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Analysis of Hydraulic Lift Force of a Fuel Assembly

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핵연료 집합체에 대한 수력적 양력의 해석

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Abstract

The exact expression for the lift force on a fuel assembly in a reactor core is derived in terms of calculable hydraulic parameters. The relation for the lift force, pressure drop, buoyancy force, viscous force, and fuel assembly weight is discussed. Based on the derived exact expression, error analysis is made for a simple expression applying COBRA IV-i to a typical PWR fuel assembly. The error analysis revealed that the error of the simple expression consists of four terms and the overall error depends on the flow rate change direction, and its magnitude is about 1%.

요 약

유체 유동상의 인자로 구성된 핵연료집합체에 걸리는 수력적 양력의 정확한 표현식은 핵연료의 건전성 설계 및 해석에 중요한 인자이다. 그러나 현재까지 이 양력에 대한 이론적인 해석이 제대로 이루어지고 있지 않아 이 분야에 혼란이 빚어지고 있다. 본 논문에서는 핵연료집합체에 걸리는 수력적 양력에 대한 성확한 표현식을 이론적인 고찰을 통하여 유도하였으며 또한 양력에 관련된 제반 힘 요소들 즉, 압력강하, 부력, 전단응력, 집합체하중, 상호 간의 관계를 검토하였다. 유도된 정확한 이론식을 이용하여 양력에 관한 간이식 오차의 특성을 분석한 결과 오차는 4가지 항으로 구성됨과 총 오차의 크기는 노심 유량의 변화 방향에 따라 달라짐을 알 수 있었다. 정량적인 분석을 COBRAIV-I를 이용하여 수행한 결과 총 오차의 크기는 약 1% 정도임이 밝혀졌다.

1. Introduction

The hydraulic lift force of a fuel assembly is the resultant force of the interaction between coolant flow and a fuel assembly and is directly related to the mechanical integrity of a holddown device of a fuel assembly. Analysis of the hydraulic lift force is one of the important works in designing a fuel assembly and analyzing the hydraulicmechanical compatibility of a transition core. In the conventional flow engineering area, the force to an object in a flow field is called drag or lift force depending on the force direction and many works on this subject are found in open literature. However, works on nuclear application area are quite few. One of the few works is Jin and Sim's work/1/. They analyzed the relation among the object shape, drag force and pressure drop for CANDU reactor refueling device analysis. Considering the importance of the lift force analysis, it is quite strange that nearly nothing has been so far found in literature on analytical investigation of the force. What has been found to the authors in literature is only an equation /2/ stating the magnitude of lift force is the product of pressure drop and area. As described above, the lift force is interrelated with pressure drop, shear stress, and body force but nothing has been clarified in Reference 2. on the relation among them and the error involved in the equation The authors feel that many people think that an equation of the type of the equation in Ref. /2/ is exact in principle. In open literature, even a single statement implying analytic works on relation of the lift force are described in a proprietary document is not found. One possible explanation to this situation is that the equation in Ref. /2/ was formulated by guessing and calibrated to measurement data. For improvement of a lift force analysis method, works are required on theoretical analysis of the interaction between fluid flow and an fuel assembly. To clarify the relationship between the hydraulic lift force and relevant parameters, an exact expression is first derived and discussion is made on the relationships among buoyancy force, pressure force, viscous force, and weight of a fuel assembly. Also the error of the simple expression is analyzed for the case of a typical PWR fuel and core operation condition using COBR-IV-i /3/.

2. Derivation of the Exact Expression by Flow Parameters

When an object of an arbitrary shape is located in a flow as shown in Fig. 1, three different forces are exerted to the object. First is the body force by the gravitational acceleration, that is, weight and second is the surface force by pressure and viscous shear stress, and third is the external force which is applied to keep the object in a fixed position.

