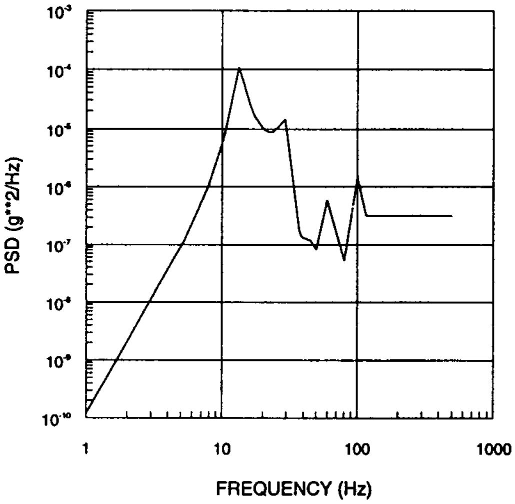


**Fig. 2. Base Excitation Power Spectral Density**

#### 3.3.1. Base Excitation

The base excitations are applied as a form of PSD at the bolt locations (Figure 2). They are applied to the model in a fully correlated fashion. The PSD is input up to 130 Hz which is enough to cover all important modes.

Bolt reactions and stress intensities are summarized in Tables 2 and 3. As shown in Table 2, there is a good agreement between full and half models for bolt reactions but each code generates a little different reactions. STARDYNE and NISA generate almost the same reactions which are different from those of ANSYS. This is not a problem for the designer' s point of view because the design bolt reactions are calculated using the maximum of all bolts for shear, axial force and moment. Stress intensities of the plate are the same between codes but the cylinder stresses show a big difference and are not acceptable for different codes and should be investigated further even though they are not used by designer. Even though it needs more work to draw a general result, the same stresses for plate are assumed to be obtained from different codes. Therefore in the pressure excitation analysis only STARDYNE runs were made for several cases.

#### 3.3.2. Pressure Excitation

The pressure PSDs are applied to the plate and cylinder (Figure 3) in various correlated cases. Since the area in each node has three different directions upon which the PSD acts, these areas

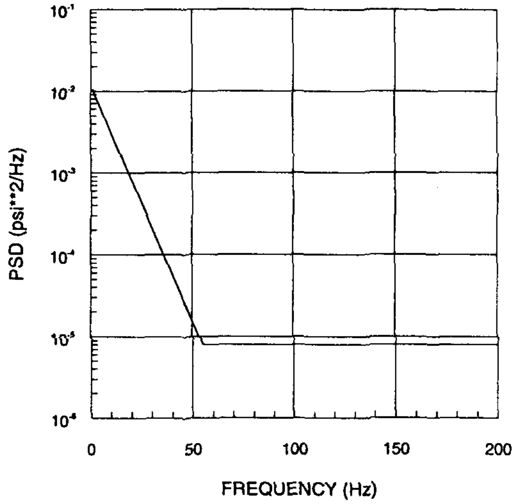


**Fig. 3. Pressure Power Spectral Density**

projected in the x, y and z directions are generated from the nodal weights which is available in modal analysis run. The following equations are used to calculate the projected areas ;

$$A_x = \frac{\omega_n}{\rho_{cyl} t_{cyl}} \cos\theta \tag{23}$$

$$A_y = \frac{\omega_n}{\rho_{cyl} t_{cyl}} \sin\theta \tag{24}$$

$$A_z = \frac{\omega_n}{\rho_{pl} t_{pl}} \tag{25}$$

where $\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$, $\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$

and $\omega_n$ : nodal weight,
$\rho_{cyl}$ : density of the cylinder,
$\rho_{pl}$ : density of the plate,
$t_{cyl}$ : thickness of the cylinder,
$t_{pl}$ : thickness of the plate.

No projected area exists for a node on the plate ($z = 0$) except for $A_x$. For a node on the cylinder, no projected area exists for the $z$ direction.

*Axial Direction Loading*

The pressure PSDs are applied to the plate in the axial direction in fully correlated, partially correlated and fully uncorrelated fashions. The bolt reactions and stress intensities are summarized in Tables 4 through 6. As anticipated, fully correlated case generates the highest response, which is a general trend for random response analysis.

