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Nonlinear Model-Based Robust Control of a Nuclear Reactor Using Adaptive PIF Gains and Variable Structure Controller

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적응 PIF Gain 및 가변구조 제어기를 사용한 비선형 모델에 의한 원자로의 Robust Control

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Abstract

A Nonlinear model-based Hybrid Controller (NHC) is developed which consists of the adaptive proportional-integral-feedforward (PIF) gains and variable structure controller. The controller has the robustness against modeling uncertainty and is applied to the trajectory tracking control of single-input, single-output nonlinear systems. The essence of the scheme is to divide the control into four different terms. Namely, the adaptive P-I-F gains and variable structure controller are used to accomplish the specific control actions by each terms. The robustness of the controller is guaranteed by the feedback of estimated uncertainty and the performance specification given by the adaptation of PIF gains using the second method of Lyapunov. The variable structure controller is incorporated to regulate the initial peak of the tracking error during the parameter adaptation is not settled yet. The newly developed NHC method is applied to the power tracking control of a nuclear reactor and the simulation results show great improvement in tracking performance compared with the conventional model-based control methods.

요 약

적응 비례-적분-추정 (PIF) gain 및 가변구조 제어기를 사용하는 비선형 모델에 의한 하이브리드 제어방법 (NHC)이 개발되었다. 이 제어기는 모델의 불확실성에 대해 robust한 성질을 가지며 단일 입-출력 비선형 시스템의 추적제어 (tracking control)에 응용된다. 이 방법의 특징은 제어기가 각각 특정 역할을 수행하는 4개 부분으로 구성되는 것이다. 즉, 적응 P-I-F 및 가변구조 제어기로 구성된다. Lyapunov의 제 2방법으로 결정된 적응 PIF gain에 의한 제어성능 확보 및 모델의 불확실도를 평가하여 피이드백 함으로써 모델의 불확실성에 대해 robust한 제어기를 구성하였다. 가변구조 제어기는 PIF gain이 적절히 결정되지 않은 상태인 초기의 오차증가를 제어하기 위해 도입되었다. 새로 개발된 NHC방법을 원자로의 출력변동 제어에 응용한 결과 기존의 모델을 이용한 제어방법 (model-based controller)들에 비해 제어성능이 크게 개선되었다.

1. Introduction

A number of model–based nonlinear control laws have been proposed in recent years. A major problem in this area is to circumvent the inevitable gap between real plant and the theory of conventional analysis and synthesis of linear, time–invariant systems[1,2,3,4,8]. Especially, the trajectory tracking control of a nonlinear plant is a class of problems where the classical linear, transfer function methods break down, since no transfer function can represent the system over the entire operating region. There are alternative methods which can be applied to the nonlinear system control that are robust and easily implementable such as variable structure control (VSC) approach and global linearizing control (GLC) method. VSC concept gives a viable way to the control of nonlinear systems with known uncertainty bounds and the construction of proper switching surface[1,4]. Recently, there has been considerable effort to design controllers in which the closed–loop response is exactly linear in a global sense. This GLC is achieved by feedback of the inverse of nonlinear terms based on plant model (see ref. 5 and the references therein). This algorithm makes use of the state and feedback transformations to bring a nonlinear (time–invariant) system into an equivalent linear system. Then well–established linear system theory can be utilized. But robustness and invariance properties against the change of operating points should be considered. The goal of this paper is to develop a robust and nonlinear model–based controller, which may be considered as a subset of GLC algorithm, for the power tracking control of a nuclear reactor. The method shall be called nonlinear model–based hybrid controller (NHC), since the control method is divided into four different terms of the adaptive PIF gains and VSC. The robustness is obtained by the feedback of estimated uncertainty and integral control action. The tracking performance is greatly improved by incorporating a gain adaptation algorithm for PIF gains.

Another difficulty in applying the model–based controller is the need for the knowledge of the unmeasurable state variables. In this paper, an open–loop observer is used to estimate the unmeasurable state variables. The effect of estimation error in the open–loop observer due to mismatches in the initial conditions and system parameters is compensated by an uncertainty estimator.

The NHC method is applied to the power tracking control of a nuclear reactor in which we use as a plant model a nonlinear model for neutronics/thermal–hydraulics and six–group precursors for a pressurized water reactor (PWR). As a control model, we use a time–invariant neutronics/thermal–hydraulics, one–group precursor model. The control model is simpler and of lower order than the plant model, but is still nonlinear.

The paper is organized as follows. Section 2 provides some mathematical preliminaries. Section 3 provides the development of NHC. Section 4 provides the NHC design method for power tracking control of a PWR and the simulation results with discussion. Finally, Section 5 provides conclusions of the study and recommendations for future work.

2. Mathematical Preliminaries

System description. We consider the following n–dimensional single–input, single–output system which is nonlinear in the state vector $x(t)$ and linear in the input $u(t)$ of the form :

$$\begin{aligned} \dot{x} &= f(x,t) + g(x,t)u, \qquad x(0) = x_0 \\ y &= h(x) \end{aligned} \tag{1}$$

where $x(t)$, $u(t)$ and $y(t)$ are state vector, control input and the output of nterest, respectively. The vector fields $f(x) : R^n \rightarrow R^n$, $g(x) : R^n \rightarrow R^{n\times 1}$ and an output function $h(x) : R^n \rightarrow R$ are assumed to be smooth and suitably well behaved.

Lie derivative. The *Lie derivative* of h with respect to f is defined by

$$L_f h \equiv \nabla h \cdot f \tag{2}$$

where ∇h is the gradient of h with respect to x.

Relative degree. The *relative degree* of the output with respect to the control input is defined as the smallest positive integer r satisfying

$$L_g L_f^{r-1} h(x) \neq 0 \tag{3}$$

which is interpreted as the minimum number of times one has to differentiate the output y, with respect to time, in order to have the derivative of y depend explicitly on $u(t)$. In this paper, we will treat systems with relative degree 1, which means that control input directly affects the first derivative of the output (nuclear reactors belong to this category).

