

A Theoretical Study on the Fluid-Structure Interaction Due to the Pump in the Pressurized Water Reactor

Kye Bock Lee and Jong Ryul Park

Korea Atomic Energy Research Institute

(Received February 21, 1995)

원자로에서 펌프에 의해 야기되는 유체와 구조물 상호 작용에 대한 이론적 연구

이계복 · 박종률

한국원자력연구소

(1995. 2. 21 접수)

Abstract

The propagation of pump-induced pressure pulsation in a reactor is important because of the potential for vibration and resultant damage of reactor internals. A hydrodynamic model has been developed to obtain the pressure fluctuation due to the operation of pumps in the annulus(between the core support barrel and reactor vessel of a pressurized water reactor) including the coolant inlet pipe. The mathematical analysis is formulated in accordance with the linearized Navier-Stokes equation by assuming a compressible, inviscid flow. Two regions are considered separately and by coupling the solutions of the inlet pipe and the annulus, the inlet nozzle pressure(pressure at pipe and annulus interface) is to be calculated without assumptions. The geometric parameter effect on the pump-induced pressure pulsation is evaluated. Comparison of predicted and measured inlet nozzle pressure values for each forcing frequency shows good order of magnitude agreement.

요 약

원자로에서 펌프에 의해 야기되는 맥동 압력은 원자로 내부 구조물에 진동과 손상을 줄 수 있기 때문에 관심이 증가되고 있다. 본 연구에서는 냉각관과 환형관(원자로 압력 용기와 노심 보호 지지대 사이)으로 구성된 기하 형태에서 펌프에 의해 야기되는 맥동 압력을 해석할 수 있는 수력학적 모델을 개발하였다.

수학적 지배 방정식은 압축성, 비점성 유체에 대해 선형화된 Navier-Stokes 방정식이다. 냉각관과 환형관을 따로 분리하여 해석하고 두영역의 커플링 영향을 고려하였다. 또한 본 기하 형태에서 펌프 맥동 압력에 영향을 미치는 주요 기하 인자에 대한 평가를 수행하였다. 본 해석 결과와 실험치를 비교하여 만족할 만한 결과를 얻었다.

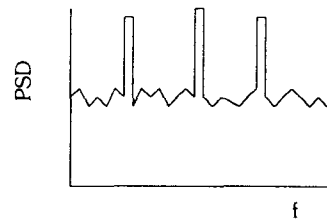
1. Introduction

Fluid-structure interaction problems have received considerable attention in recent years within the nuclear industry. A total understanding of some characteristics of fluid-structure interaction problems has not been attained. The reactor internals related to safety are designed to accommodate both static and dynamic design loads. These design loads include both static and dynamic flow induced loads related to normal operating and anticipated transients and mechanical excitation due to the operating base earthquake. Dynamic response of the reactor internals is a function of the magnitude, frequency, and spatial distribution of the flow induced loads and the modal frequencies and the mode shapes of the assemblies.

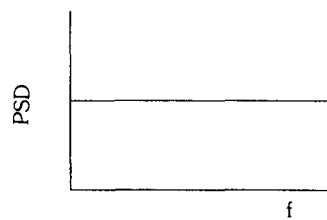
Flow induced vibrations, which have led to many problems in reactor operation, are of major concern in the structural integrity view of the reactor internals. For example, the original designed control element assembly shroud of Palo Verde unit 1 was cracked during the pre-core hot functional test. This failure was primarily caused by low frequency response of the assembly to excitations induced by adjacent structures (CEA tube bank) with secondary contributing stresses from shell mode responses due to pump pressure pulsation. To avoid fatigue failure of reactor internal components due to flow-induced loads, it is necessary to predict flow-induced hydraulic forces loaded on the surface of the structure.

Hydraulic exciting forces can be deterministic, random, or a combination of both. Fig. 1 is a idealized spectral density representation of combined random and periodic response. They may result from the pressure fluctuations generated by the pumps, vortex shedding, other flow instabilities, turbulence, etc. The forced vibration of a structure depends on the magnitude and frequency of the exciting forces and on the dynamic characteristics of the structure. The vibration can become larger if the frequency of an important force component is coincident with a main natural frequency of the structure. Because of the

a) random & periodic



b) random



c) periodic

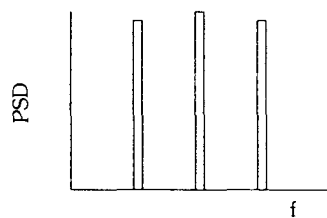


Fig. 1. Idealized Response PSDs

general complexity of this problem, the pressure fluctuations of the reactor coolant induced by pump pulsation is considered in this study.

Pump pulsation loads are due to harmonic variation in fluid pressure caused by the reactor coolant pump. These pulsations propagate throughout the system as acoustic waves, independent of fluid velocity. These pulsations occur at multiples of the pump rotor and blade passing frequencies. The magnitude and spatial distribution of these pressures are dependent on the pump, geometry of the flow path, and the temperature in the liquid.

