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Development of Radiation Shielding Analysis Program Using Discrete Elements Method in X-Y Geometry

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2차원 직각좌표계에서 DEM을 이용한 방사선차폐해석 프로그램개발

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Abstract

A computational program [TDET] of the particle transport equation is developed on radiation shielding problem in two-dimensional cartesian geometry based on the discrete elements method. Not like the ordinary discrete ordinates method, the quadrature set of angles is not fixed but steered by the spatially dependent angular fluxes.

The angular dependence of the scattering source term in the particle transport equation is described by series expansion in spherical harmonics, and the energy dependence of the particles is considered as well.

Three different benchmark tests are made for verification of TDET : For the ray effect analysis on a square absorber with a flat isotropic source, the results of TDET calculation are quite well conformed to those of MORSE-CG calculation while TDET ameliorates the ray effect more effectively than S_N calculation. In the analysis of the streaming leakage through a narrow vacuum duct in a shield, TDET shows conspicuous and remarkable results of streaming leakage through the duct as well as MORSE-CG does, and quite better than S_N calculation. In a realistic reactor shielding situation which treats in two cases of the isotropic scattering and of linearly anisotropic scattering with two groups of energy, TDET calculations show local ray effect between neighboring meshes compared with S_N calculations in which the ray effect extends broadly over several meshes.

요 약

입자수송방정식의 각분할해석법에서 입자흐름방향을 묘사하는 각방향에 대한 각분할집합이 고정된 값이 아니고 각입자속으로 조종되는 각요소법에 근거하여 방사선차폐해석 목적의 전산프로그램(TDET)를 2차원직각좌표계에 대해서 개발하였다.

산란선원항의 각의존성은 Spherical Harmonics Series Expansion으로 해석하였고 입자의 에너지 의존성은 다중에너지군으로 처리하였다.

3 종류의 Benchmark 시험을 통해서 TDET 프로그램을 검증하였다. 평판형 등방적 선원을 가진 사방흡수체에 대한 해석에서 TDET 해석결과가 MORSE-CG 해석결과에 잘 일치하고, 각분할법으로 해석할때 나타나는 Ray effect를 DOT 4.3 보다 잘 치유하고 있다. 차폐체 내에 좁은 Vacuum duct가 있는 문제의 해석에서 TDET는 MORSE-CG와 마찬가지로 duct를 통한 streaming leakage에 대해서 분명하고 현저한, 그리고 DOT 4.3 보다 매우 훌륭한 해석결과를 보여주고 있다. 원자로차폐구조물 규모에 대해, 2 에너지군, 등방적 산란 및 선형 비등방 산란의 경우에 대해서 해석한 결과 각분할법으로 계산하여 제시하고 있는 기준치에서는 여러 mesh들에 걸쳐 넓은 규모의 Ray effect를 나타내고 있는데, TDET에 의한 해석결과에서는 이웃 mesh들 간에 미세한 Ray effect를 나타내고 있다.

1. Introduction

Discrete ordinates method is an effective methodology to solve the particle transport equation numerically. It works with relatively few arithmetic operations per space–angle grid point. There are, however, some limitations to the method when applied to the problems of optically thick regions, of very little scattering media or of localized sources. Anomalies in the scalar distribution, called ray effects, arise in such problems. These ray effects are caused due to the limited number of fixed angular quadratures in the discrete ordinates method.

Mathews[1] introduced the discrete elements method to ameliorate the ray effect in discrete ordinates method. He devised the method with by retaining the essential simplicity of the S_N algorithm but replacing the fixed quadrature set of angles with a "steered" ones. He showed that the spatial differencing scheme using the steered angle elements propagates the element fluxes in these steered directions to strongly ameliorate the ray effects. He tested the discrete elements method for the problems of one–dimensional slab geometry and two–dimensional isotropic scattering medium with a mono–energetic fixed flat source.

In this study, a computational program on radiation shielding calculations is developed in X–Y geometry based on Mathews' theory but extended to multigroup transport equations with anisotropic scattering.

2. Discrete Elements Method

The angular discretization method[2,3] incorporated in most of the wellknown transport codes is based upon the method of discrete ordinates. In this method, a set of discrete directions, $\hat{\Omega}_m (m=1,2,\cdots,M)$, is chosen, and the transport equation is evaluated for these directions with suitable averaging processes. The choice of these ordinates is not arbitrary but seeks to satisfy the following conditions :

1) physical symmetries are preserved upon discretization ;
2) the spherical harmonic moments are well approximated to provide accurate representa-

tion for the sources ; and

3) derivatives with respect to the angular coordinates resulting from the streaming operator are simply approximated.

In two or three dimension, not all of the above conditions can be met exactly with a single selection of a discrete ordinates set. Thus compromises are made, such as relaxing the complete symmetry requirement so that more spherical harmonics moments can be accurately calculated or the angular derivative term remains a simple expression with minimum coupling.