3. Development of NHC Method

The model-based control with PI feedback has been proposed and applied to chemical processes[6,7] which involves specification of PI control strategy to the target trajectory using the first time-derivative of the plant output. The tracking error in the PI law is the deviation between desired trajectory and actual measured output. This algorithm can be considered as the relative degree 1 case of the GLC and an optimal control approach in the sense of forcing the plant output rate to match a reference rate. The reference rate for the plant is generated from the set point deviation as

$$r_d = k_p(y - y_d) + k_i \int_{t_0}^{t} (y - y_d) d\mu \tag{4}$$

where r_d is the desired output rate of change, y_d is the prescribed desired trajectory, and k_p, k_i are the proportional and integral (PI) gains with negative signs, respectively. The control signal is generated by setting

$$r_d = \dot{y} - \dot{y}_d \tag{5}$$

and combining (5) with (1) and (4) we get the necessary control input based on both plant model and PI settings, which is the key difference from the conventional PI controllers. The control input is then given by

$$u(t) = (L_g h)^{-1}[k_p(y - y_d) + k_i \int_{t_0}^{t} (y - y_d) d\mu - L_f h + \dot{y}_d] \tag{6}$$

A major advantage of this method is that a generic response can be specified through the choice of the gains k_p and k_i. This response is independent of the plant dynamics provided that the model perfectly depicts the plant and the exact control input can be implemented. If these conditions for the model and control input are satisfied, the closed-loop system equation can be given by

$$\dot{e} = k_p(y - y_d) + k_i \int_{t_0}^{t} (y - y_d) d\mu \tag{7}$$

where $e = y - y_d$. Eq. (7) can be written in Laplace domain as:

$$Y(s) = \frac{2\xi\tau s + 1}{\tau^2 s^2 + 2\xi\tau s + 1} Y_d(s) \tag{8}$$

and

$$k_p = -2\xi/\tau, \quad k_i = -1/\tau^2 \tag{9}$$

This result shows that the closed-loop system of a given nonlinear plant is transformed to a second order linear system, which explains the fact that the method is a relative degree 1 subset of the GLC algorithm. The performance specification of the system given by (8) can be plotted as a second order response curve and is shown in Fig. 1. Thus one can choose ξ and τ to give the desired response. Because of this effectiveness of performance specification which is independent of the plant dynamics, this control method may be a viable method provided that the model and con-

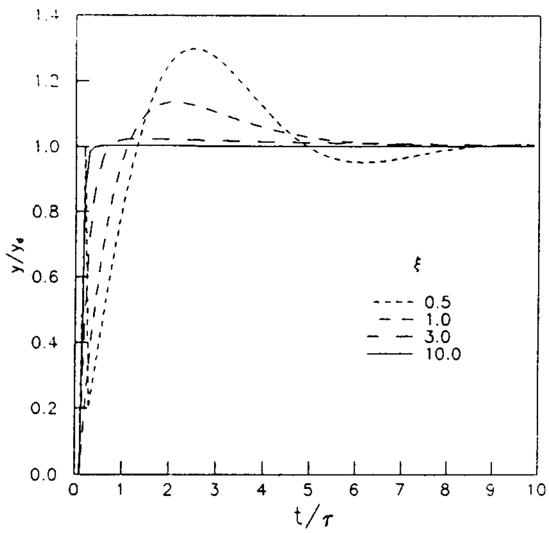

Fig. 1. Performance Specification of (8) with Parameters ξ and τ

trol are perfect.

But the modeling uncertainty and the nonidealities of the control have to be considered when this control method is applied to a real plant, which will be discussed in the following sections.

3.1. Disturbance Compensation

Many of the nonlinear control designs include integral action in the control law. The motivation for adding integral action is the robustness and disturbance rejection capability of the integral control. But, for the tracking control problems with large variation of plant parameters, the integral action only is not enough. To compensate the modeling uncertainty, Lee *et al.*[7] suggested to feedback the estimated uncertainty. Output disturbances appear explicitly in y, whereas state/input disturbances will appear in the derivative of y. Thus a simple way of the uncertainty estimation is to estimate the output derivative from the measured data of y and to use the nominal model. To explain this algorithm, rewrite (1) as

$$\dot{x} = (\hat{f} + \Delta f) + (\hat{g} + \Delta g)u \tag{10}$$

where $\hat{f}$ and $\hat{g}$ are functions representing the nominal model, and Δf and Δg are functions representing uncertain parameters and unmodeled dynamics whose values belong to some closed and bounded sets. Then (10) is transformed to the output dynamics with the following form

$$\dot{y} = L_{\hat{f}}h(x) + u(t)L_{\hat{g}}h(x) + d \tag{11}$$

where

$$d \equiv L_{\Delta f}h(x) + u(t)L_{\Delta g}h(x) \tag{12}$$

which represents the lumped uncertainty. This lumped uncertainty is estimated from the input/output measurement and nominal model, i.e.

$$\hat{d}(t) = \dot{y}(t - T) - (L_{\hat{f}}h + uL_{\hat{g}}h) \tag{13}$$

where $\hat{d}$ represents the estimated uncertainty. Here, we approximated the derivative of the measurement as

$$\dot{y}(t) \simeq \frac{y(t) - y(t - \Delta t)}{\Delta t} \simeq \dot{y}(t - T) \tag{14}$$

where Δt represents the sampling interval and T is an arbitrary but small time delay induced by the sampling effect.

Now we generate $u(t)$ combining (7),(11) and (13), which becomes

$$u(t) = (L_{\hat{g}}h)^{-1}[k_p(y - y_d) + k_i\int_{t_0}^{t}(y - y_d)d\mu - L_{\hat{f}}h - \hat{d} + \dot{y}_d] \tag{15}$$

Inserting this control input into (11), the tracking error dynamics is rewritten as

$$\begin{aligned}\dot{e} &= k_p(y - y_d) + k_i\int_{t_0}^{t}(y - y_d)d\sigma + (d - \hat{d}) \\ &= k_p(y - y_d) + k_i\int_{t_0}^{t}(y - y_d)d\sigma \\ &\quad + [\dot{y} - \dot{y}(t - T)] - [q(t) - q(t - T)]\end{aligned} \tag{16}$$

where, $q(t) \equiv (L_f^2 h + u L_g L_f h)\,|_t$. Then (16) can be written in the transfer function form:

$$Y(s) = \frac{-k_p s - k_i}{s^2 - k_p s - k_i + s^2[exp(-Ts) - 1]} Y_d(s)$$

$$- \frac{s[1 - exp(-Ts)]}{s^2 - k_p s - k_i + s^2[exp(-Ts) - 1]} Q(s) \quad (17)$$

where Y, Y_d, and Q represent the Laplace transformed variables of y, y_d, and q, respectively. Application of the final value theorem shows that the plant output asymptotically approaches y_d for any values of k_i. Since the feedback of the estimated uncertainty $\hat{d}$ would make the closed-loop system be very close to the real plant, the integral action is enough to guarantee the robustness of the controller. However, due to the effect of time-delay T, the performance will be degraded. This means that in this formulation, the performance specification of Fig. 1 is not enough. Thus, we need a constructive method to determine the feedback gains.