*Horizontal Direction Loading*

The pressure PSDs are applied to the cylinder in the horizontal direction in fully correlated, fully

**Table 5. Comparison of Bolt Reactions for Pressure Excitation in Axial Direction (Partially Correlated Case).**

| Node | Full Correlated | | | | | Fully Uncorrelated | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| no | Fx | Fy | Fz | Mx | My | Fx | Fy | Fz | Mx | My |
| 501 | 35.7 | 15.3 | 531.6 | 978.6 | 1387.1 | 10.4 | 4.6 | 86.7 | 159.7 | 225.1 |
| 501 | 33.5 | 12.8 | 361.9 | 666.2 | 942.2 | 52.3 | 91.4 | 579.9 | 89.4 | 1507.0 |
| 507 | 60.5 | 64.0 | 678.0 | 1264.2 | 1236.7 | 74.5 | 74.5 | 580.2 | 1067.6 | 1067.5 |
| 513 | 75.7 | 46.6 | 636.9 | 1659.2 | 90.6 | 91.3 | 52.3 | 579.8 | 1506.7 | 89.4 |
| 519 | 60.3 | 64.1 | 678.0 | 1264.2 | 1236.7 | 74.5 | 74.5 | 580.2 | 1067.5 | 1067.6 |
| 525 | 33.6 | 12.9 | 361.8 | 666.1 | 942.1 | 52.3 | 91.4 | 579.9 | 89.5 | 1507.0 |
| 531 | - | - | - | - | - | 74.4 | 74.5 | 580.2 | 1067.6 | 1067.5 |
| 537 | - | - | - | - | - | 91.3 | 52.3 | 579.8 | 1506.6 | 89.5 |
| 543 | - | - | - | - | - | 74.5 | 74.5 | 580.1 | 1067.5 | 1067.5 |
| Total | 263.6 | 200.4 | 2716.6 | 5519.9 | 4448.3 | 585.1 | 585.4 | 4640.1 | 7462.4 | 7463.0 |

uncorrelated, partially correlated and segment correlated fashions. The bolt reactions and stress intensities are summarized in Tables 7 through 9. As is indicated in Tables 7 and 8, partially correlated case generates the highest bolt reactions following segment correlated, fully correlated, fully uncorrelated case for full and half models. Figure 4, which shows bolt loads for full

**Table 6. Comparison of Stress Intensities for Pressure Excitation in Axial Direction.**

| Component | Half model | | | Full model | | |
|---|---|---|---|---|---|---|
| | Fully Correlated | Fully Uncorrelated | Patrially Correlated | Fully Correlated | Fully Uncorrelated | Patrially Correlated |
| Assembly | | | | | | |
| Top | 723.8 | 101.9 | 433.3 | 724.2 | 78.4 | 356.0 |
| Bot. | 680.7 | 96.2 | 408.8 | 681.1 | 74.5 | 337.7 |
| Plate | | | | | | |
| Top | 723.8 | 101.9 | 433.3 | 724.2 | 78.4 | 356.0 |
| Bot. | 680.7 | 96.2 | 408.8 | 681.1 | 74.5 | 337.7 |
| Cylinder | | | | | | |
| Top | 445.5 | 66.2 | 279.8 | 445.4 | 49.2 | 223.6 |
| Bot. | 422.0 | 62.3 | 263.8 | 421.9 | 46.2 | 210.3 |

(unit = psi)

**Table 7. Comparison of Bolt Reactions for Pressure Excitation in Horizontal Direction.**

| Halfmodel | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Node | Fully Correlated | | | | | Fully Uncorrelated | | | | |
| no | Fx | Fy | Fz | Mx | My | Fx | Fy | Fz | Mx | My |
| 501 | 14.1 | 0.4 | 17.0 | 21.0 | 100.2 | 11.3 | 5.2 | 10.4 | 20.3 | 27.7 |
| 507 | 20.3 | 20.3 | 33.8 | 141.1 | 141.1 | 50.1 | 55.5 | 14.1 | 32.0 | 29.0 |
| 513 | 0.0 | 28.2 | 34.0 | 200.3 | 0.2 | 67.3 | 14.8 | 10.0 | 34.7 | 15.8 |
| 519 | 20.2 | 20.2 | 33.8 | 141.1 | 141.1 | 50.1 | 55.5 | 14.1 | 32.0 | 29.0 |
| 525 | 14.1 | 0.4 | 17.0 | 21.0 | 100.2 | 11.4 | 5.3 | 10.5 | 20.4 | 27.7 |
| Total | 68.7 | 69.5 | 135.6 | 524.5 | 482.8 | 190.2 | 136.3 | 59.1 | 139.4 | 129.2 |

