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Influence of Vapor Phase Turbulent Stress to the Onset of Slugging in a Horizontal Pipe

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기체상의 난류 응력이 수평 유동관 내에서의 Slugging에 미치는 영향에 관한 연구

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Abstract

An influence of the vapor phase turbulent stress (i.e., the two-phase Reynolds stress) to the characteristics of two-phase system in a horizontal pipe has been theoretically investigated. The average two-fluid model has been constituted with closure relations for stratified flow in a horizontal pipe. A vapor phase turbulent stress model for the regular interface geometry has been included. It is found that the second order waves propagate in opposite direction with almost the same speed in the moving frame of reference of the liquid phase velocity. Using the well-posedness limit of the two-phase system, the dispersed-stratified flow regime boundary has been modeled. Two-phase Froude number has been found to be a convenient parameter in quantifying the onset of slugging as a function of the global void fraction. The influence of the vapor phase turbulent stress was foun d to stabilize the flow stratification.

요 약

수평 유로를 통과하는 이상류 내에서 기체상의 난류 응력이 이상류 시스템의 전반적 거동 특성에 미치는 영향을 연구하였다. 이를 위하여 이미 오랜 연구 노력으로 입증된 평균 이상류 거동 방정식을 채택하여 수평관에서 흔히 발생하는 성층류형 이상류에 대하여 적절히 모델하였다. 모델의 고유치를 계산하고 그의 특성 방정식으로 부터 이상류 시스템의 well-posedness 및 다른 이상 유동 영역으로의 천이 경계를 새로이 결정하였다. 그 결과로 지금까지 무시해 왔던 기체상의 난류응력이 이상류 시스템의 안정성에 커다란 영향을 주는 것이 발견되었다. 본 연구에서는 그 영향이 정량화 되었으며 그외의 효과도 정량화 하는 방법을 정립하였다.

본 연구는 원자로 시스템 해석 코드에 쓰이고 있는 열수력 모델 중 불확실성이 많은 모델인 수평관에서의 이상유동 영역 천이 조건식을 개선함으로써 시스템 해석 결과의 신뢰도를 높이는 데 이용할 수 있다.

1. Introduction

Two-phase flows in a horizontal pipe have dramatically different characteristics from those in a vertical pipe since the net gravity, which is normal to the flow direction, can easily induce the phase separation (i.e., the flow stratification). In many industrial applications of horizontal two-phase flow (e.g., the PWR's hot leg, the CANDU reactor coolant channel, steam generator in the VVER type reactor, etc.), the cooling/heating capability change resulting from the flow stratification is very important for safe and economic operations of the equipments.

There have been many works[1], [2], [3], [4], [5], concerning flow regime determination in a horizontal pipe. In most of adiabatic horizontal flow studies, the surface wave instability[1], [2], [5], is known to trigger the flow regime transition (i.e., the stratified-to-dispersed flow transition). In fact, the two-phase system's eigenvalues, which are the propagating speeds of perturbed two-fluid variables, are strongly related with this type of instability. In should be noted that the state-of-the-art nuclear reactor system analysis codes (e.g., RELAP5/MOD3) contain this type of flow regime transition criteria either for stratified-to-dispersed or for dispersed-to-stratified flow transitions. The major part of modeling efforts was in-depth understanding of the phasic interfacial phenomenon. Since, theoretically, the algebraic forces in the momentum balace equation do not influence this type of instability, it is believed that gradient of two-fluid variable could be included in the constitutive models, especially in the interfacial shear stress[5]. On the other hand, the gas phase turbulence is believed to play a very important role in controlling this type of instability[6]. Based upon the average two-fluid model, the stress induced by the turbulent fluctuation has been taken into account to the momentum balance through the so-called two-phase Reynolds stress. It is quite clear that the vapor phase turbulent stress strongly depends on the phasic interface geometry. Recent diabatic horizontal study[4] is based upon the quasi-static assumption. Therefore, the flow regime boundary determined in diabatic study has a different theoretical basis from the other adiabatic approaches.

In this study, the two-phase system's eigenvalues have been obtained using the generally accepted average two-fluid model[6], [7]. The phase change effect has been included in the two-fluid model. It has been found that the eigenvalues obtained from the presented constitutive relations can be consistently reduced to the surface wave stability criteria obtained by the previous worker[1] when the vapor phase turbulent stress is neglected. It is found that the two-phase Froude number is a convenient measure for the instability in the stratified two-phase flow. By using the well-posedness limit, the dispersed-stratified flow regime boundary has been obtained as a function of the void fraction. It is found that the tube diameter should be used instead of the equivalent diameter in characterizing the interfacial pressure difference for a bundle flow (e.g., the CANDU coolant channel). The Reynolds stress of the vapor phase is found to increase the regime of flow stratification. Further modeling effort is necessary to explain this result completely.