Fig. 2 is a schematic drawing of a pressurized water reactor (PWR). The region between the core support barrel and the reactor vessel is the annulus. Coolant water, supplied by the pump, enters this an-

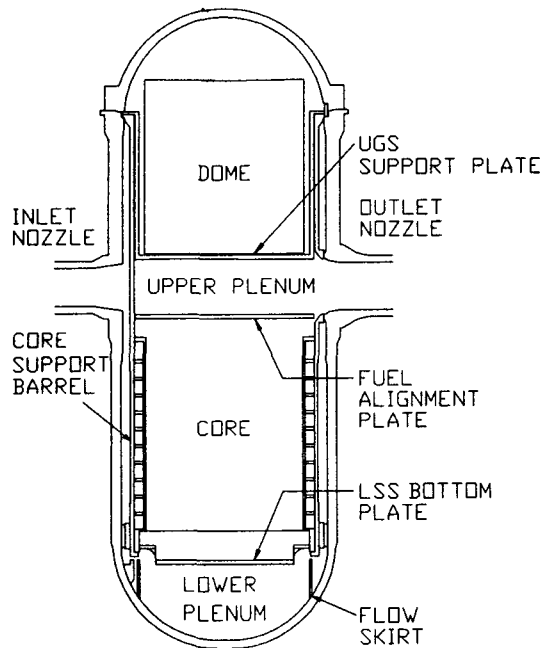


Fig. 2. Schematic Drawing of Pressurized Water Reactor

nulus through the pipe line. The core support structures have the important function of supporting and restraining the core. Therefore, the determination of their structural integrity during operation and accident conditions is of great interest. The major member of the core support structures for all ABB-CE designs is the core support barrel assembly which includes the core support barrel, and the lower support structure. The lower support structure provides support for the core as well as the orientation for the lower ends of the fuel assemblies. The core support barrel is a right circular cylinder and is supported from a ledge on the reactor vessel.

Pressure pulsations in a PWR annulus resulting from the pump excitation were first studied by Penzes[1], and later Bowers and Horvay[2]. The mathematical analysis is formulated in accordance with a three dimensional Navier-Stokes equation by assuming a compressible, inviscid liquid. This formulation of the problem determines a time dependent,

mixed boundary value problem. For this case, the separation of variables technique cannot be applied and a solution of this problem would require a lengthy procedure utilizing numerical analysis techniques. Therefore, they obtained an approximate solution by introducing the concept of time dependent body force in the governing differential equations. With this body force concept, mixed boundary value problem can be represented as a forced vibration problem with homogeneous boundary conditions. M.K. Au-Yang[3] solved the three dimensional wave equation using Green's function method, and he treated the forcing region as a point source. But in these studies, the excitation at the inlet nozzle is assumed to be known prior.

Fisher et al. [4] demonstrated an improved technique in solving one-dimensional wave propagation problems with time dependent boundary conditions. This technique introduces a transformation equation in terms of a new variable and auxiliary functions. The mathematical description is recast in the form of a non-homogeneous differential equation. The boundary conditions become homogeneous by restricting the auxiliary functions at the boundaries. Due to the homogeneity of the resulting boundary conditions, the transformed system of equations is amenable to solution by separation of variables. Lee and Chandra [5] analyzed the pump induced pulsating pressure in a reactor coolant pipe using this technique. But, they missed the constraints on the auxiliary functions to make the boundary conditions homogeneous. Lee et al. [6] developed an improved analytical model which satisfies all boundary conditions. Actually, the boundary condition at the end of pipe (inlet pipe/annulus interface) lies between open and closed condition. Thus, the piston-spring supported condition was adopted. Analytical values of inlet nozzle pressure at different forcing frequencies were calculated for an empirically determined non-dimensional spring constant.

From the literature survey, the following restrictions or assumptions are included in the previous theor-

etical analysis. The pressure at the inlet nozzle is assumed to be known prior in the prediction of the three dimensional wave propagation in the annulus and the spring constant is assumed empirically in the one dimensional wave propagation of the inlet pipe.

To predict the pump-induced pressure pulsations within a PWR, the coupling effect between the inlet pipe and the annulus must be considered. Thus, a hydrodynamic model has been developed to obtain the pressure fluctuation due to the operation of pumps in the annulus including the inlet pipe. Two separate regions (inlet pipe and annulus) are considered in this study, and by coupling the solutions of the inlet pipe and annulus, the inlet nozzle pressure, which was assumed in the previous study, is calculated.

The objectives of this study are to develop an improved analytical method of estimating the coolant pump induced pressure distribution in the annulus of PWR including the inlet pipe line and to verify the analytical method by comparing with the measurements in the Precritical Vibration Monitoring Programs (PVMP). Finally, the geometric parameter effect on the coupling effect between the inlet pipe and the annulus is evaluated. Numerical results for typical reactor steady state pressure pulsations are presented for both the inlet pipe and the annulus.

