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The Control Rod Speed Design for the Nuclear Reactor Power Control Using Optimal Control Theory

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최적제어이론에 의한 원자로 제어봉속도의 설계

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Abstract

The state feedback optimal control techniques are used in designing the reactor control system. The mathematical plant model with the temperature feedback effects is established from the one delayed neutron group point kinetics equation and the singly lumped thermal-hydraulic balance equations, and is expressed in terms of state variables. The LQR (Linear Quadratic Regulator) control system is designed, being followed by the LQG (Linear Quadratic Gaussian) design to determine the optimal conditions of rod movements for the desired reactor power responses. And two different servo control schemes, the ordinary feedback system and the order increased regulating system, are proposed for the purpose of input tracking. The general control characteristics such as stability margins and output responses are discussed. Comparing each other, it is found that the order increased regulating system has far better control characteristics than the ordinary feedback system.

요 약

본 논문에서는 최적제어기법을 이용한 원자로 출력 제어시스템을 다루었다. 시스템 변수들을 상태변수로 표시하면 관측치 뿐만 아니라 시스템 내부의 모든 상태변수를 동시에 다룰 수 있으므로 설계의 자유도가 증가될 수 있다. 따라서 본 논문에서는 원자로의 동특성식과 열수력학적 에너지 평형식을 사용하여 원자로를 모델링한 후 이를 상태변수로 나타내었다. 다음으로는 LQR 및 LQG 시스템을 설계하여 제어봉 및 출력의 거동을 동시에 만족시킬 수 있는 설계조건을 결정하였다. 또한 서보 시스템의 설계를 위해 보통의 휘드백 시스템과 차수를 증가시킨 레귤레이팅 시스템을 만들어 비교하였으며 그 결과 증가차수 레귤레이팅 시스템이 보통의 휘드백 시스템에 비해 우수한 제어 특성이 있음을 알 수 있었다.

1. Introduction

The control design techniques have been changed significantly over the last decade. Although the classical PID control has been used and proved to be powerful in various fields of the control application, new control techniques are widespread at present with the advent of the computer aided control design.

In the nuclear field, the problem of the plant control is now one of the important issues with the obsolescence of the existing control system [1]. The upgrade of the instrumentation and control is now being on its way, and the improvements are made by degrees on the existing plants as well as on the new plants to be constructed. However, such improvements have a tendency toward the mere digitalization of the PID algorithms making use of the digital technologies, and does not adopt the advantages of new control techniques [2].

In this paper, the optimal control methods are applied to the design of the reactor power control system by establishing a set of state space equations. Contrary to the classical control systems in which only the measured outputs are controllable, the optimal control techniques permit more degrees of freedom since the state variables can be controlled. Particularly, on account of the nuclear limitations on the reactor system such as flux distortions or local peakings, it is necessary to consider the control rod motions together with the output responses, and the optimal techniques provide an easy way to handle these parameters at the same time.

The contents of this study are outlined as follows. First, the mathematical reactor model is established. The one delayed neutron group point kinetics equations as well as the singly lumped thermal-hydraulic balance equations are employed to include the temperature feedback effects. Then the optimal regulating control systems of the LQR (Linear Quadratic Regulator) and the LQG (Linear Quadratic Gaussian) are designed, and adequate design conditions of the weighting matrices are determined. Finally, the servo system in which the output follows the input command signal is discussed in detail.

2. Plant Modeling

To realize an efficient control system, the plant, or the process, which is to be controlled should be defined exactly to reflect the real situation. At the same time, it is desirable that the plant be simple enough for an easy control system implementation. In this paper the reactor is modeled making use of the one delayed neutron group kinetics equation with the complementary energy balance equations to consider the temperature feedback effects within a reactor. The one delayed neutron group point kinetics equation has some limitations. First, the energy dependent effects should be taken into account since the delayed neutrons appear with somewhat lower energies than do the prompt fission neutrons. Secondly, the point kinetics equation involves the assumptions that the flux is represented by a single, time independent spatial mode. In accordance, the detail local flux distribution can not be represented by this model. However, the reactor model to be developed in this paper is a lumped one, and the overall dynamic characteristics can be described by the one delayed neutron group point kinetics equation [3].

The one group neutron kinetics equation, with neglecting the external sources, is

$$\frac{dn}{dt} = \frac{\rho-\beta}{\Lambda} n + \lambda C$$

$$\frac{dC}{dt} = \frac{\beta}{\Lambda} n - \lambda C \qquad (1)$$

where, n = neutron density, C = precursor density, ρ = reactivity, β = delayed neutron fraction, λ = precursor decay constant, and Λ = neutron effective life time.