This study shows very good possibility in utilizing the average two-fluid model in determining very important flow regime boundary in a horizontal/inclined pipe.

2. Theory

The phasic average continuity equation can be written as

$$\frac{\partial(\alpha_k \rho_k)}{\partial t}+\frac{\partial(\alpha_k \rho_k v_k)}{\partial z}=\Gamma_k \qquad (k=g,f) \tag{1}$$

where Γ_k is the mass exchange due to evaporation or condensation. It is hold that

$$\Gamma_g+\Gamma_f=0 \tag{2}$$

Averaging the local instantaneous momentum equation, the phasic average momentum equation can be obtained as[6], [7] :

$$\frac{\partial(\alpha_k\rho_k v_k)}{\partial t}+\frac{\partial(\alpha_k\rho_k v_k^2)}{\partial z}=\alpha_k\rho_k g\cos\beta-\alpha_k\frac{\partial p_k}{\partial z}$$

$$+\Delta p_{ki}\frac{\partial\alpha_k}{\partial z}+M_{ik}-\tau_i\frac{\partial\alpha_k}{\partial z}$$

$$+\frac{\partial(\alpha_k\tau_k^{Re})}{\partial z}-M_{wk}+\Gamma_k v_{ki} \tag{3}$$

where ρ_k, α_k, p_k, β and τ_k are the phasic density, the volume fraction, the pressure, the angle of inclination of the pipe and the interfacial shear, respectively. Δp_{ki} is the difference between the phasic interface and the phasic average pressure. The interfacial pressure difference (Δp_{ki}) is known to be proportional to the square of the relative velocity for dispersed two-phase flows[8]. However, when the flow is separated, the gravity head across the channel cross section governs the interfacial pressure difference. The phase average pressure for the stratified flow can be given by averaging the local pressure over the corresponding phase volume, that is:

$$p_k=\frac{1}{A_k}\iint_{A_k}p(h)dA_k \tag{4}$$

Since the local pressure in the pipe as shown in Figure 1(a) or (b) can be given by

$$p(h)=p_{ki}+\rho_k g(H-h)\sin\beta,$$

$$k=\begin{cases} g & \text{for } h>H \\ f & \text{for } h<H \end{cases}, \tag{5}$$

we may obtain the interfacial pressure differences by evaluating the integral in Eq.(4) for each phase as:

$$\Delta p_{gi}\equiv p_{gi}-p_g=(\rho_g gD\sin\beta)\left(\frac{\sin^3\theta}{3\pi\alpha_g}-\frac{\cos\theta}{2}\right) \tag{6a}$$

$$\Delta p_{fi}\equiv p_{fi}-p_f=-(\rho_f gD\sin\beta)\left(\frac{\sin^3\theta}{3\pi\alpha_f}+\frac{\cos\theta}{2}\right) \tag{6b}$$

where the angle of stratification (θ) can be related to the void fraction as

$$\pi\alpha_g=\theta-\sin\theta\cos\theta \tag{7}$$

The phasic interfacial pressures (i.e., p_{gi} and p_{fi}) are generally different from each other due to the surface tension. However, it is known that the effect of the surface tension is negligible for normal stratified flows (i.e., $p_{gi}-p_{fi}=2\sigma/R\cong 0$). There should be more rigorous consideration for the surface tension effect when the diameter of the rod in the bundle flow (Figure 1(b)) is relatively large.

As shown in Eqs.(6), the interfacial pressure difference has the void fraction dependency, which is quite different from the same term for the dispersed two-phase flows. For a horizontal bundle flows, D in Eqs.(6) should be the diameter of the flow channel.

M_{ik} is the interfacial momentum exchange which normally includes the interfacial drag and the virtual mass force. In this study, both of these terms are neglected since those are not important for the horizontally separated flows. M_{wk} is the force induced by the wall friction.

The phasic interfacial stress for the separated flow can be modeled as[6]:

$$\tau_i\frac{\partial\alpha_g}{\partial z}=\frac{\xi_i}{A_{x-s}}\tau_{gi} \tag{8}$$

where ξ_i is the wetted perimeter of the vapor phase. τ_{gi} in Eq.(8) should be understood as the interfacial shear force (i.e., the viscous shear) used by previous workers [1], [5].