2. Theoretical Analysis

The geometric representation of the inlet pipe and annulus is shown in Fig 3. It has been assumed that the inlet pipe of length L_I can be represented as being straight, rigid and having wave propagation in the x direction with speed of sound C_I . The pressure pulsations are plane waves so that their propagations in the pipe may be assumed one dimensional. The annulus with length L_{II} and speed of sound C_{II} is bounded by the core support barrel and reactor vessel. The radii a_1 , a_2 correspond to the outer radius of the core support barrel and inner radius of the reactor vessel, respectively. The area bounded by z_1 and z_2 in the vertical direction and θ_1 and θ_2 in the cir-

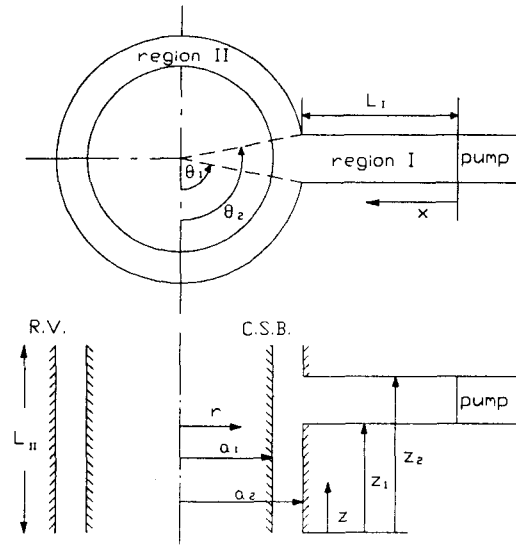


Fig. 3. The Geometry of the Inlet Pipe and Annulus

cumferential direction corresponds to the area of the inlet pipe, and is located at the interface of an inlet pipe/annulus. The core support barrel and the reactor vessel are assumed rigid wall except at the inlet pipe/annulus interface.

In deriving the governing hydrodynamic differential equations, the fluid is taken to be compressible and inviscid. Linearized versions of the equations of motion and continuity are used. The governing equations are homogeneous acoustical wave equations for pressure with non-homogeneous time dependent boundary conditions.

2.1. Analysis in the Inlet Pipe (region I)

The mathematical description of the inlet pipe region is a one dimensional acoustic wave equation.

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{C_I^2} \frac{\partial^2}{\partial t^2} \right) P_I = 0 \quad (1)$$

The excitation for the system occurs at the pump discharge ($x=0$). The boundary condition is;

$$P_I(x=0) = P_D \cos \omega_p t \quad (2)$$

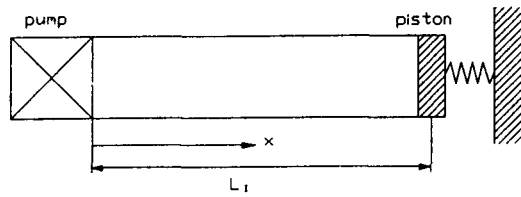


Fig. 4. Boundary Condition of Piston-Spring Supported end

where P_D can be obtained through analytical means[7], is the pressure wave at the pump discharge and ω_P is the associated pump forcing frequency.

The boundary condition at the inlet pipe/annulus interface is assumed to be piston-spring supported. Fig. 4 represents this condition. The equilibrium of forces at the piston leads to the following condition;

$$\left(\frac{\partial P_1}{\partial x} - \frac{\rho A}{K} \frac{\partial^2 P_1}{\partial t^2} \right)_{x=L_1} = 0 \quad (3)$$

where A is the pipe cross-sectional area, ρ is the fluid density, and K is the spring constant.

Eqs. (1)–(3) describe the boundary value problem with the time dependent non-homogeneous boundary condition.

An analytical method was developed for predicting a one dimensional pump-induced fluctuating pressures using a boundary operator method which reduces a homogeneous differential equation with non-homogeneous boundary condition into a non-homogeneous differential equation with homogeneous boundary condition[6].

To render the boundary condition homogeneous, the following equation is used;

$$P_1(x, t) = Q_1(x, t) + g_1(x)f_1(t) + g_2(x)f_2(t) \quad (4)$$

where $g_1(x)$ and $g_2(x)$ are auxiliary functions and Q_1 is the transform function. Substitution of Eq.(4) into Eqs.(2) and (3) yields the following homogeneous boundary condition for $Q_1(x, t)$;

$$Q_1(0, t) = 0 \quad (5)$$

$$\left(\frac{\partial Q_1}{\partial x} - \frac{\rho A}{K} \frac{\partial^2 Q_1}{\partial t^2} \right)_{x=L_1} = 0 \quad (6)$$

If we set

$$g_1(0) = 1, \quad g_1(L_1) = 0$$

$$\text{and } f_1(t) = P_D \cos \omega_P t, \quad f_2(t) = a \cos \omega_P t \quad (7)$$

then, $g_2(x)$ and $f_2(t)$ should satisfy

$$g_2(0) = 0, \quad g_2(L_1) = 1$$

$$\text{and } g_2'(L_1)f_2(t) + \frac{\rho A \omega_P^2}{K} g_2(L_1)f_2(t) + g_1'(L_1)f_1(t) = 0 \quad (8)$$

These are the only constraints placed upon the auxiliary function $g_1(x)$, $g_2(x)$ and they are arbitrary $0 < x < L_1$.

