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Digital Dynamic Compensation Methods of Rhodium Self-Powered Neutron Detector

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로듐 자기출력형 중성자 계측기의 디지탈 동적 보상방법

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Abstract

The best method is selected among the 3 digital dynamic compensation methods which are developed or applied for the Rhodium self-powered neutron detector. The three digital dynamic compensation methods are the existing Dominant Pol Tustin method of the COLSS(Core Operating Limit Supervisory System), the Direct Inversion method and Kalman Filter method. The Direct Inversion method is an improved method of D. Hoppe and R. Maletti and the Kalman Filter method is developed using the Kalman Filter. Response times of the compensated signals to achieve 90% of a step input are 28.1, 17.2 and 6.5 seconds respectively for the same noise gain telling that the Kalman Filter method is the best among the 3 methods.

요 약

로듐 자기출력형 중성자계측기에 대하여 3가지 디지탈 동적 보상방법을 개발 및 적용하여 가장 우수한 방법을 제시하였다. 3가지 디지탈 동적 보상방법은 기존 COLSS의 Dominant POL Tustin 방법과 Direct Inversion 방법 및 Kalman Filter 방법이다. 이 논문에서는 D. Hoppe와 R. Maletti의 Direct Inversion 방법을 개선하였으며 Kalman Filter를 이용한 방법을 개발하였다. 3가지 방법론을 비교한 결과 같은 Noise 증가 조건하에서 Step 중성자속 입력에 대한 90% 도달 시간이 각각 28.1초, 17.2초 및 6.5초로 나타나 Kalman Filter 방법이 가장 우수함을 알 수 있었다.

1. Introduction

The self-powered neutron detectors(SPND's) are commonly used to obtain core neutron flux distributions at nuclear reactors. Among the detectors used is the Rhodium SPND. Fixed Rhodium SPNDs are used extensively at ABB-CE PWRs to have on-line monitoring and on-demand surveillance of core power distributions. The Rhodium SPND is accurate at steady-state but responds slowly to changes in neutron flux. The slow response time of Rhodium SPND precludes its direct use for control and protection purposes. On the other hand, the extensive presence and high accuracy of the SPND makes it a desirable candidate for these applications. For example, there are 45 5-level Rhodium SPND's at YGN (YongGwang Nuclear) 3&4 plants and their accuracy is 3.5% at one sigma level. Therefore, a method to improve the response time of the slowly-responding detector can make it possible to apply it for core control and protection purposes as well as to perform core monitoring and surveillance requirements.

This paper describes 3 digital dynamic compensation methods of the Rhodium SPND which are the Dominant POL Tustin method of the COLSS(Core Operating Limit Supervisory System) of the YGN 3 &4 plants, Direct Inversion method of D. Hoppe and R. Maletti[1], and the Kalman Filter method. The Direct Inversion method of this paper is a slightly improved version of D. Hoppe and R. Maletti and the Kalman Filter method is developed using the Kalman Filter.

2. Digital Dynamic Compensation Methods

The SPND operates on the principle of activation of the detector emitter material. For delayed-responding detectors, the detector signal is generated primarily by neutron-beta reactions within the emitter. Incident neutrons are captured by nuclei in the emitter, from which energetic negative beta particles are released in a statistical decay which is delayed in time and characterized by a half-life which is uniquely identified with the type of the emitter material. The beta decay scheme of Rhodium SPND involves isotope of Rhodium-103. For Rhodium, two radioisotopes are involved as shown in Figure 1.

The predominant decay mode, contributing 92.3% of the (n, β) signal, is the beta decay of Rhodium-104 with a 42 second half-life. The remaining 7.7% Rhodium (n, β) signal results from the two-stage decay of Rhodium-104m, first by gamma emission to Rhodium-104 with a 4.4 minute half-life, followed by beta decay with a 42 second half-life. In addition to the signal generated from (n, β) reactions, a small