In the method of discrete ordinates, the angular fluxes are evaluated at discrete directions $\hat{\Omega}_m$ having components μ_m, η_m, and ξ_m ; $\mu_m^2+\eta_m^2+\xi_m^2=1$. Each discrete direction $\hat{\Omega}_m$ can be visualized as a point on the surface of a unit sphere with a surface area, W_m. The W_m denote the weights. Clearly, the sum of the weights must equal to the surface area of the unit sphere.

Considerable works[4] have been devoted to develop a suitable quadrature set for discrete ordinates codes. Nevertheless, there are an essentially invariant features in the discrete ordinates codes, using quadrature set of fixed discrete directions, although there are differences in treating the angular variables within the framework of the discrete ordinates method. In the discrete ordinates approximation scheme the transport equation is solved along a few discrete characteristics (i.e., rays). Alternatively, one can describe the discrete ordinates method as a transformation of the rotationally invariant transport equation to a finite set of coupled (via scattering) transport equation that are at most invariant under discrete ordinates. It is expected that this loss of rotational invariance will be mitigated, if the traveling directions of particles are not fixed but steered somewhat to the directions of most probable streams of the particles. By adopting this idea, the traveling direction of a particle with energies E at location r is defined as follows :

$$\hat{\Omega}_m = \frac{\int_{D_m} \hat{\Omega}\,\psi(r, E, \hat{\Omega})\, d\hat{\Omega}}{\int_{D_m} \psi(r, E, \hat{\Omega})\, d\hat{\Omega}} \tag{1}$$

where D_m is the domain of angles of m–th discrete element. Then, the direction $\hat{\Omega}_m$ is not fixed but steered by the spatially dependent angular fluxes, which is expected to reduce the loss of rotational invariance in original discrete ordinates method. The domains D_m are like wedge or cone which together form the unit sphere of solid angle and are visualized as a partitioning of the surface of a globe along lines of latitude and of longitude.

If we express the direction $\hat{\Omega}_m$ in polar cosine–azimuthal angle coordinates system (ξ, φ), where ξ is the direction cosine of polar angle θ and φ is the azimuthal angle, and assume separation of variables in the coordinates so that $\psi(\hat{\Omega})$ is of form $f(\xi)g(\varphi)$ within each discrete element m, the directional elements ξ_m and φ_m of the direction $\hat{\Omega}_m$ are expressed as a flux weighted mean in each coordinate in which the integral is done in-

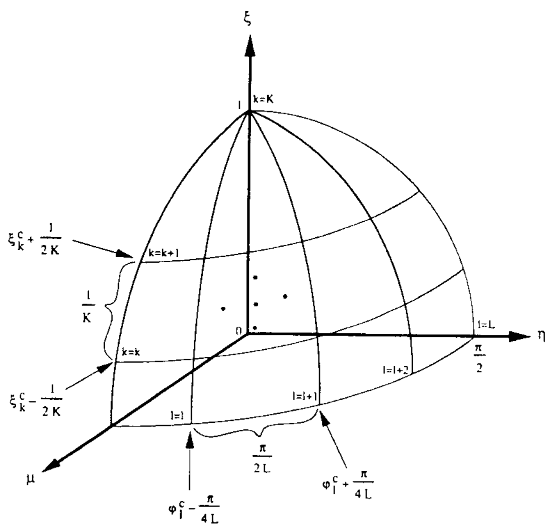

Fig. 1. Fixed Auxiliary Directions in a Typical Discrete Element on the Unit Octant of a Sphere for 3-point Gauss–Legender Quadrature Rule.

dependently. Then, the resulting equation is one dimensional in a view point of angular variables. This one-dimensional integral is approximated by the quadrature rule. Three-point Gauss-Legendre quadrature rule[5] is an effective choice. Employing this quadrature rule, each flux-weighted mean is approximated with a numerical integration form of three points of fixed direction variables and the angular fluxes in that directions. The fixed directions are associated with an offset factor bh of the associated point interval in the quadrature rule.

Figure 1 shows a set of the fixed auxiliary directions in a typical discrete element domain for the three-point Gauss quadrature rule, where it is figured on a unit octant of sphere. The numerical integration forms of the steered angle elements are as follows;

$$\xi_m = \xi_k^C + (bh)\frac{f(\xi_k^C + bh) - f(\xi_k^C - bh)}{f(\xi_k^C + bh) + 1.6f(\xi_k^C) + f(\xi_k^C - bh)} \tag{2}$$

$$\xi_k^C = (k - \frac{1}{2})(\frac{1}{K}), \quad k = 1, 2, \ldots, K \tag{3}$$

$$b = \sqrt{0.6}, \quad h = \frac{1}{2K} \tag{4}$$

$$\varphi_m = \varphi_\ell^C + (bh)\frac{g(\varphi_\ell^C + bh) - g(\varphi_\ell^C - bh)}{g(\varphi_\ell^C + bh) + 1.6g(\varphi_\ell^C) + g(\varphi_\ell^C - bh)} \tag{5}$$