Eqn.(1) is transformed to a non-homogeneous differential equation;

$$\frac{\partial^2 Q_1}{\partial x^2} - \frac{1}{C_1^2} \frac{\partial^2 Q_1}{\partial t^2} = - \sum_{i=1}^2 \left(g_i'' f_i - \frac{1}{C_1^2} g_i f_i \right) \quad (9)$$

Natural frequencies of the water can be obtained from the homogeneous expression of Eqn.(9), using the separation of variables;

$$\frac{\partial^2 X_n}{\partial x^2} + \frac{\omega_n^2}{C_1^2} X_n = 0 \quad (10)$$

$$X_n(x=0) = 0, \quad \left(\frac{\partial X_n}{\partial x} + \frac{\rho A \omega_n^2}{K} X_n \right)_{x=L_1} = 0 \quad (11)$$

Solving Eqn.(10) with the boundary conditions of Eqn.(11), we obtain the frequency equation and normal modes;

$$\tan \left(\frac{\omega_n}{C_1} L_1 \right) + \frac{K}{\rho A C_1 \omega_n} = 0 \quad (12)$$

$$X_n = \sin \left(\frac{\omega_n}{C_1} x \right) \quad (13)$$

Because of the boundary condition (Eqn. (11)), the following orthogonality relations are obtained. For $n \neq m$:

$$\frac{1}{C_1^2} \int_0^{L_1} X_n X_m dx - \frac{\rho A}{K} X_n(L_1) X_m(L_1) = 0 \quad (14)$$

$$\int_0^{L_1} \frac{dX_n}{dx} \frac{dX_m}{dx} dx = 0 \quad (15)$$

$$\int_0^{L_1} \frac{d^2 X_n}{dx^2} dX_m dx - \frac{dX_n(L_1)}{dx} X_m(L_1) = 0 \quad (16)$$

Solution of the non-homogeneous Eqn.(9) is sought in the form;

$$Q_1(x, t) = \sum_{n=1}^{\infty} X_n(x) T_n(t) \quad (17)$$

Substituting Eqn.(17) into Eqn.(9), multiplying both sides by $X_m(x)$, integrating from 0 to L_1 and utilizing the orthogonality relations of the eigenfunction, we can yield the pressure solution of region I.

$$\ddot{T}_n + \omega_n^2 T_n = \frac{1}{I_n} \int_0^{L_1} \left[\sum_{i=1}^2 (g_i'' f_i - \frac{1}{C_1^2} g_i f_i) \right] X_n dx \quad (18)$$

where

$$I_n = \frac{1}{C_1^2} \int_0^{L_1} X_n^2 dx - \frac{\rho A}{K} X_n^2(L_1) \quad (19)$$

Neglecting the natural vibration modes by assuming the existence of damping, the solution of Eqn.(18) from zero initial condition is obtained as follows:

$$T_n(t) = \frac{1}{I_n \omega_n} \sum_{i=1}^2 \left[g_i' X_n \right]_0^{L_1} \int_0^t [f_i(\tau) \sin \omega_n(t-\tau)] d\tau - \frac{\omega_n}{C_1^2} f_i(t) \int_0^{L_1} g_i X_n dx > (20)$$

Using Eqns. (4), (13), (17) and (20), the following expression for $P_1(x, t)$ is obtained as follows:

$$P_1(x, t) = \sum_{n=1}^{\infty} \frac{P_D I_{2n} + \alpha I_{3n}}{I_{1n}} \cos \omega_n t \sin \left(\frac{\omega_n}{C_1} x \right) + g_1(x) f_1(t) + g_2(x) f_2(t) \quad (21)$$

where,

$$I_{1n} = \frac{L_1}{2} + \frac{C_1}{4\omega_n} \sin \left(\frac{2\omega_n}{C_1} L_1 \right) \quad (22)$$

$$I_{2n} = \frac{C_1^2}{L_1^2 \omega_n^2 + C_1^2} \left[2.164 \left(\frac{C_1}{\omega_n} \right) \left(1 - \cos \left(\frac{\omega_n L_1}{C_1} \right) \right) - L_1 \sin \left(\frac{\omega_n L_1}{C_1} \right) \right] - \frac{C_1}{\omega_n} \left[1 - \frac{C_1}{L_1 \omega_n} \sin \left(\frac{\omega_n L_1}{C_1} \right) \right] - \frac{C_1 \omega_n}{(\omega_p^2 - \omega_n^2)} \quad (23)$$