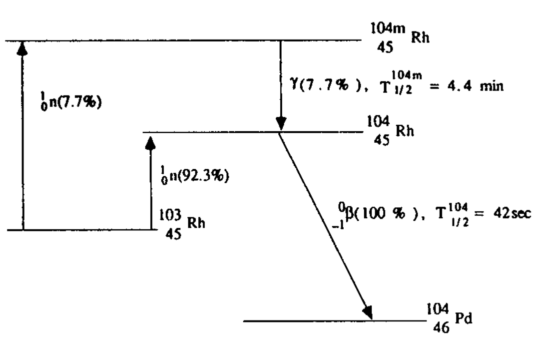

Fig. 1. Simplified Decay Scheme of Rhodium SPND [2].

but significant portion of the detector signal is generated by other reactions such as (γ, β), which are prompt. For the Rhodium SPND, the prompt component is 6.5 ±0.5 percent[3] for ANO-2 type reactors.

The rate equations can be written as (see the Nomenclature section for denotations) :

$$\frac{dRh^{104m}(t)}{dt} = \sigma^{104m} Rh^{103} \phi(t) - \lambda^{104m} Rh^{104m}(t) \quad (1)$$

$$\frac{dRh^{104}(t)}{dt} = \sigma^{104} Rh^{103} \phi(t) + \lambda^{104m} Rh^{104m}(t) - \lambda^{104} Rh^{104}(t) \quad (2)$$

$$I(t) = K^{P}(\sigma^{104}+\sigma^{104m}) Rh^{103} \phi(t) + K^{104} \lambda^{104} Rh^{104}(t) \quad (3)$$

When Eqs. (1), (2) and (3) are Laplace-transformed and rearranged :

$$\frac{I(s)}{\phi(s)} = K^{P}(\sigma^{104}+\sigma^{104m}) Rh^{103} + \frac{K^{104} \lambda^{104} \sigma^{104} Rh^{103}}{s+\lambda^{104}} + \frac{K^{104} \lambda^{104} \lambda^{104m} \sigma^{104m} Rh^{103}}{(s+\lambda^{104})(s+\lambda^{104m})} \quad (4)$$

The above Eq. (4) eventually becomes :

$$\frac{I(s)}{\phi(s)} = \frac{I_0}{\phi_0} \{ p + \frac{q}{(\tau^{104} s+1)} + \frac{r}{(\tau^{104} s+1)(\tau^{104m} s+1)} \} \quad (5)$$

2.1. Dominant Pole Tustin Method

The purpose of a digital dynamic compensation algorithm is to reconstruct the dynamic flux signal which is sensed by the detector. Therefore, we can write $G_{(s)} = \frac{\phi_{(s)}}{I_{(s)}}$ from Eq. (5) :

$$G_{(s)} = \frac{\textit{Compensated Detector Output}}{\textit{Detector Output}}$$

$$= \frac{N_2 s^2 + N_1 s + N_0}{D_2 s^2 + D_1 s + D_0} \quad (6)$$

where

$$N_2 = \tau^{104}\tau^{104m}, \quad N_1 = \tau^{104} + \tau^{104m}, \quad N_0 = 1$$

$$D_2 = p\tau^{104}\tau^{104m}, \quad D_1 = p\tau^{104} + p\tau^{104m} + q\tau^{104m}, \quad D_0 = r$$

The Tustin method[4] is based on the following substitutions on Eq. (6) to get the digital dynamic compensation :

$$\frac{1}{s} = \frac{T_c}{2}\frac{Z+1}{Z-1}$$

$$\left(\frac{1}{s}\right)^2 = \left(\frac{T_c}{2}\frac{Z+1}{Z-1}\right)^2$$

The Dominant Pole Tustin method is a spin off of the Tustin method. The difference is that one of the two digital pole-zero pairs obtained using the Tustin method is cancelled, giving a digital filter with a single pole and zero. The Dominant Pole Tustin method simplifies the computing calculations by reducing the order of the difference equation. Also, by eliminating the pole that is very close to the unit circle, the chance of inexact filter coefficients leading to an unstable filter is greatly reduced. The drawback of this method is that it results in a degradation in dynamic response. In other words, the time response of the first-order filter obtained by this method is slower than that of second order filters of the Tustin method.