$$\varphi_\ell^C = (\ell - \frac{1}{2})(\frac{2\pi}{4L}), \quad \ell = 1, 2, \ldots, L \tag{6}$$

$$b = \sqrt{0.6}, \quad h = \frac{\pi}{4L}. \tag{7}$$

In the above forms, $f(\xi_k^C)$ and $g(\varphi_l^C)$ are the auxiliary fluxes in the center directions of the solid angle element $\hat{\Omega}_m$ which have the domains of $[\xi_k^C - 1/(2K), \xi_k^C + 1/(2K)]$ and $[\varphi_l^C - \pi/(4L), \varphi_l^C + \pi/(4L)]$, respectively, and the others are the fluxes in the directions apart from the center directions by bh, where b is the associated point interval and h is half of the discrete angular mesh.

With this scheme, the element weights are equal for each k and l, and are given by;

$$W_k^\xi = \frac{1}{K}, \quad W_\ell^\varphi = \frac{1}{4L}. \tag{8}$$

In the discrete elements method, we can employ the same methodology of the spatial differencing scheme that is used in the discrete ordinates method. In this study, the diamond difference scheme is employed and the zero fix-up strategy is used in case that negative fluxes occur. The step difference scheme, the diamond difference scheme with no fix-up of negatives, and the step characteristic scheme are also programmed and tested in this study.

With original spatial differencing scheme used in the discrete ordinates method, conservation of particles is not assured in the discrete elements method. Because the steered streaming directions used in the discrete elements method are not fixed throughout the spatial mesh cells, the continuity of the currents is not assured across the cell interfaces. Conservation of particles in the discrete elements method is accomplished in two steps;

1) Within each space cell, the streaming angle for each element is assumed to be constant. It may have a different value after each iteration. While the spatial differencing scheme is performed, the flux-weighted streaming direction is treated as fixed. This assures conservation of particles within each space cell since explicitly conservative spatial differencing scheme is used as for discrete ordinates method.
2) The flux is discontinuous between neighboring cells, since the streaming direction is discontinuous across the cell interface. Then the conservation of particles across the cell interfaces is not assured automatically. Conservation of particles across cell interfaces is achieved by using the normal current out of one cell as the

normal current into the next cell.

The mathematical expressions assuring the particle conservation in the discrete elements method are given as the following;

$$\sum_{m=1}^{M} W_m \mu_{i-1,j,m} \psi_{(i-1)+\frac{1}{2},j,m,g} = \sum_{m=1}^{M} W_m \mu_{i,j,m} \psi_{i-\frac{1}{2},j,m,g} \quad (9)$$

$$\sum_{m=1}^{M} W_m \eta_{i,j-1,m} \psi_{i,(j-1)+\frac{1}{2},m,g} = \sum_{m=1}^{M} W_m \eta_{i,j,m} \psi_{i,j-\frac{1}{2},m,g} \quad (10)$$

In this notation, $\mu_{i,j,m}$ is the x–direction cosine and $\eta_{i,j,m}$ is the y–direction cosine of the discrete angle element $\hat{\Omega}_m$, and $\psi_{i,j,m,g}$ is the average angular flux of the particles of discrete angle element m with energies of group g in the space cell (i,j), where subscript i,j represents the cell–centered point of the space cell (i,j), $i-1/2,j$ (or $i,j-1/2$) represents the cell left (or bottom) boundary (i,j), $(i-1)+1/2$,j represents the cell right boundary of $(i-1,j)$ cell, and $i,(j-1)+1/2$ represents the cell top boundary of $(i,j-1)$ cell of which the spatial grids are I and J. W_m represents the weights which is defined by equation 8.

3. Source Term[3)]

To extend the Mathews' theory employed in the discrete elements method to multigroup transport equations with anisotropic scattering, it should be known how the angular dependence of the source terms is represented numerically in the transport equations. Spherical harmonics series expansion is an effective one which is commonly used in the discrete ordinates method.

The source term includes, in general, sources from an extraneous fixed source distributed throughout the mesh, fission source, scattering in source from other energy groups, and also scattering in source from other directions within the same energy group. In principle, this can be expressed as;

$$S_{i,j,m,g} = Q_{i,j,m,g} + \chi_g \sum_{g'=1}^{G} \nu\sigma_{f_{z,g'}} \sum_{m'=1}^{M} W_{m'} \psi_{i,j,m',g'} + \sum_{g'=1}^{G} \sum_{m'=1}^{M} W_{m'} \sigma_{s_{z,m';m,g';g}} \psi_{i,j,m',g'} \quad . \quad (11)$$

The first and least controversial simplification is accomplished by grouping those space cells having similar material composition into "material zones" (indicated as z in above equation), and by requiring that all cells within such zones have the same cross–sections. In this study, all the cross–sections are simplified as "material zone–wise cross sections".