3.2. Adaptation of PIF Gains

We have analyzed the robustness of the model-based controller with PI gains, but we have another problem of control perfectness which is a very important clue to the performance specification. In digital control implementation, the control input is generally given by the piecewise constant function between each sampling interval. Then the characteristics of the closed-loop systems (7) and (16) will be slightly changed according to the sampling step size and the resulting plant dynamics, which means that we cannot assign the performance specification using Fig. 1 only. This sampling effect is significant for the systems with very fast dynamics such as nuclear reactors. We observe that the adjustment of PI gains according to the error trajectory would give better performance[8]. In addition, the desired trajectory may have turning points in the real plant operation such as the load follow operation of the nuclear reactors. In these non-smooth points, the occurrence of over- and under-shoot is inevitable to some extent since only the tracking error is fed back to the PI controller, not the desired state itself. Thus incorporation of the feedforward term will be helpful for the problem of tracking the pre-defined trajectory which has non-smooth turning points. The basic idea is to anticipate the influence of the prescribed trajectory on the plant variables and to introduce suitable control action. In this context, we propose a constructive method to select PIF gains by using an adaptation algorithm based on Lyapunov's second method. To develop the adaptation algorithm, let us consider the following control law

$$u(t) = (L_g h)^{-1}(-K_p e + \hat{k}_p e + \hat{k}_i \int_{t_0}^{t} e d\sigma$$

$$+ \hat{k}_f y_d - L_f h - \hat{d} + \dot{y}_d) \qquad (18)$$

where K_p is a positive constant. The adaptive parameters $\hat{k}_p$, $\hat{k}_i$, and $\hat{k}_f$ represent estimates of the proportional, integral, and feedforward gains, respectively. Here, the proportional gain is decomposed into two parts: constant and adaptive parts (K_p and $\hat{k}_p$). This parameterization is introduced for easy stability proof.

Adaptation algorithm using Lyapunov's second method. The method is based not on a definite cost function, but an error differential equation of the system for parameter identification[9]. The adaptation law is to be so designed that the system attains a globally asymptotically stable equilibrium point. If $\hat{d}$ closely estimates d, the control law (18) gives the error dynamics

$$\dot{e} = -K_p e + \hat{k}_p e + \hat{k}_i \int_{t_0}^{t} e \, d\mu + \hat{k}_f y_d \qquad (19)$$

The construction of the adaptation law is to determine the proper functions g_1, g_2, and g_3 such that (19) and

$$\dot{\hat{k}}_p = g_1(e, \int_{t_0}^{t} e\, d\mu, y_d) \quad (20)$$

$$\dot{\hat{k}}_i = g_2(e, \int_{t_0}^{t} e\, d\mu, y_d) \quad (21)$$

$$\dot{\hat{k}}_f = g_3(e, \int_{t_0}^{t} e\, d\mu, y_d) \quad (22)$$

are asymptotically stable at $e=0$. These functions are then the adaptive laws which update the parameters k_p, k_i, and k_f. In this formulation, the uncertainty estimation error $(d-\hat{d})$ is not included. We do not consider this small quantity, instead this estimation error is compensated by the gain adaptation. To determine the stable adaptive schemes described above, we choose a quadratic form Lyapunov function candidate in the error and parameter space :

$$V(e, \hat{k}_p, \hat{k}_i, \hat{k}_f) = \frac{1}{2}(e^2 + \frac{1}{\gamma_1}\hat{k}_p^2 + \frac{1}{\gamma_2}\hat{k}_i^2 + \frac{1}{\gamma_3}\hat{k}_f^2) \quad (23)$$

where γ_1, γ_2, γ_3 are positive constants that adjust adaptation speed. Then inserting $\dot{e}$ from (19), the time derivative of V becomes

$$\begin{aligned} \dot{V} &= e\dot{e} + \frac{1}{\gamma_1}\hat{k}_p\dot{\hat{k}}_p + \frac{1}{\gamma_2}\hat{k}_i\dot{\hat{k}}_i + \frac{1}{\gamma_3}\hat{k}_f\dot{\hat{k}}_f \\ &= -K_p e^2 + \hat{k}_p(e^2 + \frac{1}{\gamma_1}\dot{\hat{k}}_p) + \hat{k}_i(e\int_{t_0}^{t} e\, d\mu \\ &\quad + \frac{1}{\gamma_2}\dot{\hat{k}}_i) + \hat{k}_f(ey_d + \frac{1}{\gamma_3}\dot{\hat{k}}_f) \quad (24) \end{aligned}$$

For only the signs of K_p, γ_1, γ_2, and γ_3 are known, the only manner in which $\dot{V}$ can be made negative semi-definite is to choose the terms in parentheses of (24) to disappear. Then we can get the adaptation laws :

$$\dot{\hat{k}}_p = -\gamma_1 e^2 \quad (25)$$

$$\dot{\hat{k}}_i = -\gamma_2\, e\int_{t_0}^{t} e\, d\mu \quad (26)$$

$$\dot{\hat{k}}_f = -\gamma_3\, ey_d \quad (27)$$

Since V is a Lyapunov function for the system of equations (19)–(22), it is obvious that the system is stable. Therefore, e, $\hat{k}_p$, $\hat{k}_i$ and $\hat{k}_f$ are bounded and from (24) and the adaptation laws (25)–(27),

$$\begin{aligned} \lim_{t\to\infty}[K_p\int_{t_0}^{t} e^2 d\sigma] &= \lim_{t\to\infty}[-\int_{t_0}^{t}\dot{V}(\sigma)d\sigma] \\ &= V(t_0) - \lim_{t\to\infty} V(t) < \infty. \quad (28) \end{aligned}$$

This means boundedness of $\int_{t_0}^{t} e^2\, d\sigma$ and hence $\int_{t_0}^{t} e d\sigma$ is bounded. In view of (19), this implies boundedness of $\dot{e}$, which means uniform continuity of e. Consequently, we conclude by Barbalat's Lemma† that

$$\lim_{t\to\infty} e(t) = 0. \quad (29)$$

Thus, we conclude that if $y_d(t)$ is bounded and $L_g^{\hat{}}h$ is bounded away from zero, the parameter adaptation laws (25)–(27) yield bounded $y(t)$, asymptotically converging to $y_d(t)$. Choosing the adaptation gains γ_1, γ_2, and γ_3 to be high, better adaptation speed would be obtained. But the effect of the measurement noise is smaller for the lower values of gains. Reasonable values of the adaptation gains have to be selected as a compromise between fast convergence and low noise immunity. This adaptation algorithm has very simple stability characteristics in the Lyapunov sense and requires only three parameters to be adapted independent of the order of the plant model. These are very desirable properties in view of the parameter convergence and consistency.