$$I_{3n} = \frac{C_1^2}{L_1^2 \omega_n^2 + C_1^2} \left[\frac{L_1^2 \omega_n}{C_1} \left(1 - \left(\frac{\omega_p}{\omega_n} \right)^2 \right) \cos \left(\frac{\omega_n L_1}{C_1} \right) - \frac{C_1}{\omega_n} \left(1 - \cos \left(\frac{\omega_n L_1}{C_1} \right) \right) \left(2.841 + 2.164 \frac{\rho A \omega_p^2 L_1}{K} \right) \right] + \frac{C_1 \omega_n \cos \left(\frac{\omega_n L_1}{C_1} \right)}{(\omega_p^2 - \omega_n^2)} \left(1 - \left(\frac{\omega_p}{\omega_n} \right)^2 \right) \quad (24)$$

$$\alpha = P_D \frac{\sum_{n=1}^{\infty} \frac{I_{2n} \cos \left(\frac{\omega_n L_1}{C_1} \right)}{I_{1n} C_1 \omega_n} (\omega_n^2 - \omega_p^2)}{\sum_{n=1}^{\infty} \frac{I_{3n} \cos \left(\frac{\omega_n L_1}{C_1} \right)}{I_{1n} C_1 \omega_n} (\omega_p^2 - \omega_n^2)} \quad (25)$$

$$g_1(x) = 1 - \frac{x}{L_1} \quad (26)$$

$$g_2(x) = \frac{\beta(e-1)-1}{(e-1)^2} e^{-\frac{x}{L_1}} - \frac{\beta(e-e^2)+e^2}{(e-1)^2} e^{-\frac{x}{L_1}} + \frac{\beta(1-e^2)+(e^2+1)}{(e-1)^2} \quad (27)$$

$$\beta = -\frac{\rho A \omega_p^2 L_1}{K} + \frac{P_D}{\alpha} \quad (28)$$

2.2. Analysis in the Annulus(region II)

Annulus region can be described by a three dimensional wave equation in cylindrical coordinate as follows;

$$(\nabla^2 - \frac{1}{C_{II}^2} \frac{\partial^2}{\partial t^2}) P_{II}(\vec{r}, t) = 0 \quad (29)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

and with radial boundary conditions

$$\left(\frac{\partial P_{II}}{\partial r} \right)_{r=a_1} = 0, \quad (30-a)$$

$$\left(\frac{\partial P_{II}}{\partial r} \right)_{r=a_2} = \gamma h(\theta) h(z) \cos \omega_p t \quad (30-b)$$

and axial boundary conditions

$$\text{open end: } P_{II}(\vec{r}, t) |_{z=0} = 0, \quad (30-c),$$

$$\text{closed end: } \left(\frac{\partial P_{II}(\vec{r}, t)}{\partial z} \right)_{z=L_0} = 0 \quad (30-d)$$

The radial boundary conditions at the reactor vessel have been described by the use of Heaviside step function, by defining $h(z) = H(z - z_1) - H(z - z_2)$ and $h(\theta) = H(\theta - \theta_1) - H(\theta - \theta_2)$. Eqns.(3) and (30-b) couple the two regions by requiring continuity of the pressure gradient at the interface of region I and region II.

$$\gamma = \left(\frac{\partial P_I}{\partial x} \right)_{x=L_1} \quad (31)$$

The solution of the Eqn.(29) is difficult in its present form. This description is transformed to obtain a non-homogeneous differential equation with homogeneous boundary conditions.

Assume a solution of the form;

$$P_{II}(\vec{r}, t) = Q_{II}(\vec{r}, t) + h(z)h(\theta)g(r)\gamma \cos \omega_p t \quad (32)$$

Eqn.(29) becomes,

$$(\nabla^2 - \frac{1}{C_{II}^2} \frac{\partial^2}{\partial t^2}) Q_{II}(\vec{r}, t) = -(\nabla^2 - \frac{1}{C_{II}^2} \frac{\partial^2}{\partial t^2}) [h(z)h(\theta)g(r)\gamma \cos \omega_p t] \quad (33)$$

Boundary conditions (30-a)–(30-d) become,

$$\begin{aligned} \left(\frac{\partial Q_{II}(\vec{r}, t)}{\partial r} \right)_{r=a_1} \\ = -h(z)h(\theta)\gamma \cos \omega_p t \left(\frac{dg(r)}{dr} \right)_{r=a_1} \end{aligned} \quad (34-a)$$

$$\begin{aligned} \left(\frac{\partial Q_{II}(\vec{r}, t)}{\partial r} \right)_{r=a_2} &= h(z)h(\theta)\gamma \cos \omega_p t \\ &- h(z)h(\theta)\gamma \cos \omega_p t \left(\frac{dg(r)}{dr} \right)_{r=a_2} \end{aligned} \quad (34-b)$$

$$\text{open end: } Q_{II}(\vec{r}, t) |_{z=0} = 0 \quad (34-c),$$

$$\text{closed end: } \left(\frac{\partial Q_{II}(\vec{r}, t)}{\partial z} \right)_{z=L_0} = 0 \quad (34-d)$$