With the assumptions of small perturbations about n_0 and C_0, the perturbed values are $\delta n(t) = n(t) - n_0$, $\delta C(t) = C(t) - C_0$, and $\delta\rho(t) = \rho(t) - \rho_0$. By inserting these relations into Eq. (1) and dividing each equation by n_0 and C_0 respectively, the reactor dynamics is described by a set of normalized linear kinetics equations as below.

$$\frac{d}{dt}\delta\bar{n} = \frac{\delta\rho}{\Lambda} - \frac{\beta}{\Lambda}\delta\bar{n} + \frac{\beta}{\Lambda}\delta\bar{C}$$

$$\frac{d}{dt}\delta\bar{C} = \lambda\delta\bar{n} - \delta\bar{C} \quad (2)$$

where and $\delta\bar{n} = \delta n/n_0$ and $\delta\bar{C}/C_0$.

Since the normalized neutron density is directly related to the normalized power, if all the neutrons are of one energy group, Eq. (2) can again be written in terms of the incremental power ratio of $\delta\bar{P}$.

$$\frac{d}{dt}\delta\bar{P} = \frac{\delta\rho}{\Lambda} - \frac{\beta}{\Lambda}\delta\bar{P} + \frac{\beta}{\Lambda}\delta\bar{C}$$

$$\frac{d}{dt}\delta\bar{C} = \lambda\delta\bar{P} - \delta\bar{C} \quad (3)$$

where $\delta\bar{P} = \delta P/P_0 = (P-P_0)/P_0$.

The above equations do not account the reactivity feedbacks. But in reality, there are many kinds of reactivity feedbacks such as fuel and moderator temperature effects, poison build-ups and fuel burn-up etc., and the dynamic characteristic times are ranging from hundredth of seconds to years [4]. Considering these time scales, only the temperature effects of the fuel and the moderator are included in the plant as in Fig. 1(a).

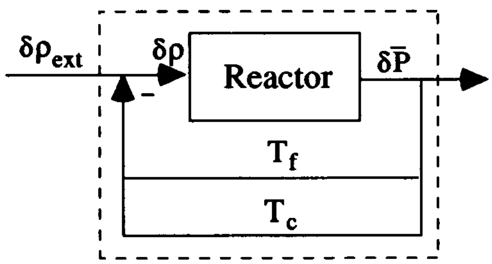

Fig. 1. (a) Plant Model With Temp. Feedbacks

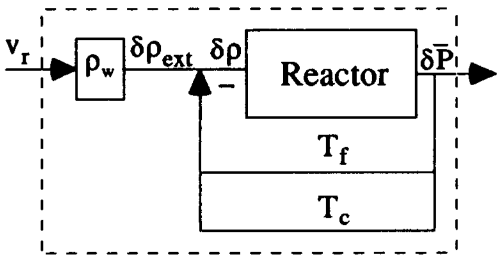

Fig. 1. (b) Plant Model With Rod Speed Input

For the complementary equations which are to be used to estimate the temperature feedback effects, the singly lumped model of Eq. (4) below is used [5].

$$M_{fe}c_{fe}\frac{dT_f}{dt} = P(t) - \frac{1}{R}[T_f(t) - T_c(t)]$$

$$M_c c_p \frac{dT_c}{dt} = \frac{1}{R}[T_f(t) - T_c(t)] - 2Wc_p[T_c(t) - T_i(t)] \quad (4)$$

where,

M_{fe} = total mass of fuel elements, i.e., total mass of fuel and cladding of the reactor,

M_c = total mass of the coolant in the reactor,

c_{fe}, c_p = specific heat capacity of fuel element and the coolant, respectively,

R = thermal resistance from fuel to coolant,

W = total coolant flow rate,

T_f, T_c, T_i = fuel temperature, average coolant temperature, and coolant inlet temperature, respectively.

For the perturbed system of $T_f = T_{f0} + \delta T_f$, $T_c = T_{c0} + \delta T_c$, $P = P_0 + \delta P$, and $W = W_0 + \delta W$, Eq. (4) becomes of

$$M_{fe}c_{fe}\frac{d}{dt}\delta T_f = P_0 - \frac{1}{R}(\delta T_f - \delta T_c)$$

$$M_c c_p \frac{d}{dt}\delta T_c = \frac{1}{R}\delta T_f - (\frac{1}{R} + 2W_0 c_p)\delta T_c + 2W_0 c_p \delta T_i + 2c_p(T_{i0} - T_{c0})\delta W \quad (5)$$

Taking the temperature feedbacks into account, the reactivity is

$$\delta\rho = \delta\rho_{ext} + \alpha_f\delta T_f + \alpha_c\delta T_c$$

or,

$$\frac{d}{dt}\delta\rho = \frac{d}{dt}\delta\rho_{ext} + \alpha_f\frac{d}{dt}\delta T_f + \alpha_c\frac{d}{dt}\delta T_c \quad (6)$$

where α_f, α_c = fuel and moderator temperature coefficient, respectively.