A less satisfactory approximation is made in fission source term. Since the fission reaction is normally treated as an isotropic process, it is possible that the fission spectrum χ is assumed to depend only upon energy, and that ν and σ_f are treated as a single unit $\nu\sigma_f$ which is function of space r and energy E. Thus, the fission term can be expressed as $\chi_g D^F_{i,j}$, where:

$$D^F{}_{i,j} = \sum_{g'=1}^{G} \nu\sigma_{f_{z,g'}} \phi_{i,j,g'} \quad (12)$$

$$\phi_{i,j,g'} = \sum_{m'=1}^{M} W_{m'} \psi_{i,j,m',g'} \quad (13)$$

The spherical harmonics expansions are introduced to represent the angular dependence of the scattering source term. We now define a type of spherical harmonic expansion in computational form;

$$C_\ell^k(\mu,\varphi) = \left[\frac{2\ell+1}{4\pi} \frac{(2-\delta_{k0})(\ell-k)!}{(\ell+k)!}\right]^{1/2} P_\ell^k(\mu) \cos(k\varphi) \quad (14)$$

where δ_{K0} is the "Kronecker delta", and the $P_l^k(\mu)$ are the associated Legendre polynomials. With these, the scattering source is approximated :

$$Q_{s_{i,j,m,g}} = \sum_{l=0}^{L} \sum_{k=0}^{l} C_{l\ m}^{k} \sum_{g'=1}^{G} \sigma_{s\ z,g;g'}^{l} \phi_{l\ i,j,g'}^{k} \quad (15)$$

$$\phi_{l\ i,j,g'}^{k} = \sum_{m'=1}^{M} W_{m'} C_{l\ m'}^{k} \psi_{i,j,m',g'} \quad (16)$$

$$C_{l\ m'}^{k} = C_l^k(\mu_{m'}, \varphi_{m'}) \quad (17)$$

where $\sigma_{s\ z,g:g'}^{l}$ represents the material zone-wise differential scattering cross section scattered from energy group g' into energy group g. If L, the series truncation index, equals to zero, the scattering is isotropic. If the scattering is linearly varying in μ (linearly anisotropic scattering), then $L=1$.

In a manner similar to that used for the scattering source term, the extraneous (or fixed) source term can be represented by a finite expansion using the spherical harmonics, when the extraneous source is inhomogeneous. In this study, however, the extraneous source term is treated simply as constants, which is true in case that the extraneous source is distributed homogeneously.

4. Development of the Program

The algorithm of the discrete elements transport program is based upon the same algorithm used in discrete ordinates transport code system.

Since the source term in S_N involves an integral over fluxes moving in other directions within the same energy group, iterations over source term must be used. The scattering integral is evaluated based on prior information or guesses if available. Once all of the fluxes for an energy group are evaluated, a new scattering integral is evaluated, and so on. This "inner" or "flux" iterations are deemed to have converged when the fluxes from two or more successive iterations are sufficiently close each other.

In S_N method, before going into the iteration process, one fixed angular quadrature set is selected. Then, the spatial differencing scheme is performed. In the discrete elements program, the process is somewhat different since the angular quadrature set is not fixed but steered by the angular fluxes. It is as follows :

1) A set of fixed auxiliary directions is determined in a discrete angle elements $\hat{\Omega}_m$ with the equations (3) and (6).
2) Using the set of fixed auxiliary directions determined in step 1, the spatial differencing scheme is performed in a cell (i,j) to determine the angular fluxes in the directions of the angle element in the cell.
3) The flux-weighted mean angles ξ_m and φ_m of the discrete angle element are determined with any quadrature rule, such as three-point Gauss Legendre quadrature rule [Equations (2),(4), (5), and (7)], Gauss Christoffel quadrature rule, etc.
4) Using equation (14), the spherical harmonic expansion functions $C_l^k(l=0,1,2,\cdots,L)$ are calculated with the flux-weighted mean angles of the discrete angle element determined in step 3.
5) Using the flux-weighted mean angles of the discrete angle element, the spatial differencing scheme is performed again in the cell to determine the main angular flux $\psi_{i,j,m,g}$ of energy group g.
6) Steps from 1 to 5 are repeated for another discrete angle element till all of the main angular fluxes $\psi_{i,j,m,g}(m=1,2,\cdots,M)$ are determined.
7) Using equation (16), the moments $\phi_{l\ i,j,g}^{k}(l=1,2,\cdots,L)$ are calculated with all of the main angular fluxes at the cell and with the spherical harmonic expansion functions obtained in step 4.