All boundary conditions are now homogeneous except Eqns.(34-a) and (34-b). However, these boundary conditions can be made homogeneous by placing the following restrictions on the auxiliary function $g(r)$;

$$\left(\frac{dg(r)}{dr} \right)_{r=a_1} = 0 \quad (35-a), \quad \left(\frac{dg(r)}{dr} \right)_{r=a_2} = 1 \quad (35-b)$$

Using the separation of variables, the pressure distribution of region II can be calculated. The detailed procedure is in Cepkauskas[8].

$$\begin{aligned} P_{II}(\vec{r}, t) \\ = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \sum_{s=1}^{\infty} \gamma C_{nms} Q_n(z) Q_m(\theta) Q_s(r) \cos \omega_p t \end{aligned} \quad (36)$$

$$\text{where, } C_{nms} = \frac{C_{II}^2 a_2 Q_s(a_2) I_1}{(\omega_{nms}^2 - \omega_p^2) I_2} \quad (37)$$

$$I_1 = \int_{\theta_1}^{\theta_2} Q_m(\theta) d\theta \int_{z_1}^{z_2} Q_n(z) dz \quad (38)$$

$$I_2 = \int_0^{L_0} \int_0^{2\pi} \int_{a_1}^{a_2} Q_n^2(z) Q_m^2(\theta) Q_s^2(r) r dr d\theta dz \quad (39)$$

$$Q_n(z) = \cos \beta_n z, \quad \beta_n = \frac{n\pi}{2L_{II}} \\ n = 1, 3, 5 \dots \quad (40)$$

$$Q_m(\theta) = \cos m\theta, \quad m = 0, 1, 2, 3 \dots \quad (41)$$

$$Q_s(r) = J_m(\tau_{nms}r) + \eta_{nms}Y_m(\tau_{nms}r) \quad (42)$$

J_m and Y_m are Bessel functions of the first and second kind.

η_{nms} are given by;

$$\eta_{nms} = - \left(\frac{\frac{d[J_m(\tau_{nms}r)]}{dr}}{\frac{d[Y_m(\tau_{nms}r)]}{dr}} \right)_{r=a_1} \quad (43)$$

$$\text{where } \tau_{nms}^2 = \frac{\omega_{nms}^2}{C_{II}^2} - \beta_n^2 \quad (44)$$

The natural frequencies, ω_{nms} , are obtained from the transcendental equation;

$$\left(\frac{d[J_m(\tau_{nms}r)]}{dr} \right)_{r=a_2} = \eta_{nms} \left(\frac{d[Y_m(\tau_{nms}r)]}{dr} \right)_{r=a_2} \quad (45)$$

The unknown constant, K , can now be obtained by requiring pressure continuity at the inlet pipe/annulus interface;

$$P_I(L_I, t) \\ = P_{II}(r=a_2, \theta = \frac{(\theta_1 + \theta_2)}{2}, z = \frac{(z_1 + z_2)}{2}, t) \quad (46)$$

Substitution Eqns.(21) and (36) into (46) results in

$$\sum_{n=1}^{\infty} \left[\frac{P_{DI_{2n}} + \alpha I_{3n}}{I_{1n}} \sin \left(\frac{\omega_n L_I}{C_I} \right) \right] + \alpha = \quad (47)$$

$$\sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \sum_{s=1}^{\infty} \gamma_{nms} Q_n \left(\frac{z_1 + z_2}{2} \right) Q_m \left(\frac{\theta_1 + \theta_2}{2} \right) Q_s(a_2)$$

where,

$$\gamma = \sum_{j=1}^{\infty} \left[\frac{P_{DI_{2j}} + \alpha I_{3j}}{I_{1j}} \frac{\omega_j}{C_I} \cos \left(\frac{\omega_j L_I}{C_I} \right) \right] \\ - \frac{\rho A \omega_j^2 \alpha}{K} \quad (48)$$

The solution of region I and region II is given by Eqns.(21) and (36) with the unknown constant K obtained by Eqn.(47).

3. Numerical Results and Discussion

Analytical results for typical reactor pressure pulsations are presented for both the inlet pipe and the annulus. Test data obtained from the ABB-CE's Precritical Vibration Monitoring Programs[9, 10, 11] are compared with predictions based on the present analysis.