3.3. Incorporation of VSC

The adaptive control algorithm discussed in the preceding section gives a constructive method to determine the PIF gains. But, we have no idea of *a priori* values of the gains (in this paper, the initial values of the PIF gains are set to zero). Especially, the feedforward term is an open-loop control ac-

†Barbalat's Lemma[11] : If $f(t)$ is a uniformly continuous function, such that $\lim_{t\to\infty}\int_0^t f(\tau)d\tau$ exists and is finite, then $f(t)\to 0$ as $t\to\infty$.

tion which is very sensitive compared with the PI actions. Thus the initial peak will appear in the tracking error until the adjustable gains are converged to the proper values. To resolve this problem, we suggest the incorporation of the VSC that regulates the initial peak of the tracking error within the pre-assigned boundary layer. Incorporating this VSC term, the control law becomes:

$$u(t) = \begin{cases} (L_{\hat{g}}h)^{-1}[-K_p e + \hat{k}_p e + \hat{k}_i \int_{t_0}^{t} e d\mu + \hat{k}_f y_d \\ \qquad - L_{\hat{f}}h - \hat{d} + \dot{y}_d], \quad \text{if } |e| < \epsilon \\ (L_{\hat{g}}h)^{-1}[-L_{\hat{f}}h + \dot{y}_d - K sgn(e)], \\ \qquad K > \lim_{t\to\infty} sup|d(t)|, \text{ if } |e| \geq \epsilon. \end{cases} \tag{30}$$

where ϵ is the pre-assigned thickness of the boundary layer. The VSC law has two structures: adaptive PIF or *signum* functions according to the error trajectory. This explains the notion of the variable structure. The signum function and relatively large gain K are used outside the boundary layer to guarantee the boundedness of tracking error. It can be easily seen that the quadratic positive definite function e^2 also becomes a Lyapunov function of the closed-loop system, if the positive parameter K always bounds the modeling uncertainty $\dot{d}$. The selection method of the large gain K can be found in Ref. 1 and 4. Within the boundary layer, smooth and fine control is achieved by the adaptive PIF controller.

4. Application to Nuclear Reactor Control

4.1. System Model

To apply the NHC method described in the previous sections, a simplified pressurized water reactor (PWR) model is developed based on the following assumptions:

1) The primary loop and steam generator of a PWR are modeled.
2) A lumped parameter nonlinear model of the primary loop is used.
3) Xenon and fuel depletion effects are not considered.
4) Single-phase heat transfer of the core coolant is considered.
5) Primary loop mass flow rate and pressure are constant.
6) Reactor power and core inlet-outlet temperatures are measured.

Then the plant model can be written as follows (see Fig. 2 and Nomenclature):

$$\frac{dP}{dt} = \frac{1}{l}[\rho_0 - \beta + \alpha_f T_f + \alpha_c T_{avg} + bu]P + \sum_{i=1}^{6} \lambda_i C_i \tag{31}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{l}P - \lambda_i C_i, \quad i = 1, \cdots, 6 \tag{32}$$

$$\frac{dT_f}{dt} = -(\frac{UA}{M_f c_{pf}})(T_f - T_{avg}) + (\frac{F}{M_f c_{pf}})P \tag{33}$$

$$\frac{dT_{avg}}{dt} = (\frac{UA}{M_c c_{pc}})(T_f - T_{avg}) - \frac{\dot{m}}{M_c}(T_{out} - T_{in}) \tag{34}$$

$$\frac{dT_{in}}{dt} = \frac{1}{\tau_{cl}}(T_{cl} - T_{in}) \tag{35}$$

$$\frac{dT_{hl}}{dt} = \frac{1}{\tau_{hl}}(T_{out} - T_{hl}) \tag{36}$$

$$T_{out} = 2T_{avg} - T_{in} \tag{37}$$

$$\frac{dT_s}{dt} = -\frac{1}{\tau_s}(T_s - T_{hl}) - D_1 L_T \tag{38}$$

$$T_{cl} = D_2 T_s - D_3 T_{hl} \tag{39}$$

Fig. 2 shows a schematic diagram of this plant. The reactor coolant system model is divided into five nodes to simulate the energy balance between fuel and coolant and the transport delays between reactor core and steam generator. The steam generator model contains heat transfer between the reactor coolant system and the secondary side. The turbine load variation is performed by changing steam flow to the turbine. The thermal part of this model is an extension of the linear, time-invariant model used by Park and Miley[10] and the nominal values used in this paper are listed in Table 1.

In addition, nonlinearity in the heat transfer between fuel and coolant is considered such that the heat transfer coefficient U is given by the following well-known correlation of *Dittus–Boelter* equation

$$U = C_v \, Re^{0.8} Pr^{0.4} \left(\frac{K_c}{D_e}\right) \tag{40}$$

where

$$\text{Reynolds number, } Re = \frac{D_e v \rho_c}{\mu} \tag{41}$$

and

$$\text{Prandtl number, } Pr = \frac{c_{pc}\mu}{K_c} \tag{42}$$

In (41) and (42), ρ_c and c_{pc} remain nearly constant which are determined mainly by the system pressure. However, μ and K_c are functions of both coolant temperature and pressure, which induce nonlinear behavior in the heat transfer coefficient. All the thermophysical properties contained in the plant model are calculated from the steam table within the range of subcooled state.

A low order, time-invariant neutronics/thermal-hydraulics and one-group precursor model is used for control model as follows:

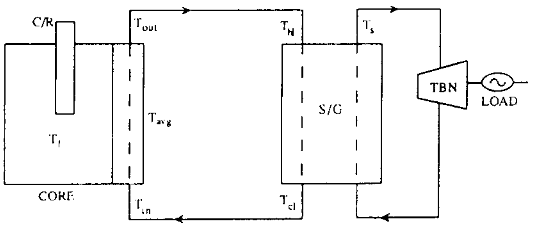

Fig. 2. Schematic Diagram of Plant Model

$$\hat{f} = \begin{pmatrix} \frac{1}{l}[\rho_0 - \beta + \hat{\alpha}_f \hat{T}_f + \hat{\alpha}_c T_{avg}]P + \lambda \hat{C} \\ \frac{\beta}{l} P - \lambda \hat{C} \\ -\frac{1}{\tau_1}(\hat{T}_f - T_{avg}) + D_4 P \\ -\frac{1}{\tau_2} T_{avg} + \frac{1}{\tau_3}\hat{T}_f + \frac{1}{\tau_4} T_{in} \end{pmatrix},$$

$$\hat{g}u = \begin{pmatrix} \frac{1}{l}\hat{b}P \\ 0 \\ 0 \\ 0 \end{pmatrix} u \tag{43}$$

where $x^T = [P,\ \hat{C},\ \hat{T}_f,\ T_{avg}]$. The control model (43) is still nonlinear, although it is of lower order than the plant model (31)–(39). Note that the control model uses nominal values for the parameters. The output function is defined to consider the turbine load and the coolant average temperature following the strategy of a PWR such that

$$y = h(x) = w_T T_{avg} + w_P P \tag{44}$$

4.2. Development of NHC for a Nuclear Reactor

The control objective is to design feedback input $u(t)$ to make $P(t)$ track $L_T(t)$ which is a prespecified power maneuvering schedule. Fig. 3 is a block diagram illustrating the NHC algorithm for a nuclear reactor. The conventional reactor control system consists of two channels: temperature deviation $(T_{avg} - T_{ref})$ and power mismatch $(P - L_T)$. The output of these two channels is used to drive the control rods. Reactor coolant average temper-

