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Optimization of the Korean Nuclear Fuel Cycle Using Linear Programming

J.I. Kim*, K.N. Chae, and B.W. Lee**
Korea Advanced Institute of Science and Technology
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선형계획법을 이용한 한국 원전연료주기의 최적화

김진일* · 채규남** · 이병휘
한국과학기술원
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Abstract

The Korean optimal nuclear fuel cycle strategy from the year 2000 to 2030 is derived using linear programming. The fuel cycle cost, the cost uncertainty, and the natural uranium consumption are used as the criteria for the optimization. These objectives are compromised by fuzzy decision-making technique which maximizes the minimum degree of satisfaction among the three objectives. The options for the back-end fuel cycle are direct disposal, reprocessing, and DUPIC. The optimal fuel cycle strategy of Korea is to start reprocessing in around 2010 and increase its capacity with the maximum of 800 tHM in around 2025, and to start DUPIC processing in 2025. The cost uncertainty and the natural uranium consumption of the optimal fuel cycle strategy are reduced by 7.1% and 6.1%, respectively, at the cost penalty of 5.4% compared with the cost-only optimal solution.

요 약

2000년부터 2030년까지의 한국 원전연료주기의 최적전략을 도출하기 위하여 선형계획법을 사용하였다. 최적화를 위한 결정인자로서는 원전연료 주기비용, 요소비용의 불확실성, 우라늄 소요량을 사용하였다. 위의 인자들을 동시에 고려하기 위하여 각각에 대한 만족도 중 최소값을 최대화하는 퍼지 의사결정 기법을 이용하였다. 사용후 원전연료에 대한 가능한 선택대안으로는 직접처분, DUPIC, 재처리를 가정하였다. 한국의 원전연료주기 전략은 2010년경부터 재처리를 시작하여 그 처리용량을 2025년경에는 800톤까지 점차로 늘려 나가고, DUPIC 처리를 2025년경부터 시작하는 것이 최적인 것으로 나타났다. 요소비용의 불확실성과 우라늄 소요량을 고려함으로써 단순히 비용만을 고려한 경우보다 총비용은 5.4% 증가하나, 요소비용 불확실성은 7.1%, 우라늄 소요량은 6.1% 감소하는 것으로 나타났다.

* 현재 (주)대우건설 원자력부 근무
** 현재 한국원자력연구소 정책부 근무

1. Introduction

The fuel discharged from light water reactors contains more fissile contents than natural uranium depending on the initial enrichment and the discharge burnup. So after chemical reprocessing or reconstitution, the spent PWR fuel can be recycled in PWRs or PHWRs. Some countries recycle spent fuel, others do not recycle or have not yet decided to. Korea is in the last case. The purpose of this study is to find out the optimal fuel cycle strategy of Korea for 2000-2030 period focusing on the back-end part.

The general options for the fuel cycle are concerned about the once-through and the recycling of spent fuel. DUPIC (Direct Use of spent PWR fuel In CANDU reactor) can be also considered as an option in Korea.

The choice of the nuclear fuel cycle is country-specific and dependent on many factors such as: fuel cycle cost; public acceptance; political environment; environmental impacts; the security of national energy supply; national policy; domestic resources (natural uranium and site); technical feasibility and so on. Among them, two quantifiable factors - the fuel cycle cost and the natural uranium consumption - are selected as the decision criteria and the cost uncertainty stemming from the variation of unit cost assumption is added.

The method used in this study to optimize the Korean nuclear fuel cycle is linear programming (LP). The LP-formulated problem can be solved by one of many commercial LP solving packages like CPLEX [1]. The material flow can be expressed as equality constraints and the various capacities as inequality constraints. The basic external variable is the nuclear fuel requirement which is calculated from the long-term electricity demand forecasting and the nuclear share.

Fuzzy linear programming which is an application of fuzzy theory to linear programming fuzzifies the optimality of the solution of ordinary linear programming with single objective. P. Silvennoinen applied LP and fuzzy LP to nuclear fuel cycle[2, 3], and later K. Yamaji used LP to analyze long-range plutonium utilization strategy[4]. The main idea of this study is somewhat similar to Silvennoinen's approach but much different in fuel cycle options such as PWR-PHWR symbiosis.