The dimensions of the inlet pipe and annulus considered are determined by the following quantities;

• Maine Yankee reactor

$$a_1 = 1.943(\text{m}), a_2 = 2.184(\text{m}), L_{II} = 8.242(\text{m}) \\ z_1 = 5.978(\text{m}), z_2 = 6.969(\text{m}), \theta_1 = 77.22^\circ, \theta_2 = 102.78^\circ \\ L_I = 8.174(\text{m}), D = 0.991(\text{m})$$

• Palo Verde reactor

$$a_1 = 2.070(\text{m}), a_2 = 2.315(\text{m}), L_{II} = 10.439(\text{m}) \\ z_1 = 6.415(\text{m}), z_2 = 7.309(\text{m}), \theta_1 = 80.65^\circ, \theta_2 = 99.35^\circ \\ L_I = 7.395(\text{m}), D = 0.762(\text{m})$$

• YGN 3&4 reactor

$$a_1 = 1.786(\text{m}), a_2 = 2.057(\text{m}), L_{II} = 10.439(\text{m}) \\ z_1 = 6.424(\text{m}), z_2 = 7.320(\text{m}), \theta_1 = 79.51^\circ, \theta_2 = 100.49^\circ \\ L_I = 7.374(\text{m}), D = 0.762(\text{m})$$

The sonic velocities of the coolant are $C_I = C_{II} = 1030.5(\text{m/s})$ at $T = 277.8^\circ\text{C}$, $C_I = C_{II} = 1516.1(\text{m/s})$ at $T = 126.7^\circ\text{C}$ and $C_I = C_{II} = 988.1(\text{m/s})$ at $T = 296.1^\circ\text{C}$.

The calculated pressure pulsations in the inlet nozzle and the corresponding Precritical Vibration Monitoring Programs data are represented in Table 1. As can be seen from the table, comparison of predicted

Table 1. Predicted Pressure Versus PVMP Pressure at the Inlet Nozzle

Plant & Test condition			Forcing Frequency								
			20 Hz		40 Hz		100 Hz		200 Hz		
Maine Yankee reactor	277.8 (°C)	PVMP data(kPa)	0.393	0.151	0.903	1.371					
		Predicted value(kPa)	0.289	0.103	0.455	0.648					
	126.7 (°C)	PVMP data(kPa)	0.496	0.510	1.771	0.868					
		Predicted value(kPa)	0.214	0.296	0.606	0.544					
	Omaha reactor	277.8 (°C)	PVMP data(kPa)	0.530	0.227	0.413	0.641				
			Predicted value(kPa)	0.214	0.100	0.620	1.991				
126.7 (°C)		PVMP data(kPa)	0.517	0.151	0.662	0.827					
		Predicted value(kPa)	0.145	0.145	1.309	2.246					
			20Hz	40Hz	120Hz	240Hz	360Hz	480Hz			
Palo Verde reactor	296.1 (°C)	PVMP data(kPa)	0.903	0.296	1.105	6.902	0.689	1.372			
		Predicted value(kPa)	0.910	0.303	4.999	13.934	1.793	2.654			
YGN 3&4 reactor	288.1 (°C)	PVMP data(kPa)	not measured								
		Predicted value(kPa)	0.883	0.131	9.198	4.247	0.862	1.641			

and measured values for each forcing frequency shows good order of magnitude agreement, considering the design safety factor. The prediction results for the inlet nozzle pressure show the same trend (peak pressure) as the measured data. For each forcing frequency and the geometry parameters, the spring constant, K , is determined by coupling of region I and region II from the pressure continuity at the interface.

The Palo Verde and YGN 3&4 reactors note an adverse trend for inlet pressure at 120 and 240 Hz respectively. The inlet pipe lengths of both Palo Verde and YGN 3&4 reactor are approximately equal.

Thus, it is evident that the radial gap size to the reactor vessel radius is important parameter. Figs. 5 and 6 show the computed acoustic pressure distribution in the annulus (inner wall, $z=L_{II}$) for each forcing frequency. The pressures were normalized to a maximum of unity. The YGN 3&4 pressure in phase lags compared with the Palo Verde pressure. From this numerical result, it is known that the coupling of the inlet pipe and the annulus is essential in predicting acoustic pressure pulsation in the reactor internals. To perform the parametric study on the coupling effect, more experimental data should be accumulated.