| Full model | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Node | Fully Correlated | | | | | Fully Uncorrelated | | | | |
| no | Fx | Fy | Fz | Mx | My | Fx | Fy | Fz | Mx | My |
| 501 | 4.2 | 0.0 | 30.2 | 0.0 | 82.0 | 10.3 | 55.3 | 15.6 | 5.8 | 40.6 |
| 507 | 3.0 | 3.1 | 30.2 | 57.9 | 57.9 | 41.6 | 41.6 | 15.9 | 29.7 | 29.7 |
| 513 | 0.0 | 4.2 | 30.2 | 81.9 | 0.0 | 55.3 | 10.3 | 15.6 | 40.6 | 5.8 |
| 519 | 3.0 | 3.1 | 30.2 | 57.9 | 57.9 | 41.6 | 41.6 | 15.9 | 29.7 | 29.7 |
| 525 | 4.2 | 0.0 | 30.2 | 0.0 | 82.0 | 10.3 | 55.3 | 15.6 | 5.8 | 40.6 |
| 531 | 3.0 | 3.1 | 30.2 | 57.9 | 57.9 | 41.6 | 41.6 | 15.9 | 29.7 | 29.7 |
| 537 | 0.0 | 4.2 | 30.2 | 81.9 | 0.0 | 55.3 | 10.3 | 15.6 | 40.6 | 5.8 |
| 543 | 3.1 | 3.0 | 30.2 | 57.9 | 57.9 | 41.6 | 41.6 | 15.9 | 29.7 | 29.7 |
| Total | 20.5 | 20.7 | 241.6 | 395.4 | 395.4 | 297.6 | 297.6 | 126.0 | 211.6 | 211.6 |

**Table 8. Comparison of Bolt Reactions for Pressure Excitation in Horizontal Direction (Partially Correlated Case).**

| Node | Partially Correlated | | | | | Segment Correlated | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Halfmodel** | | | | | | | | | | |
| no | Fx | Fy | Fz | Mx | My | Fx | Fy | Fz | Mx | My |
| 501 | 44.8 | 20.0 | 32.5 | 63.0 | 94.2 | 12.0 | 6.4 | 26.3 | 49.9 | 79.8 |
| 507 | 203.9 | 227.3 | 44.5 | 105.3 | 95.9 | 57.7 | 38.8 | 40.9 | 95.0 | 91.9 |
| 513 | 306.2 | 62.1 | 32.0 | 137.1 | 42.5 | 191.1 | 13.1 | 16.1 | 90.6 | 39.8 |
| 519 | 203.8 | 227.6 | 44.4 | 105.2 | 95.9 | 55.6 | 38.4 | 39.9 | 95.7 | 93.9 |
| 525 | 45.1 | 20.2 | 32.5 | 63.1 | 94.2 | 12.1 | 6.2 | 25.8 | 48.9 | 79.9 |
| Total | 803.8 | 557.2 | 185.9 | 473.7 | 422.7 | 328.5 | 102.9 | 149.0 | 380.1 | 385.3 |
| **Full model** | | | | | | | | | | |
| no | Fx | Fy | Fz | Mx | My | Fx | Fy | Fz | Mx | My |
| 501 | 39.6 | 237.0 | 56.9 | 23.3 | 147.9 | 13.0 | 81.8 | 47.9 | 19.1 | 126.8 |
| 507 | 170.3 | 170.3 | 56.9 | 105.8 | 105.8 | 38.0 | 37.7 | 48.2 | 88.2 | 87.7 |
| 513 | 237.0 | 39.6 | 56.9 | 147.9 | 23.3 | 81.8 | 12.9 | 47.9 | 126.8 | 19.1 |
| 519 | 170.3 | 170.3 | 56.9 | 105.8 | 105.8 | 37.7 | 38.0 | 48.2 | 87.7 | 88.2 |
| 525 | 39.6 | 237.1 | 56.9 | 23.4 | 147.9 | 13.0 | 81.8 | 47.9 | 19.1 | 126.9 |
| 531 | 170.3 | 170.3 | 56.9 | 105.8 | 105.8 | 38.0 | 37.7 | 48.2 | 88.2 | 87.7 |
| 537 | 237.0 | 39.6 | 56.9 | 147.9 | 23.4 | 81.8 | 12.9 | 47.9 | 126.9 | 19.1 |
| 543 | 170.3 | 170.3 | 56.9 | 105.7 | 105.8 | 37.7 | 38.0 | 48.2 | 87.6 | 88.2 |
| Total | 1234.4 | 1234.5 | 455.2 | 765.6 | 765.7 | 341.0 | 340.8 | 384.4 | 643.6 | 643.7 |

model, also indicates that the fully correlated and partially correlated cases give the highest reactions for axial and horizontal loadings, respectively. The differences between half and full models are not clearly clarified but it is assumed that half model gives too much conservative reactions. The stress intensities from Table 9 can be explained for plate and cylinder separately. For plate, fully correlated case gives highest stress intensities following partially correlated, segment correlated and fully uncorrelated cases. For cylinder, partially correlated case gives highest stress intensities