8) Using equation (15), the scattering source $Q_{s\, i,j,m,g}$ at a discrete angle element is calculated with the moments obtained in step 7 and with the spherical harmonic expansion functions obtained in step 4.
9) If there is a fission source, the fission source is calculated by equations (12) and (13).
10) The total source $S_{si,j,m,g}$ is calculated by equation (11). In this study, the fixed source has given as input data.
11) The outgoing angular fluxes into next cell are corrected with equations (9) and (10) to assure the conservation of particles across the cell interface. After this step, the normal current out of one cell equals to the normal current into next cell.
12) Steps from 1 to 11 are repeated to next cell.
13) If the steps from 1 to 12 described above are finished for all of the space cells ($i=1,2,\cdots,I$; $j=1,2,\cdots,J$) in one quadrant direction in the X–Y geometry, steps from 1 to 12 are repeated for another quadrantal direction till all of the four directions of the hemisphere in the geometry are executed.
14) Using equation (13), the main scalar fluxes $\phi_{i,j,g}$ are calculated in all of the space cells.
15) The maximum flux deviation is investigated in all of the space cells and is compared to the convergence criterion of the inner iteration.
16) If the maximum flux deviation is smaller than the convergence criterion, steps from 1 to 15 are repeated to another energy group till the calculations are completed for all energy groups. But if the maximum flux deviation is greater than or equals to the convergence criterion, steps from 1 to 15 are repeated with the updated source term calculated in step 10.
17) The other processes are similar to those of S_N.

5. Benchmark Calculation

A computational program of the multi–group particle transport equation, which uses the Lahey FORTRAN Compiler F77L–EM/32 Version 2.0 and the Lahey LINK–EM/32 Extended Memory Linker in PC–386(33MHz) with Intel 387 Math Co–Processor, is developed on radiation shielding problem in two–dimensional cartesian geometry using the discrete elements method. The angular dependence of the scattering source term is considered in the program as expanding the source term with the spherical harmonics expansion series functions.

It is tested in three different cases of the benchmark problems on radiation shielding.

5.1. Problem 1[1),6),7)]

We consider a well–known problem of a flat isotropic source in a square absorber (C=2/3). This problem was introduced initially by Lathrop to analyze the ray effects of the discrete ordinates transport, and has been dealt repeatedly in many papers for the purpose.

The spatial region was subdivided into 30 by 30 equally spaced mesh intervals $\Delta x=\Delta y=2/30$. The diamond difference scheme with zero fixup strategy in case that negative flux occurs is employed as spatial differencing scheme, in which 3–point Gauss Legendre quadrature rule with 3 elements in polar quadrant (K3) and 4 elements in azimuthal quadrant (L4) is used to determine the flux–weighted mean angles.

To assure the reliability of the calculation, it is compared with the calculation of MORSE–CG[8)], a Monte–Carlo transport code, and DOT 4.3, a well–known discrete orinates code. Figure 2 represents the cell–averaged scalar fluxes calculated from TDET along the uppermost raw of the cells (Y=1.967 cm) in a 30 by 30 spatial meshes which are compared with those obtained from the calculations of MORSE–CG and DOT 4.3.

The results of TDET are quite well conformed

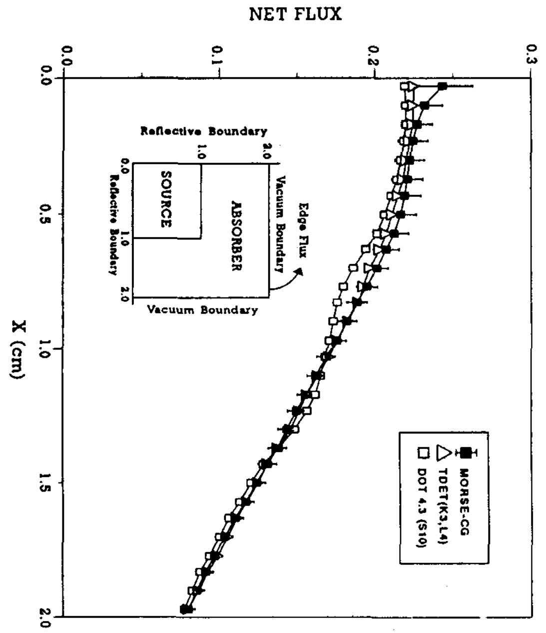

Fig. 2. Comparison of TDET Results with MORSE-CG and DOT 4.3 Results in a Square Absorber with Flat Isotropic Source(c=2/3)

to those of MORSE-CG. The results of DOT 4.3, however, show a little differences from those of MORSE-CG or TDET. These are due to the loss of rotational invariance of fixed angular quadrature. Comparing the results of TDET with those of DOT 4.3, it may be concluded that the discrete elements method with K-3, L-4 (96 angle elements in a sphere) ameliorate the ray effect more efficiently than the discrete ordinates method with S-10 (140 discrete angles in a sphere).

5.2. Problem 2[1)]

We consider a useful test case of a streaming duct in a shield, which is one of common shielding design problem. This problem consists of a thin source region along the bottom of a shield with a vacuum boundary and a centered narrow vacuum duct that nearly penetrates the shield region. This particular problem was selected as an idealized representation of an access port in a fusion reactor design. The cross-sections and source specifications for the problem 1 and 2 are given in Table 1.