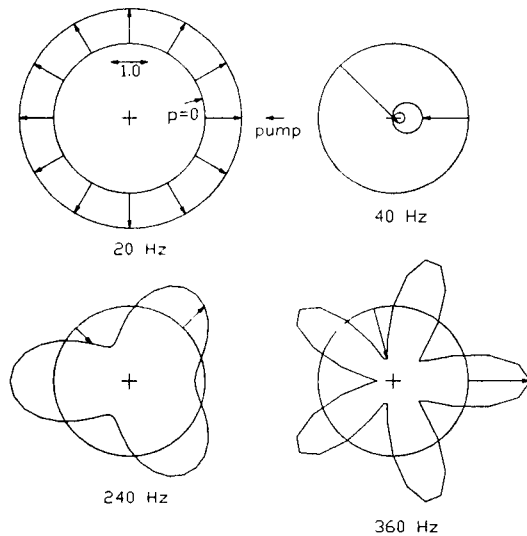


Fig. 5. Normalized Circumferential Pressure for Palo Verde

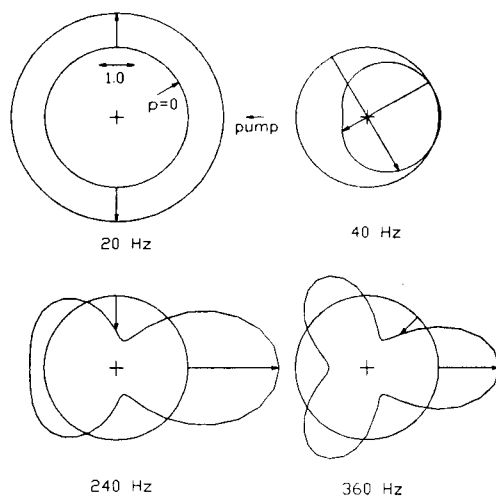


Fig. 6. Normalized Circumferential Pressure for YGN 3&4

4. Conclusions

Numerical results for typical reactor pressure pulsations caused by the pump are presented for both the inlet pipe and the annulus region. A transform-

ation technique is utilized to reduce each region to a non-homogeneous differential equations with homogeneous boundary conditions. By coupling the solution of the inlet pipe and annulus, the present solution permits determination of the inlet nozzle pressure.

The analytical results, using the present methodology, are shown to be in good order of magnitude agreement with reactor test data from the Precritical Vibration Monitoring Program. They illustrate the important effect of the coupling between the inlet pipe and annulus region on the inlet nozzle pressure. This analytical method can be applied to predict the design value for the pump-induced pulsating pressure of the reactor internals.

Nomenclature

A	area of inlet pipe
a_1	inner radius of annulus (outer radius of core support barrel)
a_2	outer radius of annulus (inner radius of reactor vessel)
C	sound velocity
C_{rms}	Fourier coefficient
D	diameter of inlet pipe
I_1, I_2	
I_{1n}, I_{2n}, I_{3n}	constants
K	spring constant
L_I	length of inlet pipe
L_{II}	length of annulus
P	pressure
Q	eigenfunction
$\vec{r} (r, \theta, z)$	location vector
t	time
T_n	time function
x	axial coordinate in the inlet pipe
r, z	radial and axial coordinates in the annulus
z_1, z_2	axial coordinates of inlet nozzle
α, β, γ	arbitrary constants

ρ	density
ω_n	natural frequency of liquid in the inlet pipe
ω_{nms}	natural frequency of liquid in the annulus
ω_p	forcing frequency
θ	circumferential coordinate

Subscripts

I	region I
II	region II

References

1. L.E. Penzes, "Theory of Pump-Induced Pulsating Coolant Pressure in Pressurized Water Reactors", *Nucl. Engrg. Des.*, **27**, 176 (1974)
2. G. Bowers and G. Horvay, "Forced Vibrations of a Shell inside a Narrow Water Annulus", *Nucl. Engrg. Des.*, **34**, 221 (1975)
3. M.K. Au-Yang, 1979, "Pump-Induced Acoustic Pressure Distribution in an Annular Cavity Bounded by Rigid Walls", *J. Sound & Vibration*, **6**, 577 (1979)
4. H.D. Fisher, M.M. Cepkauskas & S. Chandra, "Solution of Time Dependent Boundary Value Problems by the Boundary Operator Method", *Int. J. Solids Structure*, **15**, 607 (1979)
5. L. Lee and S. Chandra, "Pump-Induced Fluctuating Pressure in a Reactor Coolant Pipe", *Int. J. Pressure Vessels & Piping*, **8**, 407 (1980)
6. K.B. Lee, I.Y. Im and S.K. Lee, "Analytical Prediction on the Pump-Induced Pulsating Pressure in a Reactor Coolant Pipe", *Int. J. Pressure Vessels & Piping*, **52**, 417 (1992)
7. H.C. Simpson, T.A. Clark & G.A. Weir, "A Theoretical Investigation of Hydraulic Noise in Pumps", *J. Sound & Vibration*, **5**, 456 (1967)
8. M.M. Cepkauskas, DPVIB Computer Code Manual, Combustion Engineering Inc., SP80-STA-20 (1977)
9. Anon, Maine Yankee Precritical Vibration Monitoring Program, Combustion Engineering Inc., CENPD-93 (1973)
10. G.H. Allen, Omaha Precritical Vibration Monitoring Program, Combustion Engineering Inc., CEN-7 (1974)
11. CE, A Comprehensive Vibration Assessment Program for Palo Verde Nuclear Generating Station Unit 1, Combustion Engineering Inc., CEN-263(V)-P, Rev. 1-P (1984)