Since the three forces are in equilibrium, they satisfy Eq.(1).

$$0=\int_{V\,obj} \rho f \cdot dV+\int_{A\,obj} \sigma \cdot dA+T \qquad (1)$$

In Eq.(11), σ is the stress tenser.
When a control volume is drawn for the flow around the object as the dotted volume in Fig. 1, there are two surfaces, that is, A_i : inner control surface contacting the object, and A_o : outer control surface. The total force on the flow control volume F^f is the sum of the fluid body

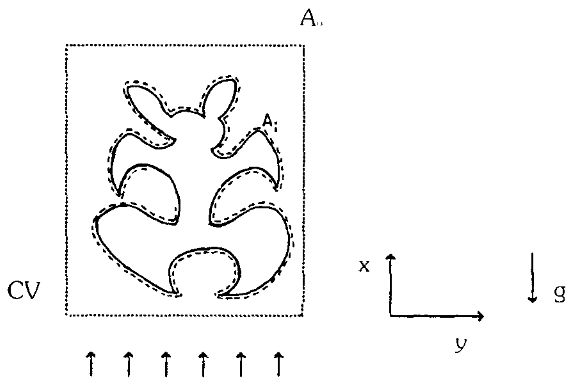

Fig 1. The Object and Control Volume

force and the surface ($A_f = A_i + A_o$) force of the viscous force and pressure force. And its expression is given by Eq.(2).

$$F^f= F^f_{Ai}+ F^f_{Ao}+ F^f_{body} \qquad (2)$$

$$=\int^f_{A_i} \sigma \cdot dA+ \int^f_{A_o} \sigma \cdot dA+ \int^f_{V_f} \rho f\, dV \qquad (3)$$

Application of the flow momentum theorem/4/to the flow control volume yields ;

$$F^f=\frac{\partial}{\partial t}\int_{V_f}\sigma V\,dV+\int_{A_f}(\rho V)\,V\cdot dA \tag{4}$$

Equating Eq. (3) and Eq. (4), the force on the inner control surface $F^f_{A_i}$ is ;

$$F^f_{A_i}=\frac{\partial}{\partial t}\int_{V_f}\rho V dV+\int_{A_f}(\rho V)V\cdot dA-\int_{A_o}\sigma\cdot dA-\int_{V_f}\rho f\,dV \tag{5}$$

Since the surface stress tensor can be decomposed into two parts of pressure and viscous shear stress as shown in Eq.(6), Eq.(5) becomes Eq.(7).

$$\sigma=-pI+\varsigma \tag{6}$$

where ς is the shear stress tensor

$$F^f_{A_i}=\frac{\partial}{\partial t}\int_{V_f}\rho V\,dV+\int_{A_f}(\rho V)V\cdot dA+\int_{A_o}pI\cdot dA-\int_{A_o}\varsigma\cdot dA-\int_{V_f}\rho f\,dV \tag{7}$$

Since the interest is only on a single force componet of a specific direction, let's call it x-direction, the x componet of the force $(F^f_T)_x$ is ;

$$(F^f_T)_x=i\cdot F^f_{A_i}$$

$$=\frac{\partial}{\partial t}\int_{V_f}\rho V_x dV+\int_{A_f}\rho V_x V\cdot dA+\int_{A_o}pi\cdot dA-i\cdot\int_{A_o}\varsigma\cdot dA-\int_{V_f}\rho f_x dV \tag{8}$$

From the force balance on the inner-surface A_i,

$$0=F^f_{A_i}+F^{obj}_{A_i} \tag{9}$$

The force to the object on the inner control surface $F^{obj}_{A_i}$ is the hydraulic lift force, which corresponds to the second term of Eq(1). Combing Eq.(9) and Eq.(8),

$$(F^{obj}_{A_i})_x=-(F^f_{A_i})_x=-\frac{\partial}{\partial t}\int_{V_f}\rho V_x dV-\int_{A_f}\rho V_x V\cdot dA-\int_{A_o}pi\,dA+i\cdot\int_{A_o}\varsigma\cdot dA+\int_{V_f}\rho f_x\,dV \tag{10}$$