2. Fuzzy Linear Programming

If there are more than one objectives in LP, one faces with the problem of compromising the attributes against each other. One of the compromising methods is fuzzy linear programming. For a constraint or an objective function, we assume the following type of membership function(μ_i) using a normalized fuzzy set:

$$\mu_i(x) = \begin{cases} 1 \text{ if } \sum_{j=1}^{n} a_{ij}x_j \le b_i \text{ is satisfied,} \\ 0 \text{ if } \sum_{j=1}^{n} a_{ij}x_j \le b_i \text{ is strongly violated.} \end{cases} \quad (1)$$

"Strongly violated" means that the constraint or the objective function value is violated such that with e_i:

$$\sum_{j=1}^{n} a_{ij}x_j \ge b_i + e_i \quad (2)$$

where e_i is the tolerance interval.

If we assume a linearly decreasing membership function in the tolerance interval, the complete membership function can be defined by:

$$\mu_i(x) = \begin{cases} 1 & \text{if } \sum_{j=1}^{n} a_{ij}x_j \le b_i \\ 1 - \dfrac{\sum_{j=1}^{n} a_{ij}x_j - b_i}{e_i} & \text{if } b_i < \sum_{j=1}^{n} a_{ij}x_j < b_i + e_i \\ 0 & \text{if } \sum_{j=1}^{n} a_{ij}x_j \ge b_i + e_i \end{cases} \quad (3)$$

A fuzzy decision in the intersection of all fuzzy sets representing either fuzzy objectives or constraints and the membership function of the intersection are calculated by applying the "MIN" operator to the membership functions of all fuzzy sets involved. The membership function of the "decision" of a problem is then characterized by its membership function:

$$\mu_D = MIN[\ \mu_1(x),\ \mu_2(x),\ \cdots, \mu_m(x)]\ . \quad (4)$$

The maximization of minimum membership function can be formulated in ordinary linear programming[5, 6, 7], that is,

$$\begin{aligned} &\textit{maximize } \mu_D \\ &\textit{subject to } \mu_D \leq 1-(\sum_{j=1}^{n} a_{ij}x_j - b_i)/e_i \quad i=1,2,\ldots,m \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad j=1,2,\ldots,n \end{aligned} \quad (5)$$

3. Application to the Korean Nuclear Fuel Cycle

3.1. Basic Assumptions

A reactor strategy is set up considering the long term electricity demand forecasting and the nuclear share[8]. The main reactor type is PWR and the complementary one is PHWR. The Korean reactor strategy used here is shown in Table 1.

The burnup of PWR fuel is increasing steadily. For simplicity, the burnup of PWR fuel is assumed to be 33,000 MWD/MTU till 1994, 45,000 MWD/MTU from 1995 till 1999, and 50,000 MWD/MTU from 2000 till 2030.

The fuel cycle options considered here are reprocessing, DUPIC and direct disposal. For the reprocessing option, the recovered Pu is recycled as MOX fuels in PWRs and the recovered uranium can be recycled in either PWRs or PHWRs. DUPIC is considered as an option for PWR-PHWR symbiotic fuel cycle. It is assumed that the reprocessing plant and interim storage facility can start operation from 2000, and the DUPIC facility from 2006 because it will take much time in studying the technical feasibility and preparing operation[9]. Figure 1 shows the simplified diagram of Korean nuclear fuel cycle.

3.2. Objective Function

The following three objective functions are considered: the fuel cycle cost (COST); the cost uncer-

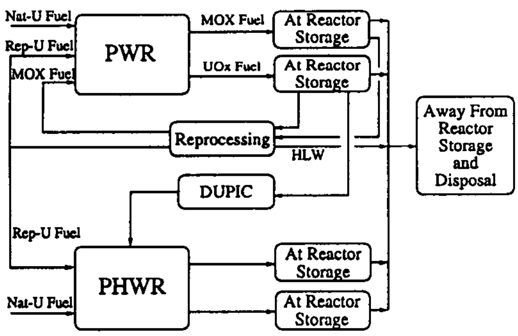


Fig. 1. Diagram of Korean Nuclear Fuel Cycle

Table 1. Reactor Strategy of Korea for 2000-2030 Period

Year	Introduced	Decommissioned	Year	Introduced	Decommissioned
2001	PWR		2015	PWR	
2002	PWR		2016	PHWR	
2004	PWR x 2		2018	PWR	
2005	PWR		2020	PWR	
2006	PWR, PHWR		2022	PHWR	
2007	PWR x 2		2023	PWR	Kori-2
2008	PWR	Kori-1	2025	PHWR x 2	Kori-3
2009	PHWR		2026	PWR x 2	Kori-4, Yonggwang-1
2011	PWR		2027	PWR	Yonggwang-2
2013	PHWR	Wolsong-1	2028	PWR	Ulchin-1
2014	PWR		2029	PWR	Ulchin-2