Table 1. Cross Sections and Sources for Benchmark Problems

	Problem I*1	Problem II*2	
		Shield	Duct
$\sigma_a(cm^{-1})$	0.25	0.75	0.0
$\nu\sigma_t$	0.0	0.0	0.0
σ_t	0.75	1.0	0.1e-05
σ_s	0.50	0.25	0.1e-05
$S(n/cm^3)$	1.0	2.0	

*1 Flat isotropic source in a square absorber
*2 Narrow vacuum duct in a square shield

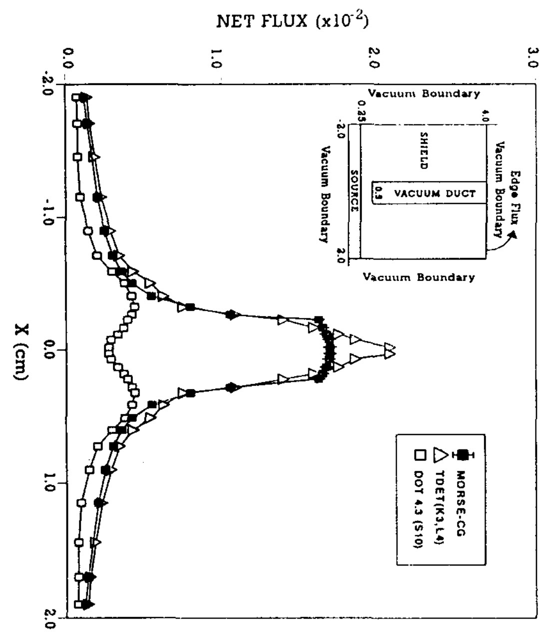

Fig. 3. Comparison of TDET Results with MORSE-CG and DOT 4.3 Results in a Streaming Analysis in a Vacuum Duct-.(duct width=0.5cm)

It is expected that the major leakage of the scalar flux out of the shield is due to the streaming of particles through the duct. Figure 3 shows the results of calculations with TDET(K–3, L–4), DOT 4.3 (S–10), and MORSE–CG.

We can find that there are considerable differences in the leakage of the scalar fluxes between TDET or MORSE–CG calculations and DOT 4.3 calculation, at the shield top through the vacuum duct with the width of the duct, 0.5 cm. It can hardly find the leakages through the narrow duct with the results of DOT 4.3 calculation. On the contrary, the leakage is conspicuous and remarkable in the results of TDET and MORSE–CG. There are sizable differences between TDET results and MORSE–CG results in the vacuum duct region, but the differences are negligible comparing to those of DOT 4.3. The differences between TDET calculation and DOT 4.3 calculation diminish as the width expands, and results of the calculations approach to those of MORSE–CG calculation. Figure 4 shows that the results of TDET well agree with those of MORSE–CG, when the width is 1.0 cm which is twice of the previous case, and the results of DOT 4.3 close similarly to those of MORSE–CG and of TDET.

The bad resolution of the S_N program in narrow vacuum duct is due to the loss of rotational invariance of the fixed angular quadrature described previously. With a few set of fixed directions, it is impossible to resolve the rapid changes of the particles streaming in such region like narrow vacuum duct. The fluxes diminish in the center of the duct in the results of TDET, like as in DOT 4.3. It may come from the loss of rotational invariance of the angular quadrature of the discrete elements method, although it is not so severe as in the discrete ordinates method. Then we conclude that TDET program analyze the transport of the particles in complicated geometry such as duct, high absorber, etc. better than S_N program.

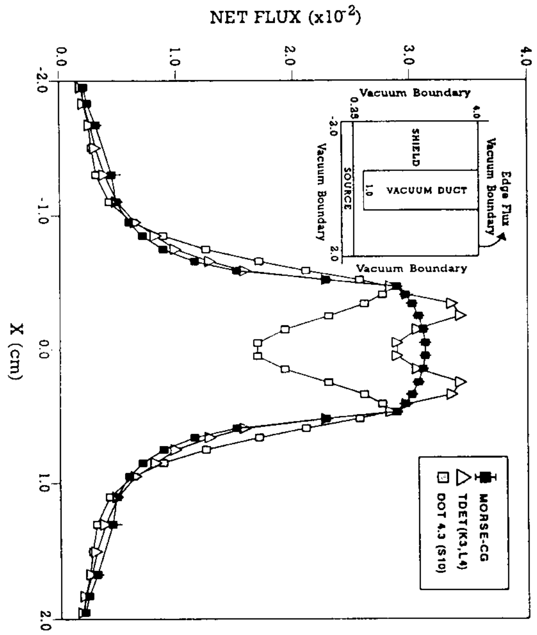

Fig. 4. Comparison of TDET Results with MORSE–CG and DOT 4.3 Results in a Streaming Analysis in a Vacuum Duct.(duct width = 1.0cm)

5. 3. Problem 3[9]

We consider a benchmark problem issued in ANL–7416 for the purpose of testing the multi–group two dimensional transport problems in X–Y geometry. The problem represents a realistic reactor shielding situation with a two–dimensional isolated source in an absorbing medium. The problem is treated in two cases : isotropic scattering and linearly anisotropic scattering. The reflective boundary conditions are employed to three surfaces and the vacuum boundary condition to the remaining one surface. The problem geometry described in the right corner of the figures from 5 to 8 shows the boundary conditions. The cross–sections and source specifications for the problem are given in Table 2 and 3.