2.2. Direct Inversion Method

By inverting Eq. (5), we can write

$$\frac{\phi(s)}{I(s)} = \frac{\phi_0}{I_0}\frac{1}{p}\left(1 + \frac{A}{s+a} + \frac{B}{s+b}\right) \quad (7)$$

where

$$a, b = \frac{p\lambda^{104} + q\lambda^{104} + p\lambda^{104m} \pm \sqrt{(p\lambda^{104} + q\lambda^{104} + p\lambda^{104m})^2 - 4p\lambda^{104}\lambda^{104m}}}{2p}$$

$$A = (a-\lambda^{104})(a-\lambda^{104m}) / (b-a)$$

$$B = (b-\lambda^{104})(b-\lambda^{104m}) / (a-b)$$

On the conditions of equidistant sampling period of T_c and rectangular approximation of current I(t), we get the result of pulse transfer function by the Z-transformation[5] :

$$Z\left(\frac{1-e^{-T_c s}}{s}\left(1+\frac{A}{s+a}+\frac{B}{s+b}\right)\right)$$

$$= 1 + \frac{A}{a}\frac{1-Z_a}{z-Z_a} + \frac{B}{b}\frac{1-Z_b}{z-Z_b} \quad (8)$$

where

$$Z_a = e^{-aT_c}, \quad Z_b = e^{-bT_c}$$

If we introduce the state variables x_a and x_b in the second and third terms of Eq. (8), it becomes :

$$x_a(n) = Z_a x_a(n-1) + (1-Z_a)\frac{A}{a}I(n-1) \quad (9)$$

$$x_b(n) = Z_b x_b(n-1) + (1-Z_b)\frac{B}{b}I(n-1) \quad (10)$$

$$\phi(n) = \frac{\phi_o}{I_o}\frac{1}{p}(I(n) + x_a(n) + x_b(n)) \quad (11)$$

For the steady-state,

$$x_a(0) = \frac{A}{a}I(0)$$

$$x_b(0) = \frac{B}{b}I(0)$$

The above method is similar with that of D. Hoppe and R. Maletti[1] but simpler than it in numerical derivations. This derivation removed a slight numerical inconsistency of using $\sigma^{104m} = 11$b[1] which should have been 11.6b to be consistent with half lives of Rhodium-104 radioisomers.

2.3. Kalman Filter Method

When Eq. (5) is Z-transformed with rectangular approximation of flux $\phi(t)$ and equidistant sampling frequency of T_c :

$$Z\left(\frac{1-e^{-T_c s}}{s}\right.$$

$$\left.\left(p+\frac{q\lambda^{104}}{s+\lambda^{104}}+r\frac{\lambda^{104}\,\lambda^{104m}}{(s+\lambda^{104})\,(s+\lambda^{104m})}\right)\right)$$

$$= p\,\{\,1+\frac{1}{p}\left(q-\frac{rg}{\lambda^{104}}\right)\frac{1-Z_g}{z-Z_g}$$

$$+\frac{rg}{(p\,\lambda^{104m})}\,\frac{1-Z_f}{z-Z_f}\,\} \tag{12}$$

where

$$Z_g = e^{-\lambda^{104}\,T_c},\quad Z_f = e^{-\lambda^{104m}\,T_c}$$

$$\text{and}\quad g = \frac{\lambda^{104}\,\lambda^{104m}}{\lambda^{104}-\lambda^{104m}}$$

If we introduce the state variables X_g and X_f in the second and third terms of Eq. (12), the equation becomes

$$x_g(n) = Z_g x_g(n-1) + \frac{1}{p}\left(q-\frac{rq}{\lambda^{104}}\right)(1-Z_g)\phi(n-1) \tag{13}$$

$$x_f(n) = Z_f x_f(n-1) + \frac{1}{p}\,\frac{rq}{\lambda^{104m}}(1-Z_f)\phi(n-1) \tag{14}$$

$$I(n) = p\,\{\,\phi(n)+X_g(n)+X_f(n)\,\} \tag{15}$$

For the steady state,

$$x_g(0) = \frac{1}{p}\left(q-\frac{rq}{\lambda^{104}}\right)\phi(0)$$

$$x_f(0) = \frac{1}{p}\,\frac{rq}{\lambda^{104m}}\phi(0)$$

If neutron flux $\phi(n)$ is assumed to be related to $\phi(n-1)$ in a stochastic process, the following equation can be written for our purpose :

$$\phi(n) = \phi(n-1) + w_1 \tag{16}$$

where w_1 is white noise with zero mean and a variance of Q_1.