Table 1. Nominal Values of Plant and Control Model

β	β_1	β_2	β_3	β_4	β_5	β_6		
0.007108	0.000216	0.001416	0.001349	0.00218	0.00095	0.000322		
$\lambda(s^{-1})$	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6		
0.078	0.0125	0.0308	0.1152	0.3109	1.24	3.3287		
$l(s)$	α_f(℃$^{-1}$)	α_c(℃$^{-1}$)	C_v	$D_e(m)$	D_1	D_2		
5.0×10^{-4}	-2.0×10^{-5}	-5.0×10^{-5}	0.0301	0.01297	3.746	0.7005		
D_3	D_4	$\tau_{cl}(s)$	$\tau_{hl}(s)$	$\tau_s(s)$	$\tau_1(s)$	$\tau_2(s)$	$\tau_3(s)$	$\tau_4(s)$
−0.2995	102.7	7.0	5.0	11.3	5.58	2.03	80.5	2.08

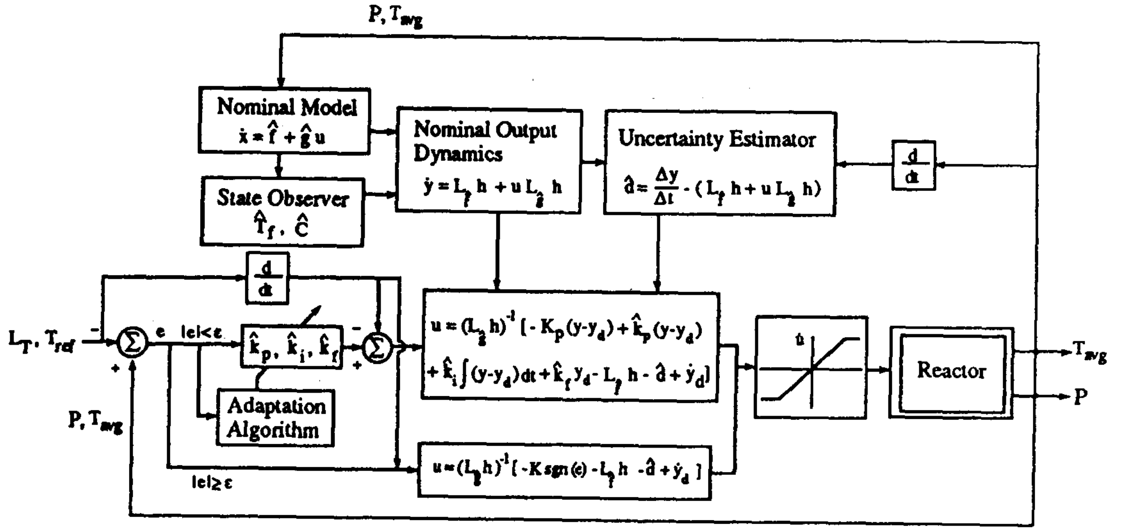

Fig. 3. Block Diagram for NHC of a Nuclear Reactor

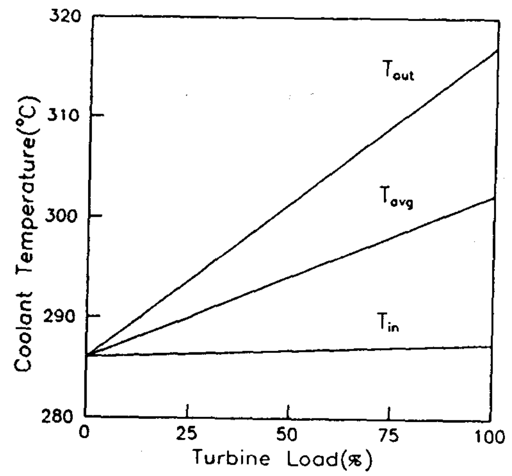

Fig. 4. Reference Temperature Setpoint vs. Turbine Load

ature signal (T_{avg}) is calculated from the measurements of inlet and outlet temperatures, and this is compared with the reference temperature (T_{ref}) which is a programmed temperature as a function of turbine load (see Fig. 4).

The inputs to power mismatch channel are the turbine load (L_T) and the nuclear power (P). To incorporate the temperature-following strategy, as is done in nuclear reactor control, the total tracking error signal sent to the rod controller is given by the weighted sum of the temperature deviation and power mismatch as in the following equation :

$$e = w_T(T_{avg} - T_{ref}) + w_P(P - L_T) \qquad (45)$$

where w_T and w_P are the weighting factors for temperature deviation and power mismatch, respectively. In this work, for simplicity, lead/lag, impulse or nonlinear gain units of tracking error generation channels are not considered, which are used in control rod system of a PWR.

From (43)–(45), we can define the tracking error dynamics of the nominal system to be controlled as

$$\dot{e} = L_{\hat{f}}h + uL_{\hat{g}}h - (w_T\dot{T}_{ref} + w_P\dot{L}_T) \qquad (46)$$

where

$$L_{\hat{f}}h = w_T(-\frac{1}{\tau_2}T_{avg} + \frac{1}{\tau_3}\hat{T}_f + \frac{1}{\tau_4}T_{in})$$

$$+ w_P[\frac{1}{l}(\rho_0 - \beta + \alpha_f\hat{T}_f + \alpha_c T_{avg})P + \lambda\hat{C}] \qquad (47)$$

and

$$L_{\hat{g}} h = w_P \frac{\hat{b}}{l} P \tag{48}$$

Then we can estimate the modeling uncertainty and generate the control input with (13) and (30).

