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Nonlinear Stochastic Stability for Steam Generator Water Level Control System

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증기발생기 수위제어의 확률론적 안정성

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Abstract

The steam generator water level control system is studied as a class of randomly sampled nonlinear control systems. The sampling interval and the loop amplification factor are considered as random variables in order to take the operator behavior in account. Stochastic stability using Lyapunov method is used without determining such Lyapunov function. The derived stability criterion is verified with time-domain simulation using the data of CANDU type nuclear power plant, Wolsung 1.

요 약

증기발생기 수위조절계통의 무작위추출 비선형 제어계통의 경우로 연구되었다. 무작위 변수로는 시간 불연속 계통의 추출시간간격이 고려되었다. Lyapunov 함수를 구하지 않는 확률론적 Lyapunov 방법이 사용되었다. 유도된 안정성 요건은 CANDU 형 원자로인 월성 1호기의 자료를 이용하여 시간 존속 모사로 검증하였다.

1. Introduction

The steam generator water level control is the one which plays fundamental roles in a nuclear power plant because it ensures sufficient cooling of the reactor and good performance of the steam separators and dryers. As a result, a good control system proves to be a determining factor in overall plant availability and safety.

There have been so many studies on the steam generator water level control. After Strohmayer[2] had introduced a simple method of modeling the dynamics of the steam generator, studies have been developed regarding with the steam generator. Optimal controllers[4], suboptimal controllers[5, 6], and PID controllers[3] on the basis of set theory have been developed for the featuring nonlinearity of the steam generator water level control system. Although the acceptability and the stability of those controllers can be thought to work well with the practical steam generator, their validities are useless in the case of the whole failure of computer system.

In practice, CANDU reactors use a dual computer system for annunciation and control. The occurrence frequency of a dual computer failure is 3×10^{-1} events per year with an unavailability of 2×10^{-4}. This means that an incident of "Dual Computer Failure" occurs every third year. It is meaningful to investigate the operator behavior thoroughly in this specific CANDU incident since the future generation of nuclear power plant would be computer controlled, a study of such incident should be performed thoroughly.

Unfortunately, however, this case of human control has not yet been studied in the field of steam-power production. Hence, in this study, the stability of the human operator's control is the main interests and some stability criterion matching this case is developed.

2. Steam Generator Water Level Control System

Among the modeling of steam generator performed by a number of studies, the result of Strohmayer[2] has been adopted in the field of control theory of nuclear power plant because of its well-fitting behavior of simulating steam generator. On the basis of his thesis, Suh & Noh[3] made their simple model of reduced order system. The linear continuous system of steam generator can be written in a state-space model as:

$$\dot{x} = Ax + Bu + Fd, \qquad x(0)=0$$

$$y = C^T x \tag{1}$$

where

$$x = (\delta L_w, \delta T_d, \delta p, \delta x_r, \delta W_r, \delta T_m)^T$$

$$u = (\delta W_{fd})$$

$$d = (\delta W_s)$$

$$y = (\delta L_w)$$

The steam flow, which is determined by overpressure protection system as safety valves, is considered as a disturbance. so the disturbance-like represented state d in Eq.(1) can be therefore neglected in the sequel. In the remainder of this study, the system matrix A may be assumed diagonal with its eigenvalues in the diagonal elements. This diagonalization does a good performance to the integration for the estimation of mean values.

3. Manual Control and Random Concept

There are 3 remarkable features in this study. These come from the particular characteristics of steam generator control system and manual control[1].

1. Task duration is not periodic but random.

At the given accident situation, the operator should control the four distinct steam generator water levels of CANDU type reactor. However, he cannot pay attention to all four signals and moreover the environments (pressurizer level, turbine generator, etc.). Hence, the operator can control the levels of four steam generators only in a sequential manner. Since the operator controls in a sequential manner, there exists some time interval between controlling steam generator water levels. In the high load condition such as dual computer failure, the operator cannot memorize the informations in his working memory. The time interval (task duration afterwards) is a delay such that if it is increased under emergent situation the rate of forgetting will be higher. Therefore every trial of a level control is different to be regarded as random.[1]