Let $\underline{x}(n) = (\phi(n)\ x_g(n)\ x_f(n))^T$

$\underline{w} = (w_1\ w_2\ w_3)^T$

$$y(n) = I(n),$$

then

$$\underline{x}(n) = \underline{A}\,\underline{x}(n-1) + \underline{w} \tag{17}$$

$$y(n) = \underline{C}^T.\,\underline{x}(n) + v \tag{18}$$

where

$$\underline{A} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{p}\left(q-\frac{rg}{\lambda^{104}}\right)(1-Z_g) & Z_g & 0 \\ \frac{1}{p}\,\frac{rg}{\lambda^{104}}(1-Z_f) & 0 & Z_f \end{pmatrix}$$

$$\underline{C}^T = (p\quad p\quad p)$$

w_2 = white noise with zero noise and a variance of Q_2 of Eq. (13),

w_3 = white noise with zero noise and a variance of Q_3 of Eq. (14),

and

v = white noise with zero noise and a variance of R of Eq. (15).

The digital Kalman Filter of Reference 6 can be applied to Eqs. (17) and (18). The solution process is

$$\hat{\underline{x}}(n/n-1) = \underline{A}\,\hat{\underline{x}}(n-1/n-1)$$

$$\underline{P}(n/n-1) = \underline{A}\,\underline{P}(n-1/n-1)\,\underline{A}^T + \underline{Q}$$

$$\underline{K}(n) = \underline{P}(n/n-1)\,\underline{C}\,\{\,\underline{C}^T\,\underline{P}\,(n/n-1)\,\underline{C} + R\,\}^{-1}$$

$$\hat{\underline{x}}\,(n/n) = \hat{\underline{x}}\,(n/n-1) + \underline{K}(n)\quad(y(n) - \underline{C}^T\,\hat{\underline{x}}(n/n-1))$$

$$\underline{P}(n/n) = (\underline{I} - \underline{K}(n)\,\underline{C}^T)\,\underline{P}(n/n-1)$$

where $\hat{\underline{x}}(n/n-1)$ = estimate of $\underline{x}(n)$ using information up to time step (n-1)

$\hat{\underline{x}}(n/n)$ = estimate of $\underline{x}(n)$ using information up to time step (n)

$\underline{P}(n/n-1)$ = Error Covariance of $E\{(\underline{x}(n)-\hat{\underline{x}}(n/n-1))(\underline{x}(n)-\hat{\underline{x}}(n/n-1))^T\}$

$\underline{P}(n/n)$ = Error Covariance of $E\{(\underline{x}(n)-\hat{\underline{x}}(n/n))(\underline{x}(n)-\hat{\underline{x}}(n/n))^T\}$

$\underline{K}(n)$ = Kalman Gain Vector, and

$$Q = \begin{pmatrix} Q_1 & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q_3 \end{pmatrix}$$

One advantage of the Kalman Filter method is that it uses the information of previous time steps through the above estimation process.

3. Simulation Results

There are two important considerations which must be taken into account in the performance evaluation of digital dynamic compensation methods of the Rhodium SPND: the response time of the compensation and the noise gain of the compensation. Experience has shown that there is an inverse relationship between the noise gain and response time of the dynamic compensation resulting from the choice of compensation coefficients, that is, the higher the noise gain, the shorter the compensation response time, and vice versa. The optimal set of digital dynamic compensation coefficients is one which yields a dynamic compensation with both a sufficiently fast time response and an acceptable noise gain.

The response time and noise gain of the Dominant Pol Tustin method for the Rhodium SPND may be varied by changing the value of the prompt fraction, p, which is used to calculate the dynamic compensation coefficients. The shortest response time occurs when the prompt fraction used to calculate the dynamic compensation coefficients is set equal to the best estimate ($p=0.065$) for the prompt component of the Rhodium SPND. The response time increases as the prompt fraction used to calculate the compensation coefficients increases from $p=0.065$. For values of less than 0.065, the response of the compensation method is overcompensated (first overshoots, then asymptotically approaches a step increase in flux from above).