From Eqs. (3), (5) and (6), the plant model with the feedback effects can be put in the following linear matrix form.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (7)$$

where, $x = \left(\delta\overline{P} \quad \delta\overline{C} \quad \delta T_f \quad \delta T_c \quad \delta\rho\right)^T$,

$u = \left(\delta T_i \quad \delta W \quad \frac{d}{dt}\delta\rho_{ext}\right)^T$,

$$A = \begin{pmatrix} -\frac{\beta}{\Lambda} & \frac{\beta}{\Lambda} & 0 & 0 & \frac{1}{\Lambda} \\ \lambda & -\lambda & 0 & 0 & 0 \\ a & 0 & -b & b & 0 \\ 0 & 0 & c & -d & 0 \\ g & 0 & h & j & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ e & f & 0 \\ e\alpha_c & f\alpha_c & 1 \end{pmatrix},$$

and $a = \frac{P_0}{M_{fe}c_{fe}}$, $b = \frac{1}{RM_{fe}c_{fe}}$, $c = \frac{1}{RM_c c_p}$, $d = c + \frac{2W_0}{M_c}$, $e = d - c$, $f = \frac{2(T_{i0} - T_{c0})}{M_c}$, $g = a\alpha_f$, $h = c\alpha_c - b\alpha_f$, and $j = b\alpha_f - d\alpha_c$.

As shown in the above equation, the plant has five state variables with input vector of coolant inlet temperature, flow rate and time rate of external reactivity. It is to be noted that the dynamic system of Eq. (7) is MIMO or MISO, depending on the number of measured variables. Therefore the model can account for the dynamics of the coolant flow rate for the case of reactor coolant pump coast down, for an example, and can reflect the effects of the secondary system transients [6]. Actually, the coolant inlet temperature varies as the power changes. However, for the purpose of simplicity, it is assumed that the coolant inlet temperature as well as the flow rate are constant, which imposes a limitation on the contents of this paper. Then the matrix **B** becomes a vector of five by one and the input vector **u** is reduced to a scala.

The external reactivity acting on the system as an input is in the form of time derivative. By introducing the total rod worth of ρ_w, the input can be expressed as [7]

$$\frac{d}{dt}\delta\rho_{ext} = \rho_w v_r \tag{8}$$

where v_r is the normalized rod velocity.

It should be noted that the above equation is far from the real situation, which presents another limitation. Since the differential reactivity worth itself is dependent on the flux distribution, the integrated reactivity worth is not linear, contrary to the linearity Eq. (8) implies. Particularly, if the fraction of the rod insertion or withdrawl is large, Eq. (8) becomes invalid.

Equation (8) indicates that the fifth state variable would be rather $\delta\rho_{ext}$ than $\delta\rho$, and the system input be v_r instead of $\frac{d}{dt}\delta\rho_{ext}$. The corresponding plant model is described in Fig. 1(b). From Eqs. (6) and (8), with the assumptions of the constant flow rate and coolant inlet temperature, the system of Eq. (7) has the final form of

$$\dot{x} = Ax + Bu \tag{9}$$

where

$x = \left(\delta\overline{P} \quad \delta\overline{C} \quad \delta T_f \quad \delta T_c \quad \delta\rho_{ext}\right)^T$,

$u = v_r$,

$$A = \begin{pmatrix} -\frac{\beta}{\Lambda} & \frac{\beta}{\Lambda} & \frac{\alpha_f}{\Lambda} & \frac{\alpha_c}{\Lambda} & \frac{1}{\Lambda} \\ \lambda & -\lambda & 0 & 0 & 0 \\ a & 0 & -b & b & 0 \\ 0 & 0 & c & -d & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \rho_w \end{pmatrix}.$$

Table 1 shows the summarized key parameters of Kori Unit 2 which are used to determine the elements of **A** and **B** [8]. It is to be noted that the nuclear properties as well as some thermal properties are subject to change through the reactor life cycle, and all the data of Table 1 is for the BOL (Beginning of Life) condition. Also it is assumed that only the rod bank A moves during the transient.

Table 1. Key Parameters of Kori Unit 2

P_0(Full Power)	876 MW	β	0.0075
M_{fe}	5638 kg	Λ	18.6 10^{-6} sec
M_c	12880 kg	λ	0.0787 sec^{-1}
c_{fe}	213.9 J/°C	α_f	−3.7 pcm/°C
W_0	8555.4 kg/sec	ρ_w	1530 pcm
R	0.2221 °C/MW		

The moderator temperature coefficient varies with the change of boron concentration and with the coolant temperature as well. At the initial period of the reactor cycle it has positive values. But as the burn-up progresses, the moderator temperature coefficient becomes negative. Figure 2 shows the modetator temperature feedback effects of the reactor. A unit step increase of the normalized rod velocity is applied to the reactor (See Eq. (8)). The figure shows that the larger negative value of moderator temperature coefficient leads to the milder power increase rate. Since the boron concentration changes abruptly during the initial period, it is assumed to be 0 pcm in this study. The overall temperature effects, moderator temperature effects plus fuel temperature effects, are also presented in Fig. 3. The input conditions are the same as Fig. 2.