The point to be noted in Eq.(10) is that it is an exact expression for the lift force to the object in a flow field and it does not contain any term which involves calculation in the inner control surface A_i, which is very complicated in a fuel assembly lift force analysis. To make Eq.(10) more convenient, K is defined as the sum of the pressure and body force terms such as Eq.(11)

$$K=-\int_{A_o}pi\cdot dA+\int_{V_f}\rho f_x\,dV \tag{11}$$

To apply the exact relation Eq.(10) to fuel assembly lift force analysis, a restriction is now imposed to the control volume and the restriction is ;

〈The shape of the outer side of the control volume is rectangular and it encloses completely the fuel assembly under analysis. The shape of the inner control surface is exactly same with the object shape and contacts the fuel assembly at every point.〉

Also new parameters A_c and x are introduced. A_c denotes the cross section area of the rectangular of the outer control volume side, and x denotes the coordinate which is normal to the cross section area.

Then, the integration of Eq.(11) results in Eq.(12).

$$K=A_c(P^s_{xin}-P^s_{xout})+\bar{\rho}_f f_x A_c\,(x_{out}-x_{in})-\bar{\rho}_f f_x V_{obj} \tag{12}$$

Where P^S is static pressure and $\bar{\rho}_f$ are defined by Eq.(13)

$$\bar{\rho}_f=\int_{V_f}\rho\,dV/\int_{V_f}dV \tag{13}$$

Since the body force acceleration in our case is the gravitational acceleration.

$$f_x=-g \tag{14}$$

Introducing this relation into Eq.(12) produces Eq.(15).

$$K=A_c[P^s_{xin}+\bar{\rho}_f g x_{in})-(P^s_{xout}+\bar{\rho}_f g x_{out})]+\bar{\rho}_f f_x V_{obj} \tag{15}$$

Plugging Eq.(15) to Eq.(10) produces Eq.(16).

$$(F^{obj}_{A_i})_x=-\frac{\partial}{\partial t}\int_{V_f}\rho V_x dV-\int_{A_f}\rho V_x V\cdot dA+i\cdot\int_{A_o}\varsigma\cdot dA$$

$$+A_c[(P^s_{xin}+\bar{\rho}_f g_{xin})-(P^s_{xout}+\bar{\rho}_f g_{xout})]+\bar{\rho}_f f_x V_{obj} \quad (16)$$

The last term of Eq.(16) is commonly called as buoyancy force. One point to note is that this so-called buoyancy force is not present in the original expression of Eq.(10) and comes to appear in Eq. (16) which was derived by converting the static pressure and fluid body force integration terms into the difference of static pressures and elevation heads. It means the so-called buoyancy force is not a generic force. Eq.(16) is an another exact expression for the force which makes a fuel assembly lift, that is, lift force. This expression will be used as the basis of further analysis in this study.

3. Error analysis of a simple expression for lift force

The exact equation for lift force, Eq.(16) is somewhat complicate for practical application and a simple expression is introduced by Eq.(17) and the lift force calculated by Eq.(17) is denoted as F^L_{Model}.

$$F^L_{Model}=A_c(P^t_{xin}-P^t_{xout})+F_b^{obj} \quad (17)$$

Where F_b^{obj} is the so-called buoyancy force of the assembly and P^t is the total pressure defined by Eq.(18)

$$P^t=\frac{1}{2}\overline{\rho V^2}+P_s+\bar{\rho}gx \quad (18)$$

and

$$\overline{\rho V^2}=\int_{A_c}\rho V^2\,dA\Big/\int_{A_c}dA \quad (19)$$

Inappopriately the first term of the RHS of Eq.(17) alone is usually called as lift force in the nuclear engineering application and the inappropriate usage of the term, "lift force" causes confusion in understanding the physics of the flow phenomena in lift force. The expression of the error E in substituting the exact equation (16) by the simple equation (17) is derived here.