At $t=t_k$ the operator begins with the control task for S/G 1 and at $t=t_k+\tau_{k,1}$ finishes with it. He moves under circumstance to the environment and perform the environment task until $t=t_k+\tau_{k,1}+\xi_{k,1}$. At $t=t_k+\tau_{k,1}+\xi_{k,1}$, he begins with the level control task for S/G 2 and finishes with it at $t=(t_k+\tau_{k,1}+\xi_{k,1})+\tau_{k,2}$. The environmental task accompanying steam generator 2 is accomplished at $t=(t_k+\tau_{k,1}+\xi_{k,1})+\tau_{k,2}+\xi_{k,2}$.

At $t_{k+1}=t_k+(\tau_{k,1}+\xi_{k,1})+(\tau_{k,2}+\xi_{k,2})+(\tau_{k,3}+\xi_{k,3})+(\tau_{k,4}+\xi_{k,4})$, the turbine operator finishes with the task accompanying S/G 4 and can begin with the task for S/G 1 again. Also, the task cycle is reinitiated. In every incident, the heat transport system of CANDU-plant can be separated in two independent loops. By choosing an appropriate numerating scheme, for example, S/G 1→Loop1, S/G 2→Loop2, S/G 3→Loop1, and S/G 4→Loop2, the duration $\{\tau_{k,1}, \tau_{k,2}\}$ of the control subtask can be made stochastically independent. Using similar argument we justify the stochastic independence of the pair $\{\xi_{k,1}, \xi_{k,2}\}$. Now we can easily see stochastic independence of the following pairs :

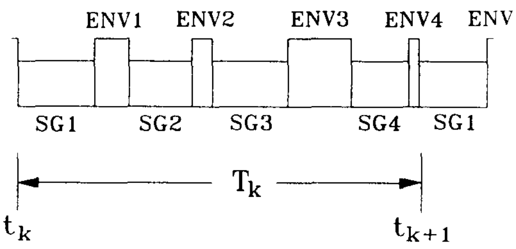

Fig. 1. The Example of S/G Water Level Control by Operator

$$\{(\tau_{k,1}+\zeta_{k,1}),(\tau_{k,2}+\zeta_{k,2})\},\{\tau_{k,2}+\zeta_{k,2}),$$
$$(\tau_{k,3}+\zeta_{k,3})\},\{\tau_{k,3}+\zeta_{k,3}),(\tau_{k,4}+\zeta_{k,4})\}$$

Especially, in case of a small perturbation from the steady state, the steam generators are only loosely coupled even in identical loop, so that we can assume the stochastic independence of the pair :

$\{(\tau_{k,1}+\zeta_{k,1}),(\tau_{k,3}+\zeta_{k,3})\}$ and also

$\{\tau_{k,2}+\zeta_{k,2}),(\tau_{k,4}+\zeta_{k,4})\}$

Now we consider $\{(\tau_{k,1,1}, \xi_{k,1,1})\}$ the duration of a task, which the operator performed at S/G 1 during the *(k+1)-th* task cycle.

Since the following equation holds :

$$t_{k+1} = t_k + \sum_{i=1}^{4}(\tau_{k,i}+\zeta_{k,i})$$

we can take the duration of a task-cycle as :

$$T_k \equiv t_{k+1} - t_k = \sum_{i=1}^{4}(\tau_{k,i}+\zeta_{k,i})$$

Since the operator accomplish, in a task cycle, a lot of tasks as ;

$\{\tau_{k,1}\},\{\zeta_{k,1}\},\cdots,\{\tau_{k,4}\},\{\zeta_{k,4}\}$

the operator who has to handle the dual computer failure is in such a stressful situation that he cannot remember what he has performed at S/G 1 a long time ago. Therefore, we have no reason to assume a strong stochastic dependence.