In this study, the vapor phase turbulent stress is to be included in the two-fluid model. The turbulent stress is induced by the vapor phase velocity fluctuation from its averaged value in the frame of reference of the liquid phase velocity. It is found that the turbulent stress can be modeled[9] as:

$$\tau_g^{Re}=C_\tau\rho_g|v_g-v_f|(v_g-v_f) \tag{9}$$

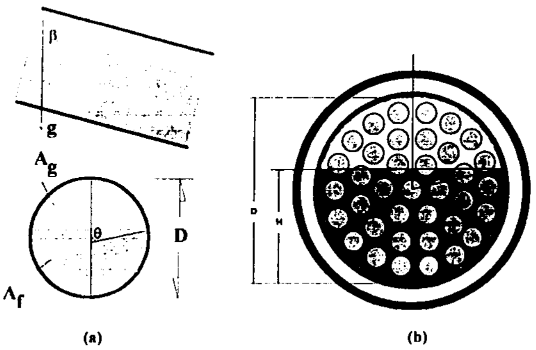

Fig. 1. A Stratified Flow Configuration

Multiplying $\alpha_{k'}$ (k' =g for k=f, k' =f for k=g) to the momentum equation of each phase (i.e., Eq.(3)) and subtracting one from the other, we obtain

$$\alpha_g\alpha_f\left(\rho_f\frac{\partial v_f}{\partial t}-\rho_g\frac{\partial v_g}{\partial t}\right)+\alpha_g\alpha_f\left(\rho_f v_f\frac{\partial v_f}{\partial z}-\rho_g v_g\frac{\partial v_g}{\partial z}\right)$$
$$=\alpha_g\alpha_f(\rho_f-\rho_g)g\cos\beta+\alpha_g\alpha_f\left(\frac{\partial p_g}{\partial z}-\frac{\partial p_f}{\partial z}\right)$$
$$-\left(\alpha_g\Delta p_{fi}+\alpha_f\Delta p_{gi}\right)\frac{\partial\alpha_g}{\partial z}-\tau_i\frac{\partial\alpha_g}{\partial z}$$
$$+\alpha_g\frac{\partial(\alpha_f\tau_f^{Re})}{\partial z}-\alpha_f\frac{\partial(\alpha_g\tau_g^{Re})}{\partial z}+\alpha_g M_{if}-\alpha_f M_{ig}$$
$$-\alpha_g M_{wf}+\alpha_f M_{iw}+\alpha_g\Gamma_f v_{fi}-\alpha_f\Gamma_g v_{gi} \qquad (10)$$

If we cast the phasic continuity equation and the combined momentum equation (Eq. 10)) into a matrix form, we obtain the system of equation as:

$$\underline{\underline{A}}\frac{\partial\phi}{\partial t}+\underline{\underline{B}}\frac{\partial\phi}{\partial z}=\underline{c} \qquad (11)$$

where

$$\underline{\underline{A}}=\begin{bmatrix}-\alpha_g\alpha_f\rho_g & \alpha_g\alpha_f\rho_f & 0\\ 0 & 0 & 1\\ 0 & 0 & -1\end{bmatrix} \qquad (11a)$$

$$\underline{\underline{B}}=\begin{bmatrix}-\alpha_g\alpha_f\rho_g[v_g-2C_\tau|v_r|] & \alpha_g\alpha_f\rho_f[v_f-2C_\tau\rho_g^*|v_r|] & -C_M+\alpha_f\tau_g^{Re}\\ \alpha_g & 0 & v_g\\ 0 & \alpha_f & -v_f\end{bmatrix} \qquad (11b)$$

$$\underline{c}=\begin{bmatrix}c'\\ \Gamma_g/\rho_g\\ \Gamma_f/\rho_f\end{bmatrix},\qquad \underline{\phi}=\begin{bmatrix}v_g\\ v_f\\ \alpha_g\end{bmatrix} \qquad (11c)$$

$$c'\equiv\alpha_g\alpha_f\Delta\rho g\cos\beta+\alpha_f M_{wg}-\alpha_g M_{wf}$$
$$+\alpha_g\Gamma_f v_{fi}-\alpha_f\Gamma_g v_{gi}-\frac{\xi_i}{A_{x-s}}\tau_{gi}-M_{ig} \qquad (11d)$$

$$C_M=\pi(gD\sin\beta)(\rho_f-\rho_g)\frac{\alpha_g\alpha_f}{4\sin\theta} \qquad (11e)$$

where the constitutive relations given by Eqs.(6) through (9) have been used.