As shown in the figure, the power increases exponentially for the case of no feedbacks while it increases linearly with the presence of temperature feedbacks, as expected.

With those values of Table 1, the elements of **A** are found to be a = 126.63, b = −0.30393, c = 0.12068 and d = 2.744. And the system eigenvalues are [−402.68, −0.8527, −0.0573, −2.7638, 0]. It should be mentioned that these eigenvalues are obtained for the case of full power and they gets smaller for the lower initial steady powers. However,

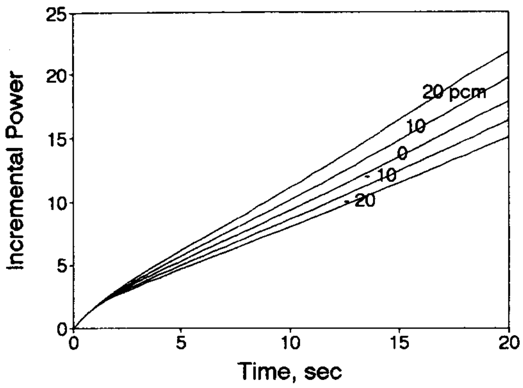

Fig. 2. Moderator Temp. Effects on Step Response of Reactor Power

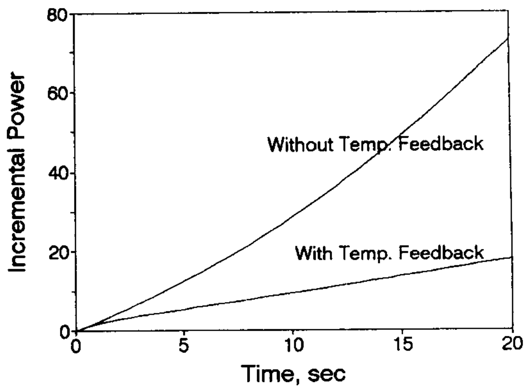

Fig. 3. Overall Temperature Effects on Step Response of Reactor Power

it was found that the plant always has four negative eigenvalues with one zero valued eigenvalue for any power level, and the reactor is assumed to be at the steady state of full power in followings.

3. The LQR System

The LQR design is to select the optimal feedback gains which minimize the constraint conditions imposed on the system. The constraint, or the cost function, is a quadratic functional of the plant states and control inputs. Since the relations between the states and the input energy, say, the rod speed, could be compromised, it also has an advantage of the compensation for the system degradation.

If $\delta\overline{P}$ is the only state variable to be sensored, the LQR design is to determine the optimal gain vector from following equations.

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}, \quad \mathbf{y} = \mathbf{Cx} + \mathbf{Du},$$

$$J = \frac{1}{2}\int_0^T \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u}\right)dt \tag{10}$$

where, C = [1 0 0 0 0], D = [0], Q = positive semidefinite matrix, and R = positive definite matrix.

The optimal control problem could be solved by various methods [9] such as Euler-Lagrange method, Hamilton-Jacobi-Bellman equation or

Pontriagin's minimum principle. By employing the minimum principle with the Riccati transformation, the sufficient condition for the optimal control is

$$\frac{-d\mathbf{P}}{dt} = \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{Q} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{11}$$

where **P** is the Hermitian matrix of the same size as **A**.

For the case of infinite horizon, this equation becomes the ARE (Algebraic Riccati Equation) of

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{Q} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} = 0 \ ,$$

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}, \ \mathbf{u} = -\mathbf{K}\mathbf{x} = \mathbf{u} \tag{12}$$

Since the rank condition of the controllability matrix is satisfied, the feedback gains could be determined from the above ARE.

The LQR control scheme is described in Fig. 4. The feedforward gain, K_1, has a function of converting the power signal to a rod velocity signal and is assumed to be unity. Since the system is a regulating system with an input command, the steady state value of the output becomes different depending on the feedback gains of **K**, and a normalizing gain K_2 is introduced to make the steady state output value equal to the input signal.

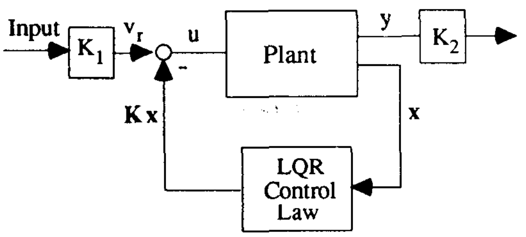

Fig. 4. Reactor Regulating System With LQR Control Scheme

The feedback controller has an effect of cancelling the heavy plant pole of -403. By letting the plant and the controller be G(s) and H(s) respectively, the open loop system is $G(s)H(s) = \mathbf{K}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$.

Since $G(s) = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$, the transfer function of the feedback controller is

$$H(s) = \frac{\mathbf{K}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}}{\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}} \tag{13}$$

For an example, with **Q** of the unit matrix of five by five and R of 500, the feedback gain vector is found to be **K**=[0.0099, 1.9444, 0.0304, 0.0028, 264.36]. And it can be found that the largest negative eigenvalue of the open loop transfer function is canceled.