On the base of this argument, we conclude that between $\{\tau_{k,1,i}, \xi_{k,1,i}\}$ and $\{\tau_{k,i}, \xi_{k,i}\}$, $i\in[1, 2, 3, 4]$ there exist no stochastic dependence. To generalize above argument, from now on we will use the stochastic independency of the pair $\{T_k, T_{k+1}\}$, i.e., the sequence of the task duration

$\{T_k, k\in N\}$

is assumed in this work to be a white process. Especially in the neighborhood of steady state, we can assume that

$$\bigwedge_{k\in N} E\{T_k\} = T_m = Const.$$

$$\bigwedge_{k\in N} E\{(T_k-T_m)^2\} = \sigma_T^2 = Const.$$

In other word, the sequence of task duration is a stationary white process.

$\{T_k, k \in N\}$

As described above, the sequence of the task duration $\{T_k, k \in N\}$ can be dealt as a Markov process since it has the properties of stochastic independence and stationary white process.

2. The amplification factor of the control variable is stochastic.

With a similar arguments, we can demonstrate that the other characteristic describing the operator behaviour, namely, the sequence of the amplification factor $\{K(k), k \in N\}$ also can be considered as a stationary white process. In Oh[9], the control system was an artificial heart, so that control input had fixed amplification factor. But in case of manual control, this varies with statistics.

In the practical sense, the human operator cannot make his control regularly. The control tool for the feedwater control is the valve positioner attached to the control dial on the control panel. In case of P-controller, it compares the output $y(k)$ with its reference $r(k)$ and subtracts the reference from the output $y(k)$, then the result $e(k) = y(k) - r(k)$, that is, the control error is multiplied with the amplification factor K. The value $K * e(k) = K * [y(k) - r(k)]$ is now the input for the actuator block :

The computing capability of the machine is obviously superior to man, especially in precision and velocity. On the other hand, the human operator makes error by subtraction operation

$$e(k) = y(k) - r(k) = y(k) - r(k) - \varepsilon(k) + \varepsilon(k)$$

$$= e_{human}(k) + \varepsilon(k)$$

$$\Leftrightarrow \quad e_{human}(k) = e(k) - \varepsilon(k)$$

By multiplication operation the human operator makes the error again :

$$[\,K + \Delta K(k)]\; e_{human}(k)$$

$$= K\, e_{human}(k) + \Delta K(k)\; e_{human}(k)$$

$$= K * e(k) - K * \varepsilon(k) + \Delta K(k) e(k) - \Delta K(k) * \varepsilon(k)$$

$$= (K + \Delta K(k)) e(k) - (K + \Delta K(k))\varepsilon(k)$$

$$= K(k)\, e(k) - K(k)\varepsilon(k)$$

Considering the above concept of the human operator, we get the following result as figure 3 :

Just before the derivation of the stability criterion we consider that the amplification factor of the control variable should have a stochastic property as follow :

$$K(k) = K_m + \Delta K(k)$$

In this section the stability criterion will be derived with this stochastic concept of amplitude variable. Before advancing this procedure, the statistics of the amplitude variable will be given.

$$E\{K(k)\} = E\{\,K_m + \Delta K(k)\,\}$$

$$= K_m + E\{\Delta K(k)\} = K_m$$

$$E\{K^2(k)\} = E\{(K_m + \Delta K(k))^2\}$$

$$= K_m^2 + E\{\Delta K^2(k)\}$$

In this case the nonlinearity $\varphi[y(k)]$ will be replaced by the stochastic variable as ;