$$E=\left[F^{obj}_{Af}\right]_x-F^L_{Model}$$

$$=-\frac{\partial}{\partial t}\int_{Vf}\rho V_x dV-\int_{Af}\rho V_x V\cdot dA+$$

$$i\cdot\int_{Ao}\tau\cdot dA\quad -A_c\left[\left(\frac{1}{2}\overline{\rho V^2}\right)_{xin}-\left(\frac{1}{2}\overline{\rho V^2}\right)_{xout}\right] \quad (20)$$

The momentum flux integral of Eq.(20) can be decomposed into three parts.

$$\int_{Af}\rho V_x V\cdot dA=\int_{(A)_x^{top}}\rho V_x V_x dA-$$

$$\int_{(A)_x^{botom}}\rho V_x|V_x|dA+\int_{(A_o)_{yz}^{side}}\rho V_x W dA \quad (21)$$

Where

$$W=\begin{bmatrix}\dfrac{V_y j\cdot dA}{|dA|} \\ \dfrac{V_z k\cdot dA}{|dA|}\end{bmatrix} \quad \text{or}$$

j, k are respectively the unit vector in the y- and zdirections. Since V_x is positive at $(A_o)_x^{top}$ and $(A_o)_{yz}^{side}$ in a condition of lift force analysis, plugging Eq.(21) into Eq.(20) produces Eq.(22).

$$E=\underbrace{-\frac{\partial}{\partial t}\int_{V_f}\rho V_x dV}_{E_I}+\underbrace{\frac{1}{2}A_c\left[(\overline{\rho V_x^2})_{xin}-(\overline{\rho V_x^2})_{xout}\right]}_{E_{II}}$$

$$\underbrace{-\int_{(A_o)_{side}}\rho V_x W dA}_{E_{III}}+\underbrace{i\cdot\int_{(A_o)}\tau\cdot dA}_{E_{IV}} \quad (22)$$

Eq.(22) shows that the error of the simple equation comes from four different sources. E_I is from the unsteady effect of a reactor coolant flow rate. It is positive when the flow rate decreases and the simple equation overpredicts the lift force at a flow rate increase but underpredicts the lift force at a flow rate decrease. E_{II} is from the algebraic simplification in equation treatment and generally positive since lift force analysis is usually made with an inlet flow rate peak. E_{III} is from the momentum flow by lateral flow convection and is generally negative since flow momentum is supposed to flow out from the fuel assembly with a inlet flow peak. E_{IV} is from the shear stress on the vertical sides of the control volume or equivalently lateral momentum diffusion. And this term is negative because of the inlet flow peak. When the sign of each error term of a general case is put together, the result is

$$E = \underset{E_I}{(\pm)} + \underset{E_{II}}{(+)} + \underset{E_{III}}{(-)} + \underset{E_{IV}}{(-)} \qquad (23)$$

From the definition of E, Eq(22), E needs to be negative for the simple equation to be conservative in lift force design analysis. Among the terms in Eq.(23), however, E_{II} is always positive and E_I can be also positive depending on the flow condition. Because of this, it is not clear whether E will be positive or negative and checking of the sign is required. One interesting point of the error terms is that the two error sources E_{II} and E_{III} counteracts to each other and makes the total error smaller than the case where only one of the two terms are present since their signs are different from each other but their magnitudes are usually close to each other as Eq.(24) shows.

$$E_{II}+E_{III} \approx \left[\frac{1}{2}\bar{\rho} A_c (V_{xin}+V_{xout}) \quad (V_{xin}-V_{xout})\right] - \left[\frac{1}{2}\bar{\rho} (V_{xin}+V_{xout}) \oint W \, dA\right]$$

$$= \left[\frac{1}{2}\bar{\rho} A_c (V_{xin}+V_{xout}) \quad (V_{xin}-V_{xout})\right] - \left[\frac{1}{2}\bar{\rho} (V_{xin}+V_{xout}) A_c \quad (V_{xin}-V_{xout})\right] \approx 0 \qquad (24)$$

Each error term was calculated to check its sign and magnitude. The calculation was made for a typical PWR fuel core at cold and hot coolant conditions. The major parameters used in the evaluation are as follows ;

.fuel rod diameter : 9.5mm

.fuel rod pitch : 12.3mm

.assembly pitch : 198.2mm

.fuel rod configuration : 16×16

.fuel rod length : 3850.0mm

.flow rate distribution at the core inlet : See Fig.2.