The weighting matrices **Q** and R are constraints on the system state variables and the rod velocity. Further it is to be noted that their relative values rather than their absolute ones have a sense [2]. The larger value of R imposes a larger constraint on the rod speed and makes the system slower with smaller feedback gains.

Figures 5 and 6 present the output of the incremental power and rod speed, respectively, for the various values of weighting martices when the power is step increased by 10 percent from the steady state of full power. The simulation was made by using MATLAB [10]. The weighting matrices of **Q** and R are assumed to be $\mathbf{Q} = q \times \mathbf{I}(5)$, and $R = r$, where $\mathbf{I}(n)$ denotes the identity matrix of size n.

As the input constraint value of r increases, the rod motion becomes milder but the system speed gets slower. By trading off the system speed and the rod speed from Figs. 5 and 6, the weighting values of q and r are determined as 2×10^{-6} and 1. The

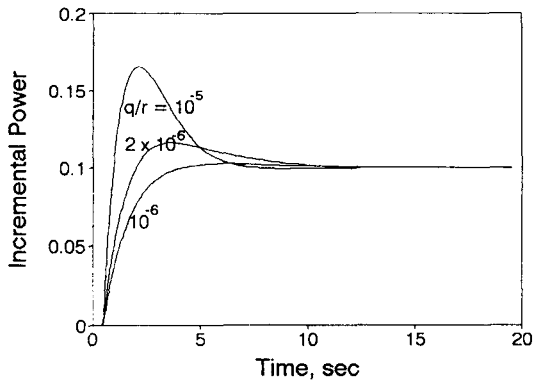

Fig. 5. Reactor Power Responses of LQR Design for Various Constraint Weighting Values

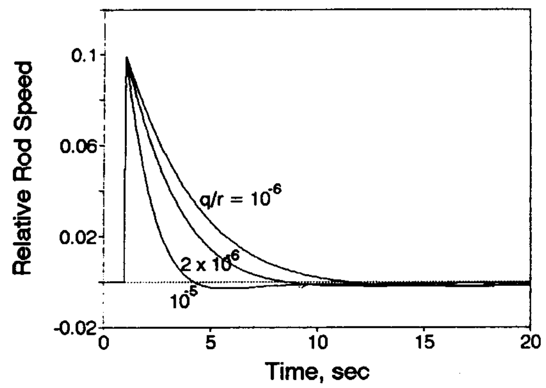

Fig. 6. Relative Rod Speeds of LQR Design for Various Constraint Weighting Values

feedback gains for this case are $\mathbf{K}$=[0.0001, 0.1488, 0.0000, 24.2855].

3. The LQG System

The basic premise of the LQR is that all states be available for the feedback. In practice, not all state variables are available for the direct measurement. In many practical cases, only a few state variables of a given system are measurable and it is necessary to estimate the unmeasured states.

An estimator, or an observer, estimates the state variables from the control inputs and the system outputs. The dynamics of the estimated states is

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) = (A - LC)\hat{x} + (B \quad L)\begin{pmatrix} u \\ y \end{pmatrix} \tag{14}$$

where $\hat{x}$ is the estimated state vector and $\mathbf{L}$ is the observer gain vector.

The observer gain could be obtained from a full order observer or a Luenberger observer, if the rank condition of the observability matrix is satisfied. However a Kalman observer is used in this work to account for the system and measurement noises.

With the presence of process and measurement noises, the system comes up to

$$\dot{x} = Ax + Bu + Gw, \quad y = Cx + v \tag{15}$$

If both noises are assumed to be white Gaussian with zero mean stationary process and uncorrelated each other, the estimated values of each variance is $E[w] = E[v] = 0$, $E[ww^T] = Q_0$, and $E[vv^T] = R_0$, where $\mathbf{Q_0}$ and $\mathbf{R_0}$ are process and sensor noise covariances respectively. The optimal estimator, or the Kalman observer is to obtain an estimate of the state which is to minimize the mean square of the estimation error, or the covariance of $J_0 = E[\tilde{x}^T\tilde{x}]$, where $\tilde{x} = x - \hat{x}$. The gain, $\mathbf{L}$, is determined from the ARE of

$$\begin{aligned} &AS + SA^T + GQ_0G^T - SC^TR_0^{-1}CS = 0, \\ &L = SC^TR^{-1} \end{aligned} \tag{16}$$

where $\mathbf{Q_0}$ is positive semidefinite and $\mathbf{R_0}$ is positive definite.