$$M(k) = K(k)\varphi[\,y(k)]$$

Then the expectation of the plant input are determined as

$$E\{M(k)\} = E\{K_m + \Delta K(k)\}\varphi[\,y(k)]$$

$$= (K_m + E\{\Delta K(k)\})\varphi[\,y(k)]$$

$$= K_m\, \varphi[\,y(k)]$$

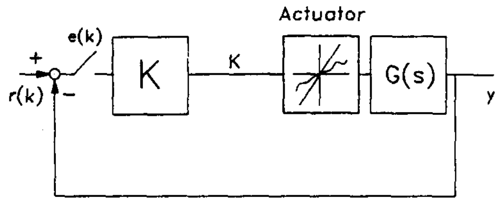

Fig. 2. Block Diagram of the Nonlinear Sampled Control System

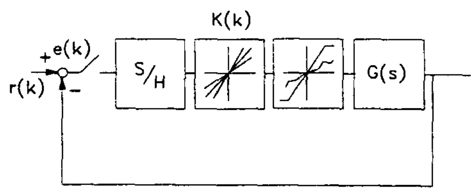

Fig. 3. Result of the Block Diagram Manipulation

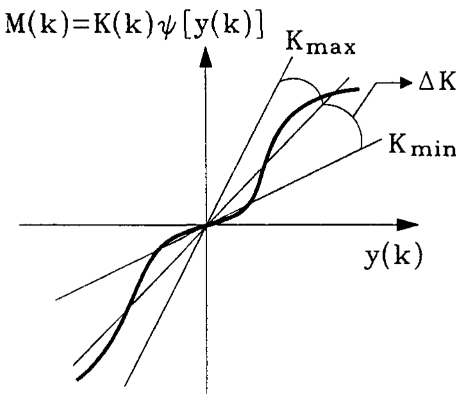

Fig. 4. Stochastic Nonlinearity

and the second moment:

$$E\{M^2(k)\} = E\{(K_m+\Delta K(k))^2\}\varphi^2[y(k)]$$
$$= (K_m^2+E\{\Delta K^2(k)\})\varphi^2[y(k)]$$

3. The Control System Contains a Pole on the Origin ($\lambda=0$).

In the steam generator water level control system, there exists an important feature in the procedure of frequency domain analysis. As already mentioned, because of the integrating property of steam generator system, the state space representation of steam generator, {A, B, c}, namely, the dynamic matrix A has a vanishing eigenvalue ($\lambda_0=0$). In other word, G (s) has a pole on the origin. Since the derived stability criterion in this paper is based on the stability of linear system, this pole on the origin of s-plane plays a key role. As seen in the criterion, this pole on the origin makes the conditions have zero denominator, which does not permit to evaluate the degree of stability any how.

In Strohmayer [2], some filters or units were applied to cancel this effect. But in this study, the view point is on the derivation of stability criterion, not on the analysis of steam generator. So, in order to handle the control system whose transfer function G(s) has a zero-valued pole, the result in Oh[9] is extended further.

4. Stochastic Stability

Eq. (1) can be discretized with one pole on the origin of s-plane as the following stochastic difference equation system[7]:

$$\begin{bmatrix} x_s(k+1) \\ \zeta(k+1) \end{bmatrix} = \begin{bmatrix} \Phi_s[T(k)] & 0 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} x_s(k) \\ \zeta(k) \end{bmatrix} - \begin{bmatrix} h_s[T(k)] \\ h_R \end{bmatrix} \varphi[y(k)]$$

$$y(k) = [c_s^T, c_R] \begin{bmatrix} x_s(k) \\ \zeta(k) \end{bmatrix} \quad (2)$$

As a candidate for the stochastic Lyapunov function, the following quadratic function is selected:

$$V[x(k)] = x^T(k)Hx(k) + \zeta(k)H_\zeta\zeta(k) \quad (3)$$

In order to derive the stability criterion for the given system, the necessary theorems are used[7]

Lemma 1

The origin is exponentially stable to the degree $\lambda > 0$ 'with probability 1' in $U(0, \delta)$, if it is stable and for all $k \in N$ and for any $0 < r < 1$

$$E\{V[x(k+1)] \,|x(k)\} - V[x(k)] \le -\gamma V[x(k)] \quad (4)$$

Lemma 2 Kalman Yakubovitch Lemma for the Sampling System

The following lemma is the principle for the derivation of the stability criterion concerned. It is related closely to the primary Kalman Yakubovitch Lemma for the continuous system. Therefore in this work it is expressed as Kalman Yakubovitch Lemma. The derivation of the lemma mentioned here is different from the early works [10, 11], and are with the help of Parseval's equation and the fundamental lemma of the variations calculation, whereby the proof proceed more formal.