Table 2. Unit Cost Estimation[10, 11, 12]

	PWR		PHWR	
	Cost	Uncertainty[1)]	Cost	Uncertainty[1)]
Uranium Ore ($/kgU)	50	40	50	40
Conversion ($/kgU)	8	0	8	0
Enrichment ($/kgSWU)	110	20	–	–
UO_2 fuel fabrication ($/kgU)	275	25	60	10
MOX fuel fabrication ($/kgHM)	1100	0	–	–
Transportation ($/kgHM)	50	10	13	0
Interim storage ($/kgHM)	230	60	–	–
Final disposal ($/kgHM)	610	80	73	27
DUPIC processing ($/kgHM)	1100	200	–	–
Reprocessing ($/kgHM)	540	180	–	–
HLW disposal ($/kg)	90	10	–	–

Note 1 : Uncertainty means the difference between the reference cost and the maximum cost estimation.

* Other assumptions[13] : Discount rate = 5%/year

Cost escalation of natural U = 1.2%/year

Uncertainty escalation of natural U = 1.13%/year

Cost escalation of MOX fabrication = −1.4%/year until 2010

tainty (RISK); and the natural uranium consumption (URAN). The estimated unit cost data are shown in Table 2.

a) Discounted fuel cycle cost

The objective for the economics of fuel cycle is to minimize the net present value of all the expenditures for the time horizon. This objective can be expressed in the following mathematical form where COST(X) is the name of objective function and X is a variable vector representing the capacities of the fuel cycle facilities and the quantities of fissile materials:

$$COST(X) = \sum_{t=2000}^{2030} c_i(t) \cdot x_i(t) \cdot (1+r)^{(2000-t)} \quad (6)$$

where $X \equiv \sum_{j=1}^{n} a_{ij}\, x_j$

c_i = cost per unit activity of x_i

r = discount rate.

b) Discounted cost uncertainty

Cost uncertainty resulting from the variation of unit cost estimation is another side of economic measurement. Instead of unit cost c_i, we use the estimates of the uncertainty in the values for given c_i's. Denoting these by d_i's, which are shown in Table 2 as the cost uncertainty, one obtains the second economic measure as follows:

$$RISK(X) = \sum_{t=2000}^{2030} d_i(t) \cdot x_i(t) \cdot (1+r)^{(2000-t)}, \quad (7)$$

where d_i = cost difference between the reference cost and the maximum cost estimation.

c) Natural uranium consumption

Uranium is not a renewable energy source and Korea is poor in her natural resources including uranium. Thus, the following objective for the consumption of natural uranium is considered as the third one:

$$URAN(X) = \sum_{t=2000}^{2030} [\ (1+f_L) \cdot F\_P \cdot LNUF(t) + (1+f_H) \cdot HNUF(t)] \quad (8)$$

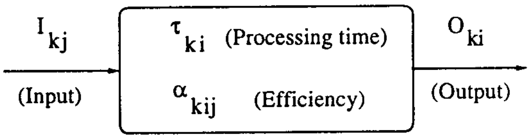
where $LNUF$ = PWR fuel from natural uranium
$HNUF$ = PHWR fuel from natural uranium
f_L = front-end loss factor for PWR fuel
f_H = front-end loss factor for PHWR fuel
F_P = feed to production ratio in the process of enrichment.

3.3. Constraints

a) Material Balance

For a given stage k in the fuel cycle, the balance of material is expressed as equality in constraints. Certain fuel cycle steps, e.g. spent fuel interim storage, are not concerned with specific residence time. However, this can be varied as a free parameter within the capacity limit which allows the discontinuous material flow across the box k. Figure 2 shows the material balance through a unit stage k.