The Riccati solution of Eq.(16) turns out to be an error covariance which has the relation of

$$tr(S) = E[\tilde{x}^T\tilde{x}] = J_0.$$

The diagram of Fig. 7 illustrates the control scheme which incorporates the Kalman observer together with the LQR control law. The converter gain K_1 and the normalizing gain K_2 are the same as Fig. 4, and the system equation is

$$\begin{aligned} &\dot{\begin{pmatrix} x \\ \hat{x} \end{pmatrix}} = \begin{pmatrix} A & -BK \\ LC & A-BK-LC \end{pmatrix}\begin{pmatrix} x \\ \hat{x} \end{pmatrix} + \begin{pmatrix} B \\ B \end{pmatrix} v, \\ &y = (C \quad 0)\begin{pmatrix} x \\ \hat{x} \end{pmatrix} \end{aligned} \tag{17}$$

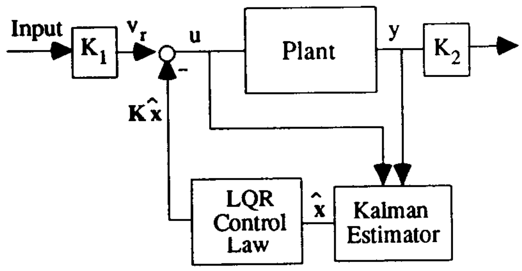

Fig. 7. Reactor Regulating System With LQR/LQG Control Scheme

Similar to the case of the LQR, the optimal observer gains are also dependent on the relative values of weighting matrices. Figure 8 shows the output of incremental power with an input of 10 percent step change from the steady state of full power. The system and measurement noise weighting matrices are assumed as $\mathbf{Q_0} = q_0 \times \mathbf{I}(5)$ and $\mathbf{R} = R = r_0$. Four cases of $q_0/r_0 = 0.1$, 1, 10 and 100 are simulated using Eq. (17). The control law feedback gains are fixed as those obtained in the LQR design and observer gains are determined from Eq. (16).

The figure explains that as q_0 becomes large, the output speed increases and is saturated above q_0/r_0 of about 10. But the observer gain, except that corresponding to the fourth variable, increases without a bound. The large value of q_0/r_0 has a merit of slightly increased phase margin (See Table 2). But this advantage is masked by the undesirable high gain. On the other hand, the low value of q_0/r_0 results in the sluggish responses as shown in Fig. 8. By trading off these properties, q_0/r_0 is determined to be unity.

It is well known that the LQG is susceptible to the model uncertainties, and the robustness as well as the stability margins might be lost. Therefore it is preferable to check the frequency response characteristics of the LQG system. The gain and phase margins of the open loop are found for various values of q_0/r_0, and are summed up in Table 2. The gain margins range from 60 to 65 dB and the phase margins stay around 80 degrees. These margins are sufficient for the system stability.

Table 2. Gain and Phase Margins of the LQR Design for Various Weighting Values

q_0/r_0	GM(dB)	PM(deg)	ω_G	ω_P
0.1	65	8	22.4	0.21
1.0	62	78	31.3	0.35
10.0	61	81	52.0	0.41
100.0	61	83	91.1	0.43

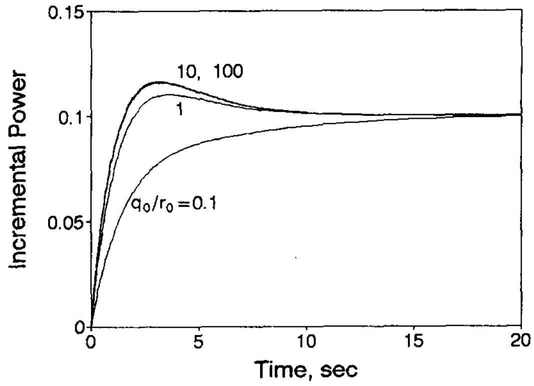

Fig. 8. Reactor Power Responses of LQG Design for Various Weighting Values

4. The Servo System

Since those systems of Fig. 4 and 7 are regulating systems with command inputs without feedback, the steady state values of the output are not the same as the command signals and are subject to variation depending on the design conditions. Therefore, it is necessary to build the servo system in which the output follows the command input signal. In the servo system, it is generally required that the system have integrators within the closed loop to eliminate the steady state errors.

The ordinary system is outlined in Fig. 9. The outdinary servo system with a feedback loop and the other is the regulating system of increased order. It should be noted that these terminologies are not of general use, but they are used in this study for the convenience of comparison.