Given the globally controllable and observable sampling system:

$$x(k+1) = \Phi x(k) - h\, m(k)$$
$$y(k) = c^T x(k) \quad (5)$$

where

$$m(k) \in l_2[0, \infty] \equiv \left\{ (f(k), k \in N) \sum_{k=1}^{\infty} f^2(k) < \infty \right\}$$

$$\max_i |\lambda_i[\Phi]| < 1, \quad \lambda_i[\Phi] = \textit{eigenvalue of } \Phi$$

There exist a matrix $H > 0$, a vector u and a real number ρ^2 in such a way that :

$$\Phi^T H\Phi - H = -uu^T, \qquad \rho^2 = \frac{1}{K} - h^T Hh,$$

$$h^T H\Phi - \frac{1}{2}c^T = -\rho u^T \tag{6}$$

if and only if the condition, for all $|z| = 1$:

$$\frac{1}{K} + Re\{G(z)\} > 0, \text{ with } G(z) = c^T(zI-\Phi)^{-1}h \tag{7}$$

is satisfied.

Proof

I) Sufficiency

The discrete-time difference of $x^T(k+1)\ Hx(k)$ along the trajectory of (5) is :

$$\begin{aligned} x^T(k+1)Hx(k+1) - x^T(k)Hx(k) &= x^T(k)[\ \Phi^T H\Phi - H]\ x(k) \\ &\quad -2[\ h^T H\Phi - \frac{1}{2}c^T]\ x(k)m(k) \\ &\quad -[\ \frac{1}{K} - h^T Hh]\ m^2(k) \\ &\quad -[\ y(k) - \frac{m(k)}{K}]\ m(k) \end{aligned} \tag{8}$$

If $m(k) \in$ in l_2 $(0, \infty)$ and $x(0) = 0$, Eq.(8) can be summed to infinity as ;

$$\begin{aligned} 0 &= x^T(\infty)Hx(\infty) - x^T(0)Hx(0) \\ &= \sum_{k=0}^{\infty}\left(x^T(k+1)Hx(k+1) - x^T(k)Hx(k)\right) \\ &= \frac{1}{2\pi}Re\Big(\oint\Big[\ x^T(z)[\ \Phi^T H\Phi - H]\ x(z) \\ &\quad -2\left(h^T H\Phi - \frac{1}{2}c^T\right)x(z)m(z) \\ &\quad -\left(\frac{1}{K} - h^T Hh\right)m(z)m(\bar{z}) \\ &\quad -\left(y(z) - \frac{m(z)}{K}\right)m(\bar{z})\Big]\ \frac{dz}{jz}\Big\}, \text{for all } |z| = 1, \end{aligned} \tag{9}$$

after the application of Parseval's equation [8].

After selecting H such that

$$\Phi^T H\Phi - H = -uu^T \tag{10}$$

whereby $\underline{u}$ is determined through the following factorization :

$$\begin{aligned} &\frac{1}{K} + Re[\ c^T(zI-\Phi)^{-1}h] \\ &= (\rho + u^T(zI-\Phi)^{-1}h)(\rho + \overline{u^T}(zI-\Phi)^{-1}h), \end{aligned} \tag{11}$$

and inserting z-transformation of (5) into (9), Eq. (9) becomes :

$$\begin{aligned} &\frac{1}{2\pi}Re\Big\{\oint\Big[\ \frac{1}{K} + c^T(zI-\Phi)^{-1}h\Big]\ m(z)m(\bar{z})\frac{dz}{jz}\Big\} \\ &= \frac{1}{2\pi}\oint\Big\{[\ \rho + u^T(zI-\Phi)^{-1}h] \\ &\qquad [\ \rho + \overline{u^T}(zI-\Phi)^{-1}h] \\ &\qquad -2Re\Big[\ h^T H\Phi - \frac{1}{2}c^T + \rho u^T\Big]\ (zI-\Phi)^{-1}h \\ &\qquad + \Big[\ \frac{1}{K} - h^T Hh - \rho^2\Big]\ \Big\}m(z)m(\bar{z})\frac{dz}{jz}, \end{aligned}$$

for all $|z| = 1$.