4.3. Consideration on Open-Loop Observer

To complete the NHC design, we need the estimate of unmeasured states of fuel temperature and precursor concentration. The construction of a closed-loop observer may be beneficial in view that the observer states would converge to the actual values. But, it is not guaranteed that the observer would yield physically meaningful estimate of the system states during the entire observation process. It turns out that the observer states may exhibit excursions outside the range of the states that the reactor can achieve. This can be avoided by properly choosing the initial conditions and using an open-loop observer. The point here is that we can do better than just using blind estimate. Good initial estimates can be obtained based on the quantitative arguments regarding the reactor dynamics and the remaining estimation error can easily be compensated by the lumped uncertainty estimator discussed in the previous section. In this paper, precursor concentration and fuel temperature are calculated by the control model (43) with nominal parameters and adjusted initial conditions. The initial conditions are adjusted intentionally from the steady state conditions to reflect the fact that the initial conditions are unknown.

4.4. Results

In this section, we illustrate the power tracking problem of a nuclear reactor with NHC simulated for a prespecified desired trajectory. A rapid power maneuvering problem is considered to demonstrate the applicability of the control method. The reference plant (31)–(39) is simulated using fifth-order Runge–Kutta method with adaptive time step sizes to deal with stiffness inherent in nuclear reactor dynamics. Here, the simulation time step sizes are far smaller than the sampling time intervals used for the generation of control law. Thus input signal u is piecewise constant in the plant simulation time steps until next control input is generated. In this paper, the sampling interval of $\Delta t = 0.4$ *sec* is used for control input generation and the control rate constraint $|\dot{u}| \leq 1$ *step/sec* is used (~ 70 *steps/min* is a typical value for PWR plants).

The system, initially at the steady state of 50 % power level, is required to track the desired trajectory $L_T(t)$ which is defined by

$$L_T(t) = \begin{cases} 50 + 0.04t, & 0 \leq t < 500 \\ 70, & 500 \leq t < 800 \\ 70 - 0.02(t - 800), & 800 \leq t < 1200 \\ 62, & 1200 \leq t < 1500 \end{cases} \tag{49}$$

To consider the uncertainties in temperature feedback parameters, the plant and control model parameters are assigned as $\alpha_f = -2\times10^{-5}$, $\hat{\alpha}_f = -1\times10^{-5}$, $\alpha_c = -5\times10^{-5}$ and $\hat{\alpha}_c = -3\times10^{-5}$. The plant and nominal values for differential rod worth is given in Fig. 5. As shown in the figure, control rod worth of the plant model (b) is assumed to have local variations along the core height due to the bank overlapping or skewed power distribution as in real PWR plants. The nominal value of control rod worth ($\hat{b}$) is given by a smooth function along the rod steps. The parameter changes of $\frac{M_f c_{pf}}{UA}$ and $\frac{M_c c_{pc}}{UA}$ are shown in Fig. 6, which are due to the variation of the heat transfer coefficient induced by the change in thermophysical properties of the reactor coolant, and are compared with the nominal parameters τ_1 and τ_3 used in the control model.

Figures 7(a) and 7(b) plot the behavior of open-loop observers for precursor concentration and fuel temperature. To consider the unknown initial values, $\hat{C}(0)$ and $\hat{T}_f(0)$ are calculated by the steady state conditions of (43) and $\hat{T}_f(0)$ is overestimated by 50℃. As shown in the figures, they give biased

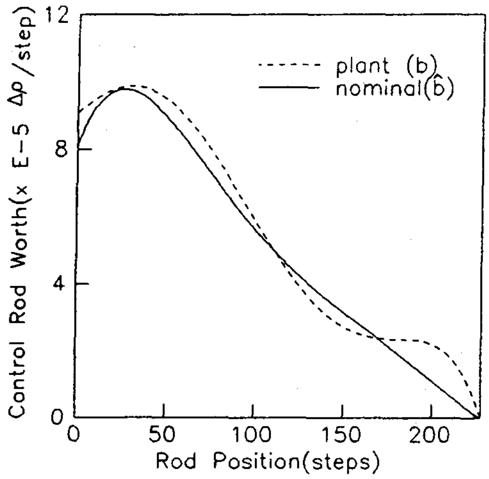

Fig. 5. Differential Rod Worth of D-Bank vs. Steps Withdrawn (113 Steps Overlapped and Moving with C-Bank)

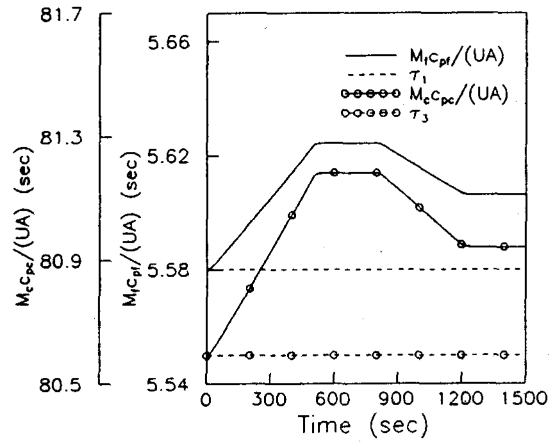

Fig. 6. Variation of Plant Parameters M_fc_{pf}/(UA) and M_cc_{pc}/(UA) Due to the Change of Heat Transfer Coefficient U Determined by (40)

results as expected. But these estimation errors are lumped and compensated by the uncertainty estimator $\hat{d}$.

The tracking error is induced by the temperature variation of reactor coolant system due to the change in steam flow and the power mismatch ($P-L_T$). The amount of reactivity required to offset the tracking error is generated by control rod

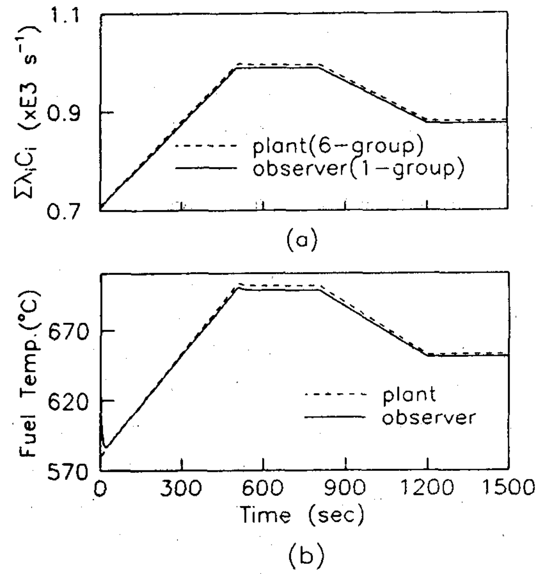

Fig. 7. Decay Rate of Precursor and Fuel Temp. with Observer Initial Condition Overestimated by 50℃