The ordinary system is outlined in Fig.9. The output is feedbacked to generate the error signal, which is integrated by an integrator. This scheme has an advantage of the simplicity. But since the overall open loop is of non-minimum phase, there is a limitation on the feedforward gain K_1. For an example, by letting the transfer function from point A to point B of the figure be $K_1M(s)$, the characteristic equation of Fig. 9 is

$$1+K_I M(s) = 0 \tag{18}$$

where

$$M(s) = \frac{822.6s^3+2571.9s^2+853.1s+51.6}{s^6+406.7s^5+1633.0s^4+2433.8s^3+3090.9s^2+885.0s+51.6}$$

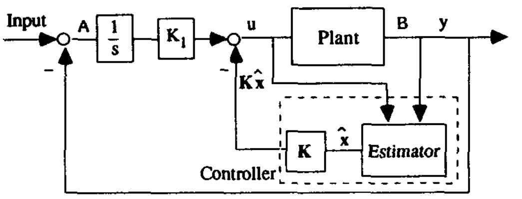

Fig. 9. Tracking System With a Feedback Loop

From the root locus diagram, it can be found that the system is of the non-minimum phase and the upper limit of K_I for the stability is about 183. The range of K_I is large for this specific case of the reactor system. But, in general, the range might be small and there could be a possibility of becoming unstable if the setting point drift or the system degradation occurs. To check the sensitivity of the system to the feedforward gain, the damping factors are calculated as in Table 3.

Table 3. Damping Factors of Feedback Servo System by Feedforward Gains

K_I	ξ	K_I	ξ
0.1	0.82	2	0.22
0.3	0.56	5	0.14
0.4	0.49	10	0.1
0.5	0.44	50	0.03
1.0	0.32	100	0.01
1.5	0.26	180	0.0002

The damping factors are very sensitive to the feedward gain, which are not desirable with respect to the system robustness. Figures 10 and 11 show the incremental power and relative rod speed when the reactor at the steady state of full power is placed to the 10 percent step increase. The output overshooting becomes larger with the increase of K_I. The control rod motion also becomes large. However the maximum value of rod speed is less than that of the regulating system.

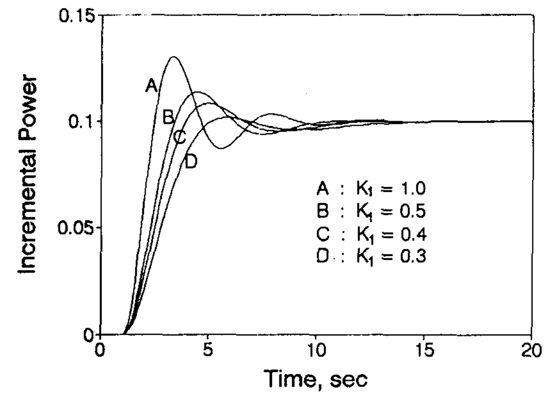

Fig. 10. Reactor Power Responses of Feedback System for Various Feedforward Gains

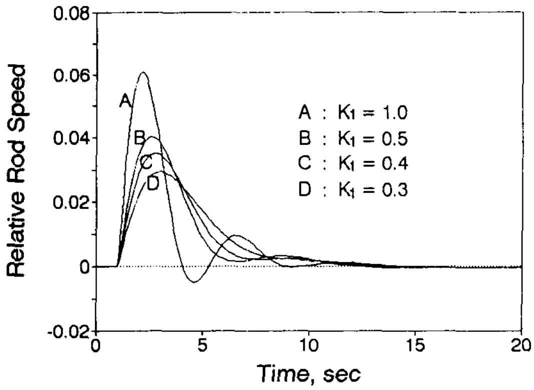

Fig. 11. Relative Rod Speeds of Feedback System for Various Feedforward Gains

The other scheme, the order increased regulating system, is shown in Fig. 12. In this scheme, the error signal is not treated separately but is augmented to the system as an additional state variable.

Since $\dot{x} = Ax + Bu$, $\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$, $\dot{z} = r - y$, and $u = -K\hat{x} - K_Z z + K_I r$, the system dynamics is

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A & -BK & -BK_Z \\ LC & -A-BK-LC & -BK_Z \\ -C & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \\ z \end{pmatrix} + \begin{pmatrix} BK_1 \\ BK_1 \\ 1 \end{pmatrix} r \quad (19)$$

$$y = \begin{pmatrix} C & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \\ z \end{pmatrix}$$

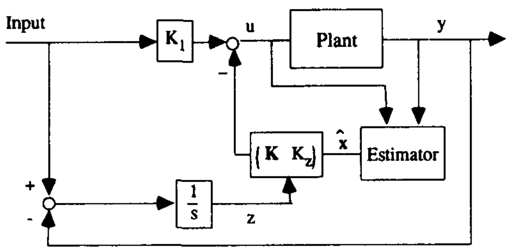

Fig. 12. Tracking System of Order Increased Regulating System

Equation (19) indicates that the feedback kain K_1 has no relation with the system properties. Since all the eigenvalues of the system are negative, the above system is always stable with the minus values of K_Z. The dynamic system of Eq. (19) is a regulating system with the increased order and the problem is to determine the control gain of $[K\ \ K_Z]$ by the LQR design.