Hence it follows :

$$\begin{aligned} &\frac{1}{2\pi}Re\Big\{\oint[\ c^{*T}(zI-\Phi)^{-1}h + \rho^{*2}] \\ &\qquad m(z)m(\bar{z})\frac{dz}{jz}\Big\} = 0 \end{aligned}$$

for all $|z| = 1$. (12)

If (12) holds for all $m(z) \in \{f(z) | \xi[f(k)], f(k) \in l_2[0, \infty]\}$, the parameters give :

$$c^{*T} = 0 \Rightarrow -\rho u^T = h^T H\Phi - \frac{1}{2}c^T$$

$$\rho^{*2} = 0 \Rightarrow \rho^2 = \frac{1}{K} - h^T Hh$$

Now the condition that $H > 0$ should be proven.

Assuming that the matrix H is not positive definite and then selecting a point $x(0) \in [x(0) | x^T(0)\ Hx(0) < 0]$, for the initial condition, then the time difference equation becomes :

$$x^T(k+1)Hx(k+1) - x^T(k)Hx(k) = -(u^T x(k))^2$$

because of the infinitesimal input variable $m(k) = 0$ from (8) and (10). Since (c^T, Φ, h) has been globally controllable and observable, the triple (u^T, Φ, h), which is determined from (11), must show equivalent property.

Otherwise a reduction should be taken place in the right hand side of (11). So it means a contradiction to the assumed global controllability and observability of (c^T, Φ, h).

Because of the global observability of pairs (u^T, Φ), $u^Tx(k)=0$ does not hold to the trajectory of (5). So from (8′):

$$\lim_{k\to\infty}\|x(k)\| \to \infty$$

is obtained which however is a contradiction to the assumption:

$$\bigwedge_i |\lambda_i[\ \Phi]\ | < 1$$

Therefore the assumption $H < 0$ is proven to be a contradiction so that

$$H > 0.$$

II) Necessity

Because

$$\Phi^T H\Phi - H = -uu^T, \qquad \rho^2 = \frac{1}{K} - h^T Hh,$$

$$h^T H\Phi - \frac{1}{2}c^T = -\rho u^T$$

Eq. (8) becomes:

$$x^T(k+1)Hx(k+1) - x^T(k)Hx(k)$$

$$= -\left[\ u^T x(k) - \rho\frac{m(k)}{K}\right]\ m(k)$$

If $\{m(k),\ k\in l_2[0,\ \infty]$ and $x(0)=0$, above equation is assumed to infinity as:

$$0 = x^T(\infty)Hx(\infty) - x^T(0)Hx(0)$$

$$= \frac{1}{2\pi}Re\left\{\oint\left[\ -|u^T(zI-\Phi)^{-1}h+\rho|^2 + \frac{1}{K} + c^T(zI-\Phi)^{-1}h\right]\ m(z)m(\bar{z})\frac{dz}{jz},\right.$$

for all $|z|=1$.

Therefore holds:

$$Re\left\{\frac{1}{K}c^T(zI-\Phi)^{-1}h\right\} = |u^T(zI-\Phi)^{-1}h+\rho|^2$$

Applying these lemmas to the system equation (2) with the previously defined stochastic properties yields the following stability criterion for the steam generator water level control system with manual control. Following Theorem 1 can be used to satisfy lemma 1 with help of lemma 2.

Theorem 1 Stability Criterion

The control system of mathematical model (2) is asymptotically

stable w.p.1 if the following conditions are satisfied

1) for a triple of positive numbers (δ, ρ^2, σ^2), the two (n×n) matrix are positive definite:

1a)

$$\left[\ \frac{\frac{1-\rho^2}{1-\delta} - E\{\Delta\phi_{ii}(k)\Delta\phi_{jj}(k)\}}{\rho^2 - \phi_{mii}\phi_{mjj}}\ \right] \geq \underline{0}$$

1b)

$$\left[\ \frac{\frac{\sigma^2}{1-\delta^{-1}} - \frac{E\{\Delta\phi_{ii}(k)\Delta\phi_{jj}(k)\}}{(1-\phi_{mii})(1-\phi_{mjj})}}{\rho^2 - \phi_{mii}\phi_{mjj}}\ \right] \geq \underline{0}$$

where

$$\phi_{mii} \equiv E\{e^{\lambda_i T(k)}\}$$

$$\Delta\phi_{ii}(k) \equiv e^{\lambda_i T(k)} - E\{e^{\lambda_i T(k)}\}$$

$$\lambda_i < 0\ ,\quad i=1,2,\cdots,n$$

2)