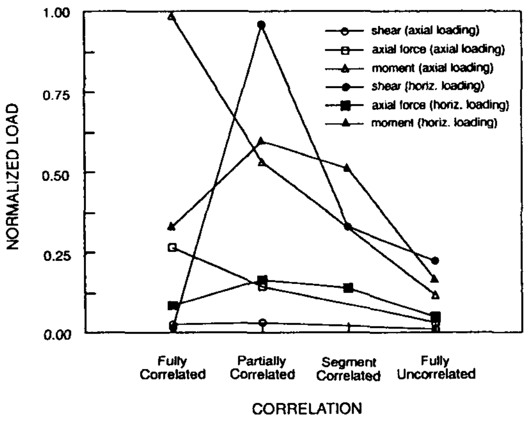


**Fig. 4. Comparison of Bolt Reactions Between Correlations**

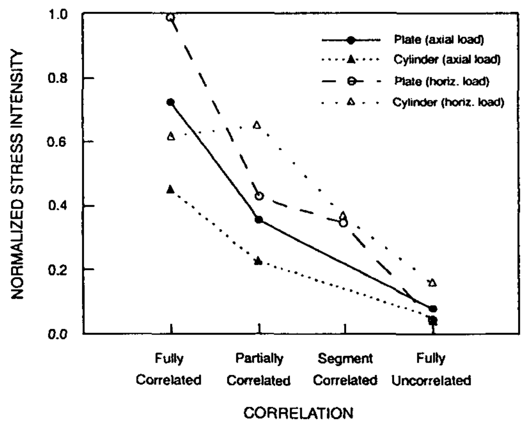


**Fig. 5. Comparison of Stress Intensities Between Correlations**

**Table 9. Comparison of Stress Intensities for Pressure Excitation in Horizontal Direction.**

| Component | Half model | | | | Full model | | | |
|---|---|---|---|---|---|---|---|---|
| | Fully Corr. | Fully Uncor. | Part. Corr. | Segm. Corr. | Fully Corr. | Fully Uncor. | Part. Corr. | Segm. Corr. |
| Assembly | | | | | | | | |
| Top | 13.14 | 3.42 | 13.75 | 9.06 | 12.84 | 2.52 | 10.53 | 5.16 |
| Bot. | 12.43 | 3.68 | 14.76 | 8.37 | 12.24 | 2.62 | 10.97 | 5.69 |
| Plate | | | | | | | | |
| Top | 13.14 | 1.90 | 7.50 | 6.65 | 12.84 | 1.52 | 6.20 | 5.16 |
| Bot. | 12.43 | 2.17 | 8.61 | 6.38 | 12.24 | 1.67 | 7.14 | 5.02 |
| Cylinder | | | | | | | | |
| Top | 8.19 | 2.50 | 10.43 | 6.01 | 7.41 | 1.89 | 7.81 | 4.41 |
| Bot. | 9.72 | 3.24 | 13.20 | 7.27 | 7.03 | 2.13 | 8.87 | 5.69 |

(unit = psi)

following fully correlated, segment correlated and fully uncorrelated cases. It is noticeable that the highest stress intensities in plate and cylinder are obtained for fully correlated case and partially correlated case, respectively (Figure 5). It was anticipated that fully correlated loading generate the highest responses for all case but if it is applied in the horizontal direction it may compensate each other. This is a good explanation for cylinder to have a maximum stress in a partially correlated fashion. As in the case of the bolt reactions, the half model is assumed to generate too much conservative stresses.

## 4. Concluding Remarks

The structure like top hat, which is composed of flange, cylinder and top plate is studied for the dynamic responses due to random loading. The modal characteristics between full and half models are investigated. Random responses such as reactions and stresses are obtained using several commercial computer codes. The following conclusions were reached:

1. Half model shows similar modal characteristics as full model except high frequency range which is negligible.
2. Maximum responses are obtained for the fully correlated case of axial pressure excitation random loading.
3. Fully correlated case generates the maximum responses in plate for horizontal pressure excitation random loading.
4. Partially correlated case generates the maximum responses in cylinder for horizontal pressure excitation random loading.
5. Half model gives almost the same responses as full model for the loadings of base excitation and fully correlated axial direction pressure excitation.
6. The different stresses between computer programs are obtained, sometimes which are significant, and this should be studied in the future.

Based on these conclusions, the followings are suggested for the generation of design load due to random loading :

1. For pressure excitation the fully correlated axial loading and partially correlated horizontal loading are applied to the cylinder, and the fully

correlated axial loading and fully correlated horizontal loading are applied to the plate.

2. Half model of the cylinder and plate may be used for the loading of base excitation to save the computing time.
3. Care should be taken to get the stress values from the post-processing run of the ANSYS code.

## References

1. GMC, STARDYNE User Information Manual, General Microelectronics Corporation, 1987.
2. SASI, ANSYS User' s Manual for Revision 5.0, Swanson Analysis Systems, Inc., February 15, 1994.
3. EMRC, NISA User' s Manual, Engineering Mechanics Research Corp., December 31, 1994.