movement calculated from the nominal model and hybrid control strategy. Fig. 8 plots the desired turbine load (Fig. 8(a)) and the tracking error trajectories controlled with the various PI gain values. Fig. 8(b) shows the result with the conventional PI control law which is model independent. The result shows relatively large tracking error and oscillatory behavior which are mainly due to the ignorance of plant dynamics and large sampling step size. To reduce these undesirable phenomena, conventional PI controller requires additional artificial techniques such as control deadband, lead/lag, and nonlinear gain units. Also, for fast power maneuvering, the nonlinearity and uncertainty of reactor dynamics are significant, which limits the ramp rate increase in PI controllers. Thus the model-based control method with robustness property must be used in this case. Fig. 8(c) shows the result with the gain values which are used in conventional PI controllers (k_p : $-1.0\sim-3.0$, k_i : $-0.01\sim-0.1$). That gives relatively good but rather slower tracking performances than expected with the gain selection strategy in Fig. 1. The neutron flux of a nuclear reactor is very sensitive and has fast response to the variation of input reactivity. So the sampling effect may destroy the desired performance specification when large feedback gains are used to get the fast tracking capability. Fig. 8(d) is the result with high PI gains (the case of the fastest response in Fig. 1 with $\xi=$ 10.0, $\tau=1.0$, $k_p=-20.0$, , $k_i=-1.0$) which shows very fast tracking capability but also high frequency oscillatory behavior. This phenomenon, which is due to the excessive control input in each control interval, is highly undesirable in view of the structural integrity of control rods. This power and control chattering problem would be reduced if very small sampling interval is used but it is not an easy task.

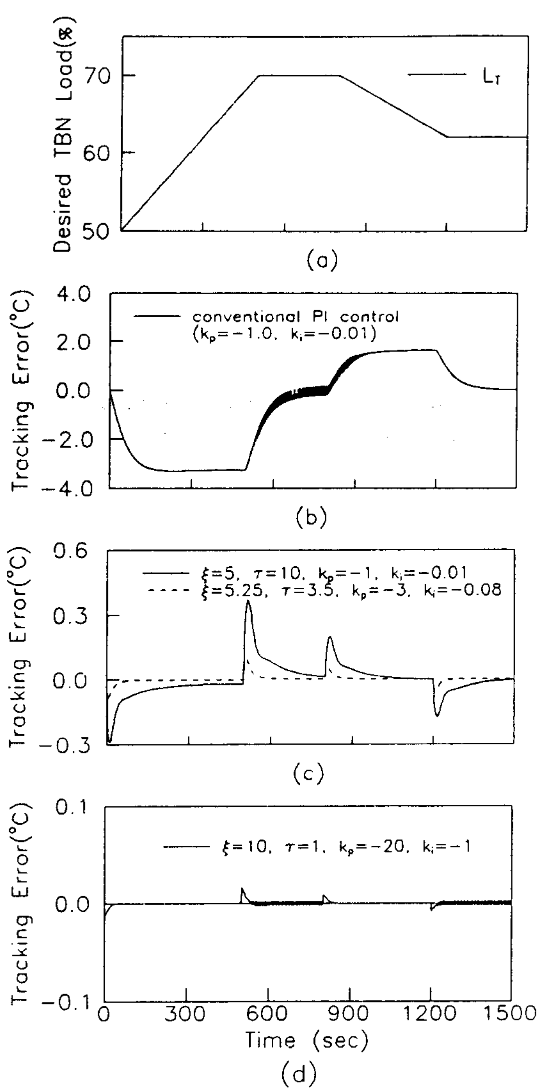

Fig. 8. Desired TBN Load and Tracking Error $e=W_T(T_{avg}-T_{ref})+W_P(P-L_T)$ with $W_T=5.0$, $W_P=1.0$(℃/%) for Various PI Gains

Fig. 9(a) represents the variation of PI gains with the adaptation laws (25) and (26). Fig. 9(b) shows very fast and smooth tracking performance controlled with adaptive PI gains in comparison with the case of $k_p=-3.0$, $k_i=-0.08$. But, the result still gives the over– and under–shoot phenomena at the turning points of the desired trajectory.

Fig. 10 plots almost perfect tracking control capability of the hybrid controller which suppresses the over–or under–shoot. Fig. 11 represents the variation of the PI and feedforward gains with

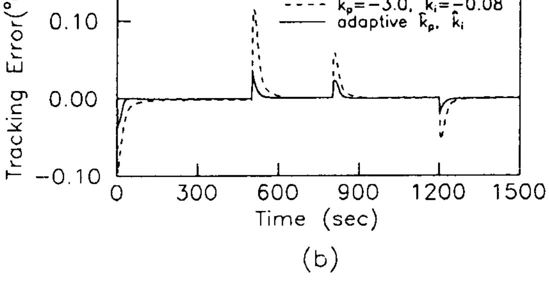

Fig. 9. Tracking Error $e=W_T(T_{avg}-T_{ref})+W_P(P-L_T)$ with $W_T=5.0$ and $W_P=1.0$(℃/%) and Adaptive PI Gains

the adaptation laws (25)–(27) and the control law (30) where $K=10^4$ and $K_p=8.5$ are used. The variation of the feedforward gain (Fig. 11(b)) indicates that it plays a role only when the desired trajectory is varying and becomes zero during the steady operation.

Figures 12(a),(b) show the nuclear power response and temperature-following characteristic of NHC. Fig. 13 plots the control rod step change performed by NHC and the variation of control rate which remains sufficiently small within the upper bound.

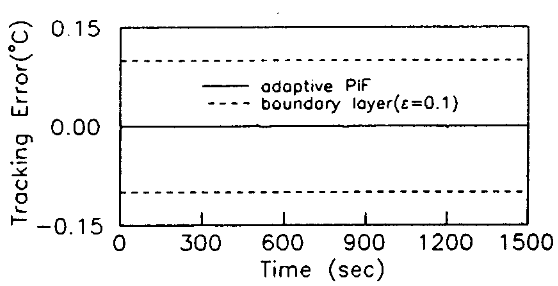

Fig. 10. Tracking Error $e=W_T(T_{avg}-T_{ref})+W_P(P-L_T)$ with $W_T=5.0$, $W_P=1.0$(℃/%) and Adaptive PIF+VSC