2a) for all $z\in\{z\in C\,|\,|z|=\rho\}$ and $i\in[\ 1,n]$:

$$\phi_{mii}^2 < \rho^2 < 1$$

2b)

$$\frac{1}{K} - h_1 + Re\{c^T(zI-\Phi_m)^{-1}h_m\} > 0$$

where

$$h_1 \equiv \frac{c_\xi h_\xi}{2(1+\sigma^2)}\left[\ 1 + E\left\{\frac{\Delta h_\xi^2(k)}{h_\xi^2}\right\}\right]$$

$$\Phi_m \equiv E\{e^{\Lambda T(k)}\}$$

$$h_m \equiv E\{\Lambda^{-1}(I - e^{\Lambda T(k)}b)\}$$

3)
The nonlinearity $\varphi[y(k)]$ satisfies the sector condition of

$$0 < \varphi[y]\ y < \frac{K}{(1+\sigma^2)(1+E\{\Delta K^2(k)\})}\ y^2$$

5. Result

Already mentioned in the previous section, the task duration T_k is not actually a constant, because the human operator has inexact behavior. The exact time values cannot be predicted in this analysis, so that the randomized task duration will be used as the principal concept in this work. Mentioned once more, a random set involves any other set in nature including the human behavior set.

Also, the human operator cannot fix the positioner in an exact amount. So, in this section the amplification factor is taken as random set. The probability distribution function is assumed as a Laplacian having its lower and upper limits to be 0.5 and 1.5, as

$$K(k) = K_{const} \cdot A_{[0.5,1.5]}$$

where A is a indicator function

Figure 5 shows the time behaviors of the state variables when the task duration and the amplification factor are set as machine, where the following figure 6 shows the case when these are treated as random as if human controls. Then figure 7 represents state variables when modification is performed in control according to the derived stability criterion.

6. Discussion

A sufficient stability criterion is derived for steam generator water level control system in CANDU-type reactor as a class of randomly sampled nonlinear system. The derived conditions of the criterion were examined for steam generator data of Wolsung 1 nuclear reactor and compared with time-domain simulation.

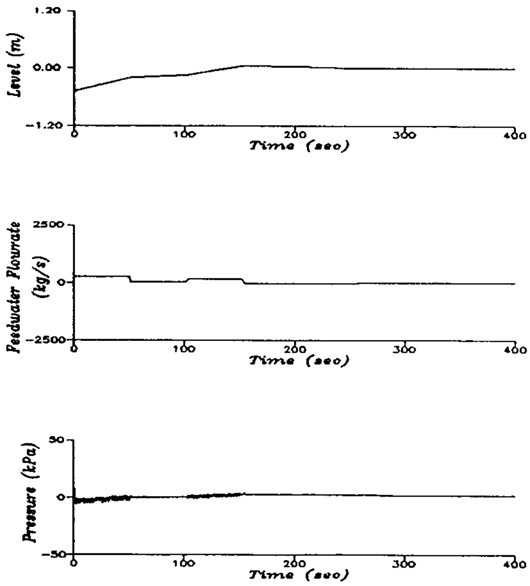

Fig. 5. State Variables At Periodic Task Duration T=50 sec

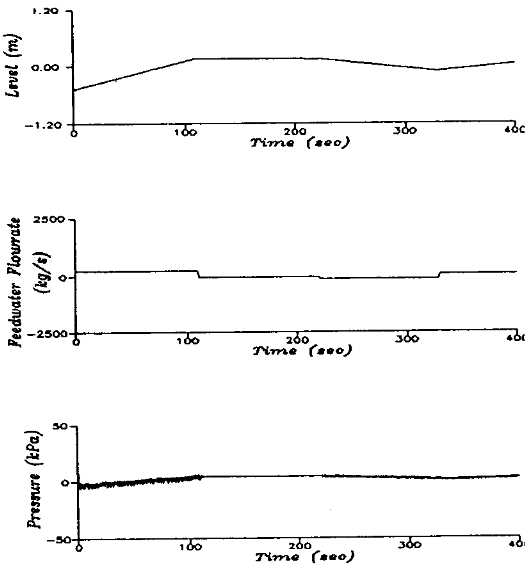

Fig. 6. State Variables at Randomized Task Duration with Random Amplification Factor