The noise gain of the dynamic compensation method is inversely proportional to the value of p which is used to calculate the dynamic compensation coefficients. The noise gain is smallest when $p=1.0$ and increases as the value of p decreases. As a results of the above considerations, a compromise has to be made in choosing a value of p which yields a dynamic compensation with both response time and noise gain which are acceptable.. The response times and noise gains of the Dominant Pol Tustin method for various values of the prompt fraction are given in Table 1. The choice of an appropriate set of the dynamic compensation coefficients was made based on the experience of previous ABB-CE plants for the Dominant Pol Tustin method of YGN 3&4 COLSS which has sampling time of 2 seconds for the Rhodium SPND's. Based on the experience, the set of dynamic compensation coefficients chosen for the ABB-CE plants and YGN 3&4 is one with $p=0.15$ which gives response time of 28.1 seconds and noise gain of 5.91.

Table 1. Response Times and Noise Gains of Dominant Pol Tustin Method for Various Prompt Fractions

Prompt Fraction	Response Time	Noise Gain
0.065	8 Seconds	12.8
0.10	18 Seconds	8.6
0.15	28 Seconds	5.9
0.20	38 Seconds	4.5
0.50	92 Seconds	1.9
1.00	176 Seconds	1.0

Note: 1. Sampling Interval = 2 Seconds
2. Response time is the time taken for the compensated signal to achieve 90% of an input step change.

The prompt fraction value was changed for the Direct Inversion method to get the same noise gain of the YGN 3&4 COLSS. The Q and R of the Kalman Filter method were changed to get the same noise gain of the COLSS. The results are shown in Table 2. As shown in Table 2, the prompt fraction value was chosen as $p=0.175$ for the Direct Inversion method and Q and were chosen as

$$Q = \begin{pmatrix} 0.05 & 0.0 & 0.0 \\ 0.0 & 0.0082 & 0.0 \\ 0.0 & 0.0 & 0.00082 \end{pmatrix} \quad \text{and}$$

R = 0.00085 for the Kalman Filter method. Figures 2 and 3 show simulation results of Dominant Pol Tustin method, Direct Inversion method and Kalman Filter method for step input and ramp input.

The Dominant Pol Tustin method gives 6.28 times better response time than the no compensation case at the cost of noise gain of 5.91. The Direct Inversion method and Kalman Filter method give 1.63 and 4.32 times better response time than the Dominant Pol Tustin method with the same noise gain of the Dominant Pol Tustin method.

Table 2. Performance Comparison of Digital Dynamic Compensation Methods for the Rhodium SPND

Method	Response Time	Noise Gain
No Compensation	176.5 Seconds	1.0
Dominant Pol Tustin (p = 0.15)	28.1 Seconds	5.91
Direct Inversion (p = 0.175)	17.2 Seconds	5.88
Kalman Filter (*)	6.5 Seconds	5.89

Note: 1. Sampling Interval = 2 Seconds

2. Response time is the time taken for the compensated signal to achieve 90% of an input step change.

*.

$$Q = \begin{pmatrix} 0.05 & 0.0 & 0.0 \\ 0.0 & 0.0082 & 0.0 \\ 0.0 & 0.0 & 0.00082 \end{pmatrix}, \quad R = 0.00085$$

4. Conclusions

Three Digital Dynamic Compensation methods, namely, Dominant Pol Tustin, Direct Inversion and Kalman Filter methods are presented in this paper.

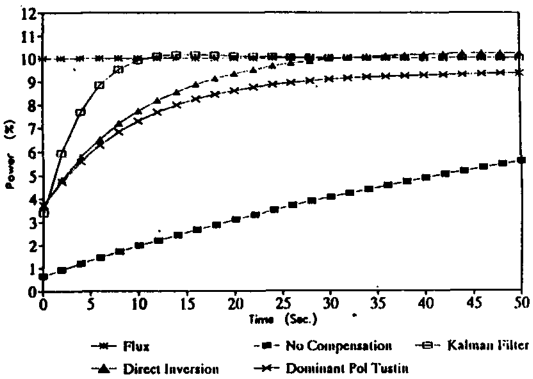

Fig. 2. Responses of 3 Digital Dynamic Compensation Methods of Rhodium SPND for Step Input and 2 Second Sampling Interval.