In determining the gain, the cost function is modified as

$$J = \frac{1}{2}\int_0^T \left((x\ z)^T \bar{Q} (x\ z) + uRu \right) dt$$

$$\bar{Q} = \begin{pmatrix} I(5) & 0 \\ 0 & z_0 \end{pmatrix} \quad (20)$$

The feedback effects can be controlled by z_0. That is, if z_0 is large, the feedback effect becomes more salient and the system speed increases. Figure 13, with the feedforward gain of unity, explains the effect of z_0 on the system responses. With z_0 of 5×10^5, the system gives a satisfying result of the faster speed

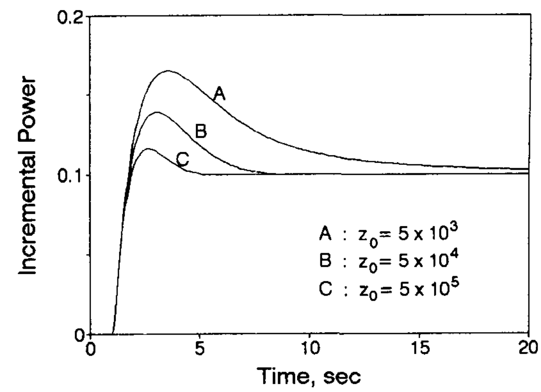

Fig. 13. Effects of Feedback Weighting of Augmented Variable

and lower overshooting.

As mentioned above, the feedforward gain K_1 has no effect on the system properties, but only has a function of amplifying the input signal. Figures 14 and 15 show the system output and control energy for various feedforward gains. As the feedforward gain increases, the initial speed becomes fast, but only at the expense of a large overshooting. Also the settling time is independent from the feedforward gain.

Although the feedforward gain K_1 boosts the initial speed, it is not desirable with respect to two criteria. The first one is the unfavorable output responses, and the second one is the transition of the rod motion. Particularly, the larger feedforward gain leads to the rapid rod motion, which results in the adverse effects such as rapid flux distortions. Therefore, taking these problems into considerartion, it would be better to drop off the feedforward gain to get the more desirable responses. These are explained in Figs. 14 and 15.

5. Conclusion

The optimal control techniques are used in designing the reator control system. The mathematical plant model with the temperature feedback effects is

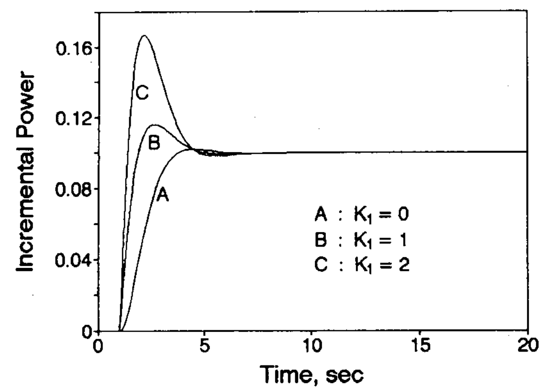

Fig. 14. Reacor Power Responses of Order Increased Regulating System for Various Feedforward Gains

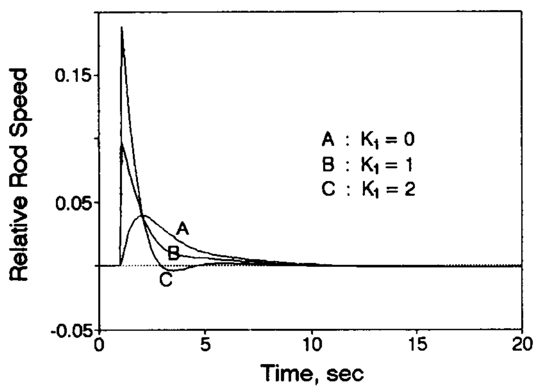

Fig. 15. Relative Rod Speeds of Order Increased Regulating System for Various Feedforward Gains

established from the one delayed neutron group point kinetics equation and the singly lumped thermal-hydraulic balance equations. The LQR control system is designed, being followed by the LQG design. Finally two control schemes, say, the ordinary feedback system and the regulating system of increased order are proposed for the servo system. The general characteristics of the control system such as stability margins and output responses are discussed.

Comparing those two schemes each other, it is found that the regulating system of increased order is superior to the ordinary feedback system in tracking the command signal. The output responses and the rod motions of the order increased regulating system are much better than those of the ordinary feedback system.

Although the order increased regulating system has an additional control law gain, the feedforward gain could be dropped, making the total number of gains be equal to those of the ordinary feedback system. Further, since the augmented gain is not so sensitive to the state weighting matrix, it is more robust than the ordinary feedback system which has a sensitive feedward gain.

Related to this study, a further work of incorporating other systems might be proposed. Since the reactor model developed in this study is of multi input system, the pump transients as well as the secondary dynamics could be included to make a larger control system. In addition, the problems such as the non-linear elements of speed programmer, and the dynamics of the various initial power levels should be studied in future works. Finally, for the case of a large commercial reactor, there is no actual problem in the power control because of a large thermal-hydraulic feedback and small neutron leakage. Therefore, for the verification, it is desirable to apply the control model developed in this study to a small experimental reactor.

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