The system's eigenvalues can be found by solving the characteristic equation, that is:

$$\det(\underline{\underline{B}}-\lambda\underline{\underline{A}})=0 \qquad (12)$$

where

$$\underline{\underline{B}}-\lambda\underline{\underline{A}}=\begin{bmatrix}\alpha_g\alpha_f\rho_g(\lambda-v_g+2C_\tau|v_r|) & -\alpha_g\alpha_f\rho_f(\lambda-v_f+2C_\tau\rho_g^*|v_r|) & -C_M+\alpha_f\tau_g^{Re}\\ \alpha_g & 0 & -(\lambda-v_g)\\ 0 & \alpha_f & \lambda-v_f\end{bmatrix} \qquad (13)$$

It should be noted that the algebraic terms in the momentum equation, such as the interfacial shear, the momentum transfer due to phase change and so on (see c in Eq.(11c)), do not affect the eigenvalue of the system.

Rearranging Eq.(12), we obtain

$$\alpha_f\rho_g(\lambda - v_g)^2 + \alpha_g\rho_f(\lambda - v_f)^2 + 2C_\tau\rho_g v_r(\lambda - \alpha_f v_g - \alpha_g v_f) - C_M + \alpha_f\tau_g^{Re} \doteq 0 \tag{14}$$

Solving Eq.(13) for λ, we obtain the dimensionless characteristics as

$$\lambda_\pm^* = \frac{\rho_g^*(\alpha_f - C_\tau)}{\rho_g^*\alpha_f + \alpha_g}Fr_{2\varphi} \pm \sqrt{D^*} \tag{15}$$

where

$$D^* = \frac{\pi(1-\rho_g^*)\alpha_g\alpha_f}{4\sin\theta(\rho_g^*\alpha_f + \alpha_g)} - \rho_g^* Fr_{2\varphi}^2 \left[\frac{\alpha_f\alpha_g - \alpha_f C_\tau(\rho_g^*\alpha_f + \alpha_g) - C_\tau^2\rho_g^* + 2\alpha_f C_\tau\rho_g^*}{(\rho_g^*\alpha_f + \alpha_g)^2}\right] \tag{16a}$$

$$\lambda_\pm^* = \frac{\lambda_\pm - v_f}{\sqrt{gD\sin\beta}}, \qquad Fr_{2\varphi} = \frac{v_r}{\sqrt{gD\sin\beta}} \tag{16b}$$

The two-phase system is hyperbolic when the eigenvalues are real or equivalently, the determinant of the quadratic equation is positive (i.e., $D^* > 0$). Let us define a critical two-phase Froude number ($Fr_{2\varphi,crit}$) such that D^* vanishes when $Fr_{2\varphi} = Fr_{2\varphi,crit}$. By setting $D^* = 0$, we obtain the critical two-phase Froude number for a stratified flow as

$$Fr_{2\varphi,crit}^2 = \frac{\pi\alpha_g\alpha_f(1-\rho_g^*)(\rho_g^*\alpha_f + \alpha_g)/(4\rho_g^*\sin\theta)}{\alpha_f\alpha_g + \alpha_f C_\tau(\rho_g^*\alpha_f + \alpha_g) - C_\tau^2\rho_g^* - 2\alpha_f\alpha_g C_\tau(1-\rho_g^*)} \tag{17}$$

Since the reality of eigenvalues implies that the two-phase system is stable against infinitesimal perturbations, we find the stable flow stratification is possible when $Fr_{2\varphi} < Fr_{2\varphi,crit}$. If we use the following assumptions in Eq.(15):

$$C_\tau = 0 \text{ and } \rho_g^*\alpha_f \ll \alpha_g \tag{18}$$

we obtain the critical relative velocity as:

$$|v_{r,crit}| = \sqrt{\frac{\pi\alpha_g(\rho_f - \rho_g)gD\sin\beta}{4\rho_g\sin\theta}} \tag{19}$$

It should be noted that Eq.(19) is the same as the surface wave wave instability criteria previously used as a flow regime boundary by Taitel and Dukler[1]. This criteria is being used in many nuclear reactor system analysis codes, such as RELAP5/MOD3. In fact, the stratified-dispersed flow regime boundary is believed to have some hysteresis effect. Unfortunately, however, the current state-of-the-art knowledge is not mature enough to quantify such effect.