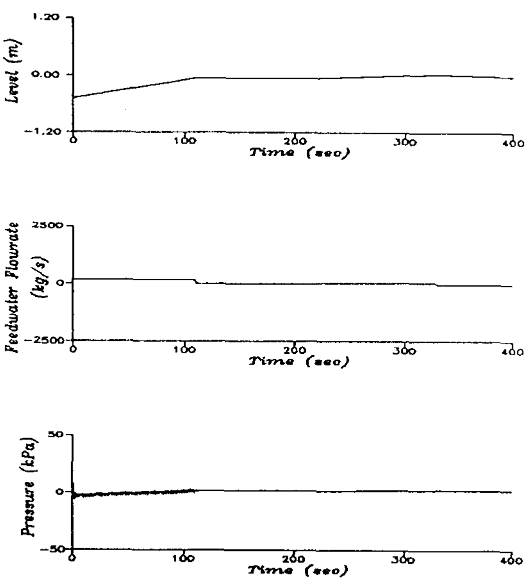

Fig. 7. State Variables at Randomized Task Duration with Random Amplification Factor with Modified Upper Limit

As the condition 3) of theorem 1 implies, the control by a human operator should be done more carefully than automatic control. Because of human operator's randomness, the upper limit of sector interval should be reduced by the factor $1/(1+E\{\Delta K^2(k)\})$.

The modeling of steam generator water level control system and the modeling of manual controller (human operator) are both known as difficult tasks. Therefore this kind of study dealing with human control has not been done until now. As shown, however, this study gives a bright inside and technical tool to handle the randomness of human operator quantitatively. So the reader is recommended to use the methodology derived in this study for the stability of any control system the controller of which is human operator. As mentioned in introduction, since manual control is needed in various control system in spite of the effort to control them automatically, this kind of stability may be a useful tool for control engineer.

For control engineer of a control system controller of which is human operator, additional study is recommended. In this study, unfortunately parameters for probability distribution function has not yet been studied. So further study should supplement these parameters for probability distribution function.

For steam generator water level control, the modeling in this study considered only the most important components. But in practice, the additional components such as valves and pipe length, etc., may be considered. Most of all, valves have important characteristics. A valve plays an integrator in transfer function in form of a pole. This pole consists double pole in series on the imaginary axis with a pole on the imaginary axis from steam-water property. So finally, extension study on a modified criterion for double pole case is recommended.

Nomenclature

x, $x_s(\cdot)$	: state variable(continuous, discrete)
u	: input variable(continuous)
d	: disturbance(continuous)
y, $y(\cdot)$	: output(continuous, discrete)
A, B, F, C^T	: system matrices(continuous)
L_w	: water level
T_d	: temperature at downcomer
p	: system pressure
x_r	: steam quality
W_r	: riser mass flow rate
T_m	: metal temperature
W_{fd}	: feedwater mass flow rate
W_s	: steam mass flow rate
t_k	: k-th time point
$\tau_{k,i}$	: k-th task start duration with i-th SG
$\xi_{k,i}$	: k-th task duration with i-th SG
T_k	: k-th Task cycle duration
$e(\cdot)$, $e_{human}(\cdot)$	: error, human error
$r(\cdot)$	: reference
$K(\cdot)$, K	: amplitude

$E\{\cdot\}$: expectation
$\varphi[\cdot]$, $m(\cdot)$: nonlinearity, input
$\xi(\cdot)$: state variable(discrete, related to pole)
$\Phi_s[T(\cdot)]$, Φ, $h_s[T(\cdot)]$, h_R : system matrices(discrete)
c^T, c_S, c_R : system matrices(discrete)
$V[\cdot]$: Lyapunov function
H : arbitrary positive matrix
λ_i : eigenvalues
$\rho_i\delta_i\sigma_i$: arbitrary numbers
ρ_T^2 : variation of duration

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