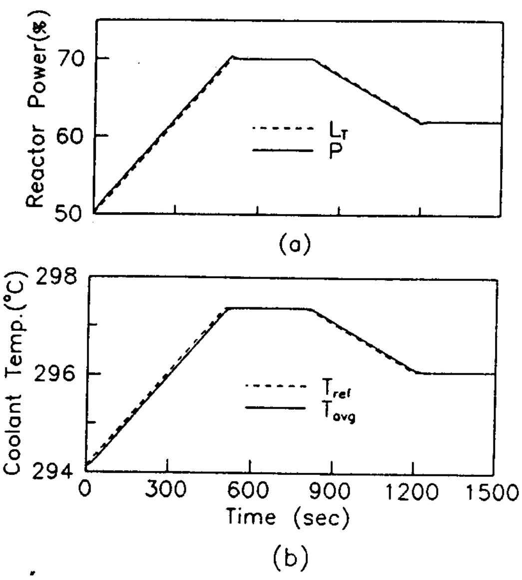

Fig. 12. Power and Temperature Trajectories Controlled with Control Law (30)

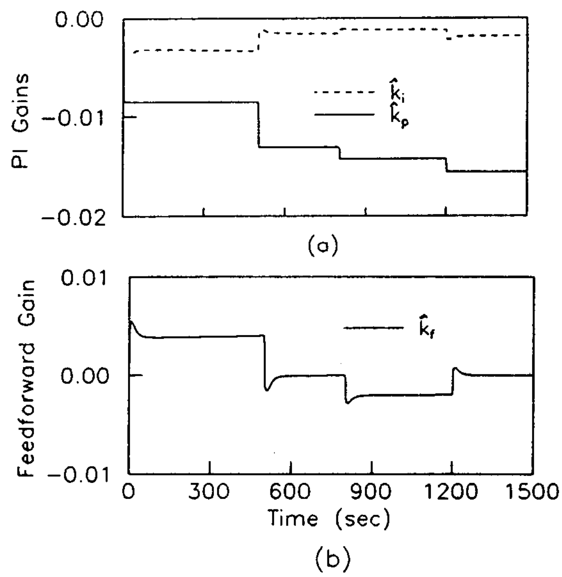

Fig. 11. Variation of PIF Gains with Control Law (30), $K_p=8.5$

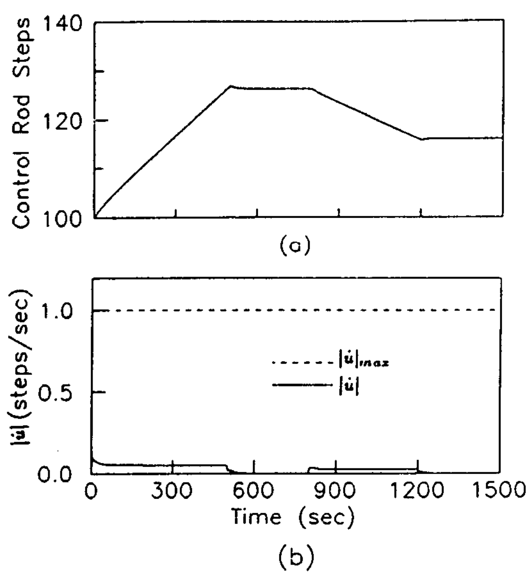

Fig. 13. Control Rod Position and Control Rate ($|\dot{u}|_{max}=1$ Step/sec)

5. Conclusions and Recommendations

A new method for the nonlinear robust digital control of nuclear reactor power has been developed using the model–based controller with adaptive PIF and VSC. The need of the robustness is due to the uncertain, time–varying parameters, unmeasurable states, and unmodeled dynamics. The results of application to power tracking control of a nuclear reactor indicate that the controller has excellent robustness and performance features. It is shown that the use of a simple uncertainty estimator and the hybrid control algorithm gives not only robustness against modeling uncertainty but also very fast and smooth performance behavior. The use of open–loop observers for fuel temperature and precursor concentration gives slightly biased results but is a good scheme in relaxing the need for the knowledge of unmeasurable state values, if the estimation error is compensated by the uncertainty estimator properly. The uncertainty estimation algorithm used in this paper may lead to a model independent controller. But the use of a nominal model and open–loop observers would be desired in a way that it will reduce the calculational burden of the uncertainty estimator.

It is recommended that explicit conditions guaranteeing boundedness and smoothness of the tracking error for a sampled–data system be derived and the actuator–sensor dynamics be augmented to develop more practical control methods.

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Nomenclature

A	=effective heat transfer area (m^2)
b	=differential control rod worth ($\Delta\rho\cdot$step^{-1})
$\hat{b}$	=estimate of differential control rod worth ($\Delta\rho/\cdot$step^{-1})
C_i	=core averaged *i*th group precursor concentration for % power
$\hat{C}$	=estimated core averaged one–group precursor concentration for % power
$c_{pf}(c_{pc})$	=fuel(coolant) specific heat (MW·s·ks^{-1}·℃$^{-1}$)
C_v	=a constant determined by water–fuel volume fraction of the lattice
D_1,D_2,D_3,D_4	=nominal parameters for reactor and steam generator model
D_e	=equivalent diameter of the lattice of coolant channel (m)
F	=conversion factor (MW·%$^{-1}$)
K_c	=thermal conductivity of reactor coolant (MW·m^{-1}·℃$^{-1}$)

L_T = turbine load(% steam flow to turbine)

l = neutron generation time (s)

$M_f(M_c)$ = fuel(coolant) mass (kg)

$\dot{m}$ = core mass flow rate ($kg \cdot s^{-1}$)

P = core averaged neutronic power (%)

Pr = Prandtl number

Re = Reynolds number

T_{avg} = core averaged coolant temperature $= \frac{1}{2}(T_{out} + T_{in})$(℃)

T_{cl} = cold leg temperature (℃)

T_f = core averaged fuel temperature (℃)

$\hat{T}_f$ = estimate of core averaged fuel temperature (℃)

T_{hl} = hot leg temperature (℃)

T_{in} = core inlet temperature (℃)

T_{out} = core outlet temperature (℃)

T_s = steam generator steam temperature (℃)

U = convective heat transfer coefficient ($MW \cdot m^{-2} \cdot ℃^{-1}$)

u = external input from control rod (step)

v = average velocity of the coolant ($m \cdot s^{-1}$)

Greek

$\alpha_f(\alpha_c)$ = fuel(moderator) temperature coefficient of reactivity ($\Delta\rho \cdot ℃^{-1}$)

β = total effective delayed neutron fraction

β_i = ith group effective delayed neutron fraction

λ = effective delayed neutron decay constant (s^{-1})

λ_i = ith group delayed neutron decay constant (s^{-1})

μ = viscosity of the coolant ($kg \cdot m \cdot s^{-1}$)

ρ_c = density of the coolant ($kg \cdot m^{-3}$)

ρ_0 = total core reactivity at initial state

τ_{cl}, τ_{hl}, τ_s = time constants for coolant loop and steam generator (s)

τ_1, τ_2, τ_3, τ_4 = time constants for control model (s)