Annual fuel requirement is calculated on the basis of the reactor strategy and the characteristics of reactors and fuels. The Korean standard PWR is assumed to be an Ulchin-3, 4 type reactor and the reference PHWR is assumed to be a Wolsong-1 type reactor.



$$O_{ki}(t) = \sum_j \alpha_{kij} I_{kj}(t-\tau_{kj})$$

Fig. 2. Description of a Model Stage

The annual energy generated in PWRs, $HEAT_L(t)$, can be expressed as:

$$HEAT_L(t) = BU_L \cdot [\ LNUF(t) + LRUF(t) + LRMF(t)\ ] \quad (9)$$

where BU_L = burnup of fuels in PWRs,
$LRUF$ = PWR fuel from recycled uranium, and
$LRMF$ = MOX PWR fuel.

The use of MOX fuel in PWRs is limited to 50% of the total core loading because of safety consideration, that is,

$$LRMF(t) \le 0.5 \cdot [\ LNUF(t) + LRUF(t) + LRMF(t)\ ]\ . \quad (10)$$

PHWRs can be loaded with three kinds of fuel, namely, natural U fuel, recovered U fuel and DUPIC fuel. Some of the reference studies[14, 15] show the burnup of recovered uranium fuel and DUPIC processed fuel[16] in PHWRs varying with their discharge burnup in PWRs. The burnup of these fuels is shown in Table 3. The total energy generated in PHWRs in year t, $HEAT_H$, can be expressed as:

$$HEAT_H(t) = BU_{NU} \cdot HNUF(t) + BU_{RU} \cdot HRUF(t) + BU_{DP} \cdot HRMF(t) \quad (11)$$

where BU_{NU} = burnup of natural U fuel in PHWRs,
BU_{RU} = burnup of recovered U fuel in PHWRs,
BU_{DP} = burnup of DUPIC fuel in PHWRs,
HRUF = PHWR fuel from recycled uranium, and
HRMF = DUPIC fuel.

Table 3. Burnup Data

Discharge burnup of PWR fuels (MWD/MTU)	33,000	45,000	55,000
Burnup of DUPIC fuel in PHWRs (MWD/MTHM)	20,000	15,700	14,100
Burnup of recycled U fuel in PHWRs (MWD/MTU)	11,800	8,900	7,300

The inventory of spent fuels in the cooling ponds is just the accumulation of the difference between the spent fuels from the reactors and the fuels transported to reprocessing, interim storage or DUPIC processing facilities depending on the type of fuel and the option. The inventory of spent PWR fuels is also calculated by simple accumulation. Because the negative term is not allowed in linear programming, the spent fuels in the interim storage are assumed not to go back to the cooling ponds.

The reprocessing plant recovers plutonium and uranium from spent PWR fuels and generates high-level wastes(HLW). The recovered plutonium assumes to be automatically recycled in PWRs and the recovered uranium can be recycled in either PWRs or PHWRs. A mathematical equation relates the fuels to be reprocessed and the fuels to be recycled in terms of fissile content.

The reconstitution of spent PWR fuels is carried out in the DUPIC process by repeating oxidation and reduction. The processed spent PWR fuels are directly recycled to PHWRs after certain processing time.

b) Capacity Limits

The capacity of a cooling pond is limited to the amount equivalent to the 10 years' discharged fuel in order to prevent the unrealistic piling up of spent fuels in the cooling pond when this situation is preferred. The cooling pond capacities of Kori-1, 2, 3, 4, Yonggwang-1, 2 and Ulchin-1, 2 assume to be saturated.

The annual capacities of the reprocessing and the DUPIC processing are limited to 800 tHM, respectively, in order to rule out the exceptionally large quantity of fuel being processed.

3.4. Formulation of Fuzzy Linear Programming

To compromise the three objectives (i.e., COST, RISK, URAN), ordinary optimizations with single objective are performed three times and the degree of satisfaction (membership function in the fuzzy theory) are defined on the basis of the result of three sub-optimal solutions. Finally the fuzzy decision making is reduced to an ordinary linear programming with the objective of maximizing the minimum degree of satisfaction.

The degree of satisfaction of each objective function is defined as a linear function between the maximum and the minimum objective values. The minimum value is that of the optimal solution for its own objective function. The maximum value is the largest one among the optimal solutions for the other two objective functions.