Table 2. Cross Sections and Sources for Benchmark Problem Situation ID.5–A1 (Isotropic Scattering Assumed)

Isotropic (cm^{-1})		
	Group 1	Group 2
σ_a	0.061723	0.096027
$\nu\sigma_f$	0.0	0.0
σ_t	0.092104	0.100877
$\sigma_{s0g\,;\,g}$	0.006947	0.004850
$\sigma_s 0_{g-1\,;\,g}$		0.023434
Source Density (n/cm^3)		
	0.006546	0.017701

Table 3. Cross Sections and Sources for Benchmark Problem Situation ID.5–A2 (Linearly Anisotropic Scattering Assumed)

Isotropic (cm^{-1})		
	Group 1	Group 2
σ_a	0.061723	0.096027
$\nu\sigma_f$	0.0	0.0
σ_t	0.101080	0.108529
$\sigma_{s0g\,;\,g}$	0.015923	0.012502
$\sigma_s 0_{g-1\,;\,g}$		0.023434
Linear Anisotrpic (cm^{-1})		
$\sigma_{s1g\,;\,g}$	0.008976	0.003914
$\sigma_s 1_{g-1\,;\,g}$		0.009016
Source Density (n/cm^3)		
	0.006546	0.017701

Twenty equally spaced intervals were used between X=0.0 and 65.0, with 21 equally spaced intervals between X=65.0 and 133.0. In the Y direction, 18 equally spaced intervals were used between 0.0 and 60.0, with 24 between Y=60.0 and 140.0. The diamond difference scheme with zero fix–up and the K–3, L–4 discrete angle elements were used, with a global error of 10^{-4} for the convergence criterion.

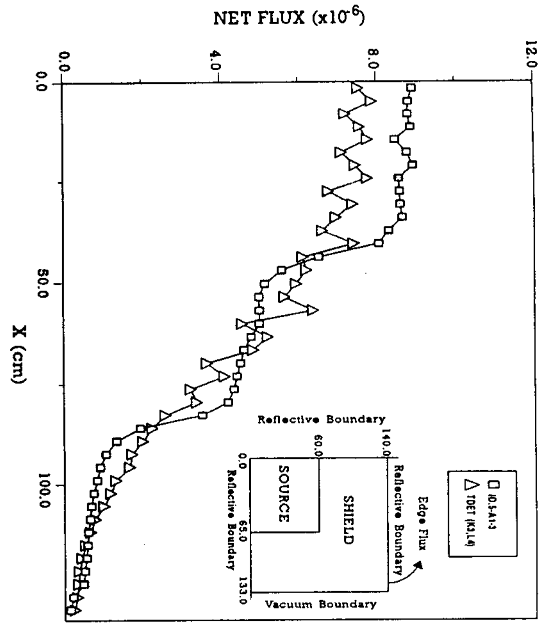

Fig. 5. Comparison of TDET Results with the Benchmark Problem Solution ID.5–A1–3 in Which Two Group Isotropic Scattering is Assumed.(group 1, y=139.2cm)

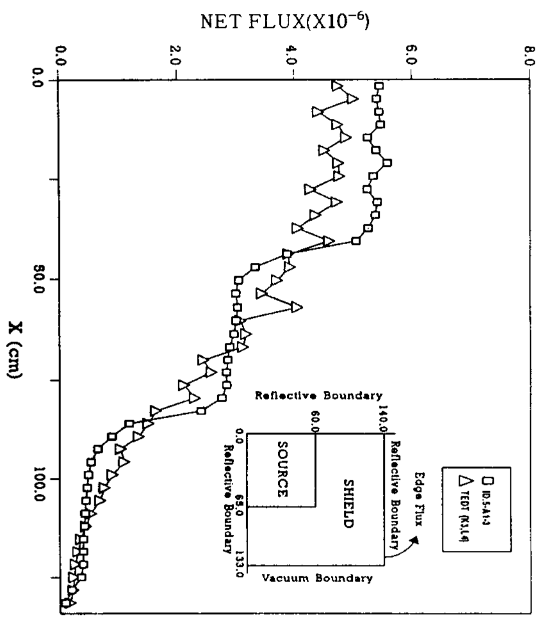

Fig. 6 Comparison of TDET Results with the Benchmark Problem Solution ID.5–A1–3 in Which Two Group Isotropic Scattering is Assumed.(group 2, y=139.2cm)