.average flow velocity in a free bundle region : 6.0m/s

.core inlet temperature & axial power distribution

: 289.8℃ and 1.55 chopped cosine

: 50℃ and iso-thermal uniform

.transient rate of the total flow : 7%/sec

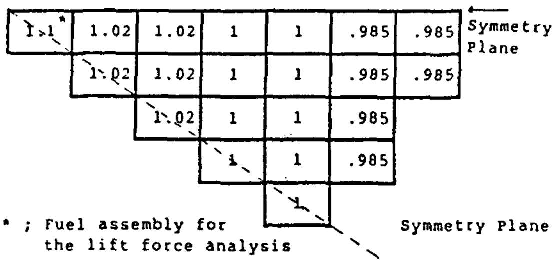

Fig. 2 Relative Flow Rate Distribution at the core Inlet

In calculating error term E_I, the maximum flow transient rate at the flow rate around 100% is taken to be about 7~9% per second considering the required total time for achieving the 100% flow rate at a plant startup and the flow change rate at the loss-of-flow event of KNU 7&8 /5/. Calculation of E_{II} and E_{III} is somewhat straighforward. The shear stress the for E_{IV} was calculated based on the model implemented in COBRA-IV-I as described in Eq.(25) since the thermal-hydraulic field calculation was made by the code.

$$\tau_{yx} = \mu_T \frac{\partial V_x}{\partial_y} = C_T \cdot \beta_y \cdot \overline{\rho V_x} \cdot \Delta V_x \qquad (25)$$

where C_T and by β_y are repectively the inverse of the turbulent Prandtl number and the product of the proportionality coefficients for the mixing length and mixing velocity/6/

The calculation results are shown in Table 1. The magnitude of the each error term is less than 2%. Among the error terms, the algebraic simplification error E_{II} and lateral momenmtum flow term E_{III} are relatively large compared with other terms. Also one can find that it is better to keep the coefficient 1/2 of the dynamic head term of Eq.(18) in using the simple equation Eq.(17) instead of using the coefficient 1.00 from the exact equation Eq.(20) unless one can not eliminate the error of E_{III} completely. The shear stress error term E_{IV} is nearly zero and the unsteady effect term is about 0.3 to 0.4% depending on the flow change rate. The overall error is estimated to be about + 1% at a flow rate decrease condition and less than 1% at both conditions of flow rate increase and steady flow. Since typical cases were selected for the error analysis, it is deduced that the magnitude of the total error in applying the simple equation Eq.(20) can be considered to be generally about 1%.

Table 1 Magnitude of the Errors

Condition	Semi-[1] Lift Force	Error(%)[2]				
		E(I)[3]	E(II)	E(III)	E(IV)	Total
50℃ Iso-Thermal Condition	9.17 (KN)	±.33	1.35	-.71	-.02	.95 .24
289.8℃ Power Operation Condition	6.48 (KN)	±.32	.74	-.79	-.00	.27 -.37

NOTES

1) Semi-Lift Force $= A_c(P'_{xin} - P'_{xout})$
2) Error relative to the semi-lift force in %
3) + ; for a flow rate decrease
   − ; for a flow rate increase

Conclusion

The exact expression for the lift force on a fuel assembly in PWR core has been derived in terms of calculable hydraulic parameters, and the relation for the lift force, pressure drop, buoyancy force, viscous force, and fuel assembly weight has been discussed. the error of a simple equation has been analyzed and the error analysis revealed that the error of a simple expression consists of four terms and the overall error depends on the flowrate change direction and its magnitude is about 1%.

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