As shown in the definition of the two-phase Froude number, the phasic relative velocity (v_r) is the key two-fluid variable in controlling the well-posedness, or equivalently, the stability of the stratified flow. When the system's two-phase Froude number exceeds the critical Froude number given by Eq.(17), the flow regime transition may occur. It is interesting to note that when the pipe is inclined to the vertical position (i.e., $\beta \to 0$), we obtain $Fr_{2\varphi} \to \infty$ from Eq.(16b), meaning that there is no chance of gravity induced flow separation (i.e., stratification is not possible).

3. Results and Conclusion

The eigenvalues and the critical values of the relative velocity have been calculated for a bundle flow. The geometric and thermo-mechanical conditions have been obtained from the CANDU-6 core fuel channel which is one of important applications of horizontal two-phase flow. The imaginary value of the system's eigenvalues supposedly signals the flow regime transition from the stratified flow to the dispersed flow. Generally, during many system transients, some of the coolant channels may undertake flow regime transitions. Among these, the dispersed-to-stratified flow transition dramatically changes the heat transfer characteristics of the fuel bundles in the coolant channel. All in all, the the uncovery of the fuel element, which is possible when the flow is stratified, should be avoided.

The two-phase system's eigenvalues are shown in Figure 2. Physically, this type of eigenvalues have been understood as the propagating speed of infinitesimal perturbation of two-phase variables, such as the void fraction[10], [11]. As shown in Figure 2, when the void fraction is reduced to a certain value (say, cutoff void fraction), the eigenvalues vanish quickly and become imaginary values (the imaginary values can not be shown in Figure 2). When the turbulent stress is included (i.e., $C_\tau = 0.5$), the cutoff void fraction is reduced. Since the instability occurs at the cutoff void fraction, we find that the turbulent stress stabilizes the stratified flow. It is an interesting result that since the two eigenvalues have almost an equal magnitude and the opposite sign, we find that the waves propagate in both direction with the smae speed in the moving frame of reference of the liquid phase velocity.

The critical Froude number given by Eq.(17) has been plotted in Figure 3. For the purpose of comparison, the critical Froude numbers at atmospheric pressure are also plotted. The stratified flow is presumably stable when two-phase system's Froude number has a value smaller than the critical Froude number. Therefore, Figure 3 can be understood as a flow regime map based upon two important two-phase parameters, that is, the two-phase Froude number and the void fraction. As shown in Figure 3, it is clear that the flow stratification is more likely when

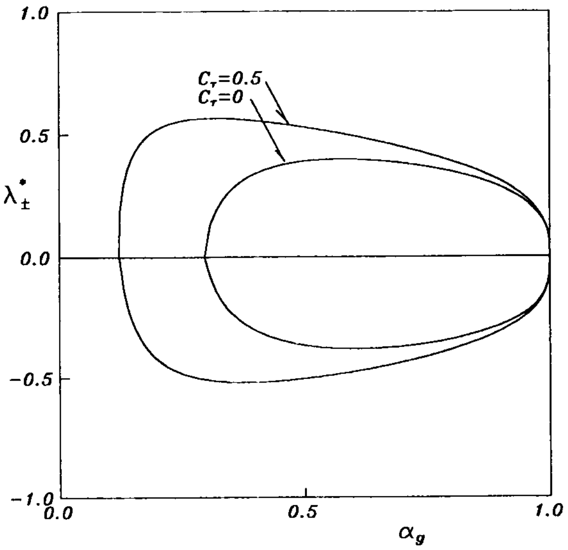

Fig. 2. Eigenvalues of Horizontal Two-Phase System in a Fuel Bundle with and Without Vapor Phase Turbulent Stress (p=10 Mpa, D=0.1 m).

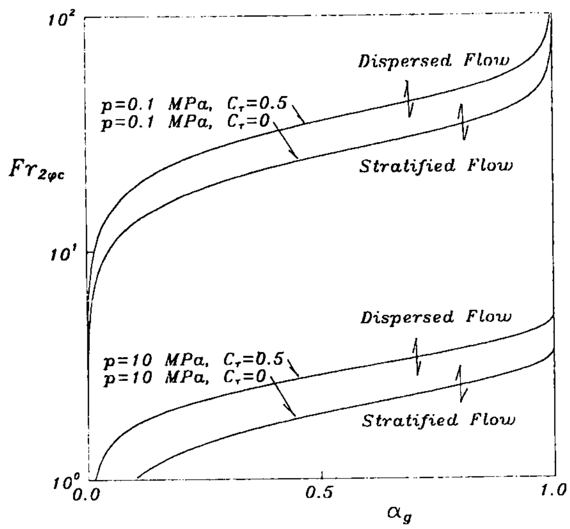

Fig. 3. Critical Froude Number with and Without Vapor Phase Turbulent Stress Under Atmospheric and Pressurized Conditions

the system pressure is reduced. This is mainly due to larger density ratio(ρ_g *) at higher system pressure. As we observed in Figure 2, we find that the vapor phase turbulent stress stabilizes the stratified flow.