Figures 5 and 6 show the results of TDET calculations with the benchmark problem solution ID. 5–A1–3. The benchmark problem solution is calculated by H. Greenspan and E.M Gelbard with the mathematical model of the discrete ordinates transport using S–8 order of the symmetric set of directions and weights, the diamond difference model for the spatial differencing scheme, 81 by 84 mesh intervals, and 10^{-6} for the convergence criterion. The distribution of the fluxes at the uppermost edge (y=139.2 cm) of the medium by the discrete elements transport program shows finely fluctuated phenomena in group one of the particle energy, as well as in group two. The phenomena of the fluctuation may be reduced in fine–mesh calculation such as for the benchmark calculation. We can find that the ray–effect of the discrete elements transport program presents imminently but finely between the neighboring meshes, although not so severe as that of the discrete ordinates method in which it extends broadly over several meshes. Such ray effect in discrete elements transport program may come from the algorithm of the particle conservation between cell interfaces such as : the different angle quadratures between neighboring meshes induce to correct the outgoing angular fluxes of previous cell calculation to maintain the balance equation between the cell interfaces, so the differences of scalar fluxes which are the summation of the angular fluxes for all of the angular quadratures of the unit sphere are present between neighboring meshes, especially in deep medium in which particles travel several mean free paths. In a small medium like as problem 1 of this paper, these phenomena do not occur. The ray effects occur broadly between several meshes, on the contrary, in the discrete ordinates method. These are due to

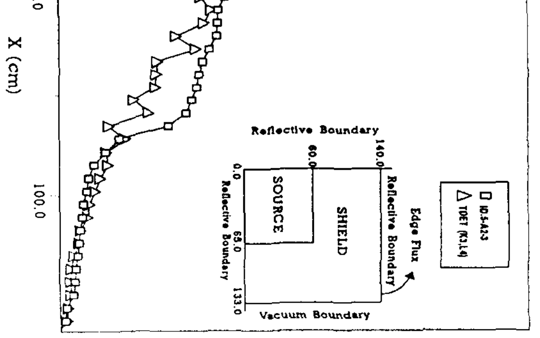

Fig. 7. Comparison of TDET Results with the Benchmark Problem Solution ID.5–A2–3 in Which Two Group Linearly Anisotropic Scattering is Assumed.(group 1, y=139.2cm)

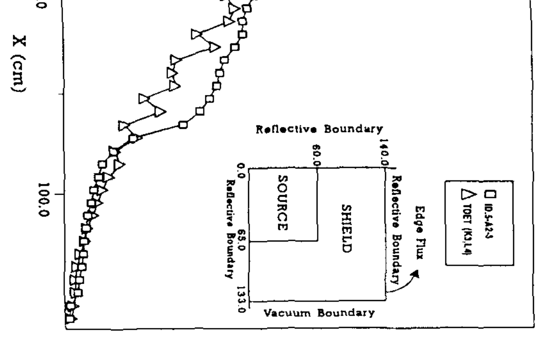

Fig. 8. Comparison of TDET Results with the Benchmark Problem Solution ID.5–A2–3 in Which Two Group Linearly Anisotropic Scattering is Assumed.(group 2, y=139.2cm)

the loss of rotational invariance of the fixed angular quadrature.

The averaged distribution curve of the penetrated scalar fluxes of the discrete elements calculation are fairly good agreement with, somewhat better than in accuracy, that of the S_N calculation.

In linearly anisotropic scattering problem, which is shown in Figures 7 and 8, such local ray-effect between neighboring meshes also exist, as happens to the isotropic scattering problem calculated by discrete elements transport program, and the penetrated scalar fluxes are comparatively well consistent with the benchmark values of S_N.

6. Conclusions

In ray effect analysis on a square absorber with a flat isotropic source, the results of TDET calculation with K-3, L-4 which has 48 discrete elements in unit hemisphere are quite well conformed to those of MORSE-CG and ameliorate the ray effect efficiently compared to those of DOT 4.3 calculation with S-10 symmetry angular quadrature which has 70 directions. TDET program analyzes quite well the streaming leakages through the narrow vacuum duct in a shield. Comparing to DOT 4.3, TDET shows remarkable simulation results on that problem as well as MORSE-CG does. In the huge shielding problem such as reactor shielding, TDET shows local ray effect between neighboring meshes which is different to the benchmark solution with S_N method. This ray effect is understandable because there is a step of correcting the angular fluxes across the cell interface between each neighboring two cells to balance the current term.

We conclude that TDET program is reliable sufficiently to analyze the multi-group particle transport equation on shielding problem by using the steered angular quadratures which are weighted by the spatially dependent angular fluxes and considering the anisotropic scattering phenomena of the particle streaming in X-Y geometry. It shows that the ray effect of the discrete ordinates program diminishes remarkably in discrete elements transport program, and TDET gives fairly good results in accuracy.

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