The degrees of satisfaction are:

$$\mu_C(X) = \frac{C_{MAX} - COST(X)}{C_{MAX} - C_{MIN}} \quad \text{for fuel cycle cost,} \tag{12}$$

$$\mu_R(X) = \frac{R_{MAX} - RISK(X)}{R_{MAX} - R_{MIN}} \quad \text{for cost uncertainty,} \tag{13}$$

$$\mu_U(X) = \frac{U_{MAX} - URAN(X)}{U_{MAX} - U_{MIN}} \quad \text{for natural uranium consumption,} \tag{14}$$

where $C_{MAX} = MAX\,[COST(X_R), COST(X_U)]$,
$C_{MIN} = COST(X_C)$
$R_{MAX} = MAX\,[RISK(X_C), RISK(X_U)]$,
$R_{MIN} = RISK(X_R)$,
$U_{MAX} = MAX\,[URAN(X_C), URAN(X_R)]$,
$U_{MIN} = URAN(X_U)$,
X_C, X_R, X_U: Sub-optimal solutions for cost, risk and uranium, respectively, and
$COST(X)$, $RISK(X)$, $URAN(X)$: Cost, risk and uranium as a function of X.

The objective function of the compromising LP problem is the minimum value of the degree of satisfaction $\mu(X)$, which is defined by

$$\mu(X) = MIN[\mu_C(X), \mu_R(X), \mu_U(X)] \tag{15}$$

Finally the maximization of the degree of satisfaction is reduced to an ordinary linear programming problem as follows:

$$\begin{aligned} &\textit{maximize} \ \mu(X) \\ &\textit{subject to} \ \mu(X) \leq \mu_i(X),\ i = C, R, U \qquad (16) \\ &\qquad AX \leq B \ \text{(Material balance and capacity limits)} \\ &\qquad X \geq 0 \ \text{(Non-negative constraints).} \end{aligned}$$

4. Results and Discussions

The optimization results are strongly dependent on some vital assumptions such as the capacity limits, the cost estimations and the reactor strategy. Thus, the sensitivity analysis for the reprocessing and DUPIC processing capacities is carried out to cope with the uncertainty.

When the two capacities are limited to 800 tHM/year, respectively, it is shown that the optimal fuel cycle strategy is to start reprocessing in 2010 reaching the maximum capacity of 800 tHM in 2025. It is also shown that the DUPIC processing would start in 2023 and end in 2026, but it is difficult to regard this as a realistic commercial operation mode because of its short operating period. Figure 3 shows the fuel composition in this case and Figure 4 and 5 show annual reprocessing and DUPIC processing requirements, respectively. The total discounted fuel cycle cost would be 12.28 billion dollars with an increase of 5.4% compared with the minimum cost for the cost objective alone, and the degree of satisfaction would be 0.697. The cost uncertainty would be 4.28 billion dollars with the degree of satisfaction of 0.710, and this amount is equivalent to about 35% of the total fuel cycle cost. The natural uranium consumption would be 94,292 tons with the degree of satisfaction of 0. 710.

Meanwhile, when both of the two processing capacities are limited to 400 tHM/year, the reaching time for the reprocessing capacity to the maximum limit is shifted 15 years ahead. And DUPIC processing would start in 2020 with the capacity of about 200 tHM/year and continue its operation until the end of time horizon. Figure 6 shows the fuel composition and Figure 7 and 8 show the annual reprocessing and DUPIC processing requirements for 400 tHM/year case, respectively. The total fuel cycle cost of this case is 12.51 billion dollars, which is not so