In the previous studies, they used the relative volume velocity which can be related with the two-phase Froude number used in the presented study as:

$$j_{gf}^{*} = \sqrt{\frac{\rho_g}{\rho_f - \rho_g}\alpha_g \alpha_f Fr_{2\varphi}}, \tag{20}$$

Two significant previous theoretical results[1], [12] have been compared with the presented model in Figure 4. It is known that the Wallis result has a good agreement with his own experimental data except for low values of void fraction. As shown in Figure 4, we find the two extreme values of vapor phase turbulent coefficient (i.e., C_τ) can comprise all previous results. One can, however, find that the slope of the lines in Figure 4 generated by the presented model is quite different from that of the previous workers. Especially, when the global void fraction is low, the disagreement is large where the slugging phenomena is very sensitive to the phasic relative velocity. The reason for this difference is due to the inclusion of the vapor phase Reynolds stress which supposedly plays more important role when the global void fraction is low. In fact, based on the conventional experimental technique, the slugging phenomena at low global void fraction is hard to correlate with the basic two-fluid variables. Therefore, the experimental data sets have a wide error range at low global void fraction[12]. Since the vapor phase Reynolds stress is very important at low global void fraction, more carefully designed experiment should be done to obtain data with tolerable errors. For medium values of global void fraction, many models including the presented model(with $C_\tau = 0.5$) agree well with the Wallis' experimental data. It should be pointed out here that in many horizontal flow experiments, the data has been compiled by using the volume or mass flow of each phase[1] with the visual observation of the flow rate regimes. This type of experimental approach has its limit in understanding slugging phenomena mechanistically. When the global void fraction is high(i.e., α_g) 0.75), the slugging is not easy to be observed in normal laboraory conditions and the data is not available from the previous workers.

It should be noted that the vapor phase turbulent stress model used in this study is a little too much simplified. Thus, the emphasis of further study should be on detailed modeling of vapor phase turbulent stress coefficient for different liquid phase flow conditions. Moreover, the effect of the vapor phase turbulent stress in a horizontal bundle flow can be better understood by modeling the vapor phase turbulence in bundle flow geometry directly.

This study has revealed that the vapor phase turbulence is very important and should be handled properly to determine the flow regime boundary correctly. Further experimental study is needed in confirming the validity of this technique.

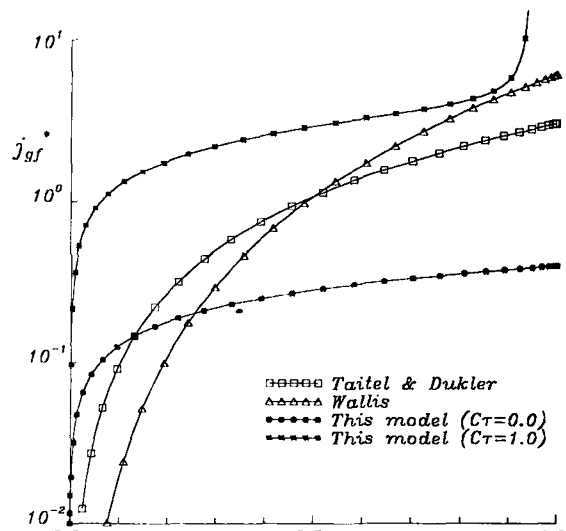

Fig. 4. Comparison of Slugging Criteria with and without Vapor Phase Turbulent Stress for Air-Water Mixture under Atmospheric Condition

Alphabet

A area
D diameter of a pipe/channel
g specific gravity
H height of liquid level
$\dot{h}$ vertical position in a channel
M momentum per unit volume
v velocity
p pressure

Subscript

g gas phase
f liquid phase
i phasic interface
w wall

Superscript

Re Reynolds stress

Greek

Γ phasic mass transfer
Δ difference
α volume fraction
β angle of pipe from the direction of gravity
λ eigenvalue
ρ density
ρ_g^* density ratio (ρ_g/ρ_f)
ξ wetted perimeter
θ stratification angle
τ sgear stress

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