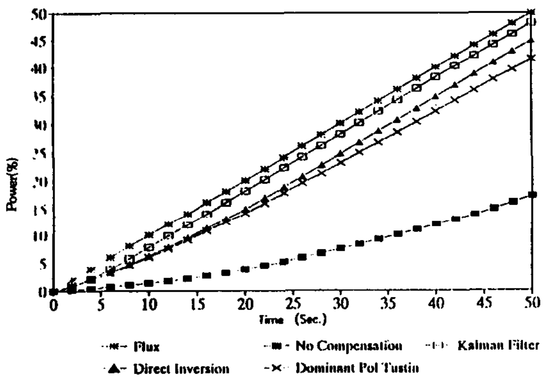

Fig. 3. Responses of 3 Digital Dynamic Compensation Methods of Rhodium SPND for Ramp Input and 2 Second Sampling Interval.

The Direct Inversion method is slightly improved from the previous version and the Kalman Filter method is developed.

The simulation results show that the Direct Inversion method is better than the Dominant Pol Tustin method, but the best compensation results can be obtained from the Kalman Filter method. The Direct Inversion method gives better results than the Dominant Pol Tustin method since it does not contain the assumption of a single pole and zero. The Kalman Filter method is the best among the 3 methods since it uses the information of previous time steps through its estimation process.

Nomenclature

Rh^{103}, Rh^{104}, Rh^{104m} = Atomic densities of ${}^{103}_{45}Rh$, ${}^{104}_{45}Rh$, ${}^{104m}_{45}Rh$

λ^{104}, λ^{104m} = Decay constants of ${}^{104}_{45}Rh$ and ${}^{104m}_{45}Rh$ (0.0165 sec^{-1}, 0.002626 sec^{-1})

τ^{104}, τ^{104m} = Time constants of ${}^{104}_{45}Rh$ and ${}^{104m}_{45}Rh$ (60.573 sec, 380.871 sec)

σ^{104} = Absorption cross section of ${}^{103}_{45}Rh$ to produce ${}^{104}_{45}Rh$(139b)

σ^{104m} = Absorption cross section of ${}^{104}_{45}Rh$(11b)

I(t) = Detector current at time t

ϕ(t) = Neutron flux at time t

p = Fraction of Rhodium detector signal which is prompt at a steady-state condition (0.065 ± 0.005)

q = Fraction of Rhodium detector signal of which half-life is 42 seconds at a steady-state condition (0.928 $(1-p)$ = 0.863)

r = Fraction of Rhodium detector signal of which half-life is 4.4 minutes at a steady-state condition (0.77 $(1-p)$ = 0.072)

K^p = Probability that a neutron absorption of ${}^{103}_{45}Rh$ leads directly to current carrying an electron

K^{104} = Probability that a ${}^{104}_{45}Rh$ decay leads to current carrying an electron

I_0 = Steady state detector current at time zero

ϕ_0 = Steady state neutron flux at time zero

s = "s" parameter in Laplace transformation

z = "z" parameter in Z-transformation

T_c = Sampling time interval in digital dynamic compensation

References

1. D. Hoppe and R. Maletti, "Improved Techniques of Analog and Digital Dynamic Compensation for Delayed Self-Powered Neutron Detectors," Nuclear Science and Engineering, Volume III, August 1992, PP. 433–436.
2. J.M. Harrer and J.G. Beckerley, "Nuclear Power Reactor Instumentation Systems Handbook," Vol. 1, US Atomic Energy Commission, 1973, pp. 55–56.
3. M.L. Kantrawitz and J.P. Thompson, "Reactor Transient Tests at ANO–2," EPRI NP–1709, Electric Power Research Institute, 1982, P. A–14.
4. J.S. Rosko, "Digital Simulation of Physical Systems," Addison-Wesley Publishing Company, 1972, P. 223.
5. E.L. Gruenberg, "Handbook of Telemetry and Remote Control," McGraw-Hill Book Company, 1967, P. 15–51.
6. A. Gelb, "Applied Optimal Estimation," MIT Press 1979, PP. 107–119.