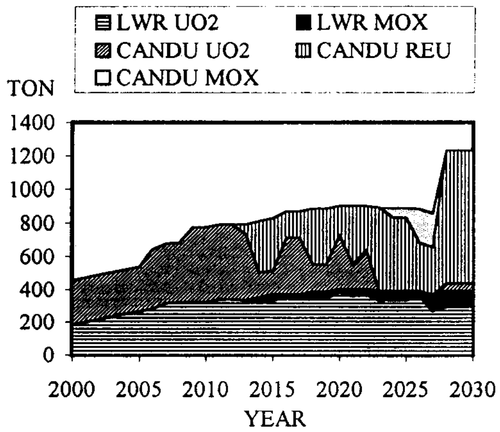


Fig. 3. Fuel Composition for the 800tHM/year Case

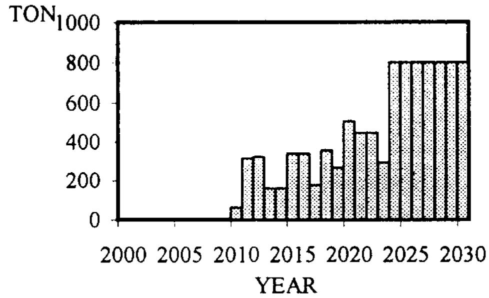


Fig. 4. Reprocessing Requirement for the 800tHM/year Case

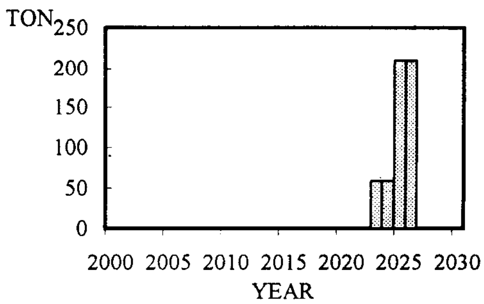


Fig. 5. DUPIC Requirement for the 800tHM/year Case

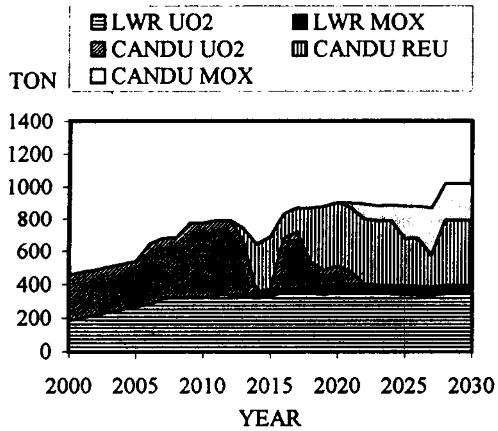


Fig. 6. Fuel Composition for the 400tHM/year Case

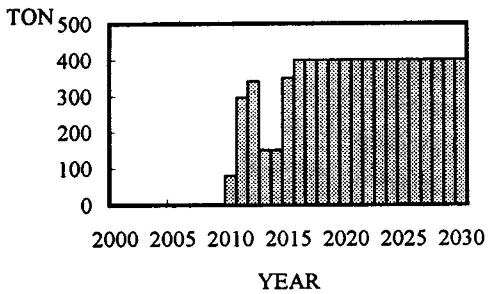


Fig. 7. Reprocessing Requirement for the 400tHM/year Case

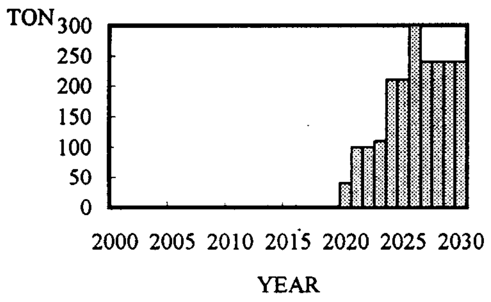


Fig. 8. DUPIC Requirement for the 400tHM/year Case

different from the case above, and the cost uncertainty and the natural uranium consumption are nearly on the same level, too. Thus, the capacity of reprocessing might be determined by other consideration like the environmental policy and the public safety.

5. Conclusions and Recommendations

The optimization is performed to establish the optimal nuclear fuel cycle strategy of Korea for the period of 2000-2030 using linear programming and fuzzy decisionmaking technique.

Based on the result of the study, the reprocessing of PWR spent fuel is recommended to start operation in around 2010 with the capacity of 400 tHM/year and from around 2025 with the upper limit of the capacity constraint of 800 tHM/year. The recovered uranium would be recycled in PHWRs due to the penalty of U-236 in PWRs, which is not the case in PHWRs.

The result such as annual reprocessing and DUPIC processing requirements shows somewhat idealistic features which reflect the characteristics of linear programming. This result can be used as a basis of decisionmaking for the nuclear fuel cycle with a sense of reality. The fluctuation of the reprocessing and DUPIC processing can be levelized over some period of years.

As the nuclear fuel cycle is strongly correlated with the reactor strategy, the optimization of fuel cycle alone is not enough in the view of the overall nuclear energy system. Thus, the fuel cycle optimized simultaneously with the reactor strategy would be more reliable and meaningful.

FBRs are expected to be introduced in around 2030. As FBRs require much plutonium, the reprocessing of spent fuel should be necessary. Considering the introduction of FBRs, the optimization of nuclear fuel cycle should be focused on the optimal reprocessing strategy.

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