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An Observer-Theoretic Approach to Estimating Neutron Flux and Precursor Spatial Distributions

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중성자속과 프리커서의 공간분포 추정을 위한 옵저버 이론 방법

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Abstract

This paper describes a method for estimating the flux and precursor spatial distributions using only limited flux measurements. It is based on the Luenberger observer in control theory, extended to the distributed parameter systems such as the space-time reactor dynamics equation. The results of the application of the method to simple reactor models showed that the flux distribution could be estimated by the observer very efficiently using information from only a few sensors.

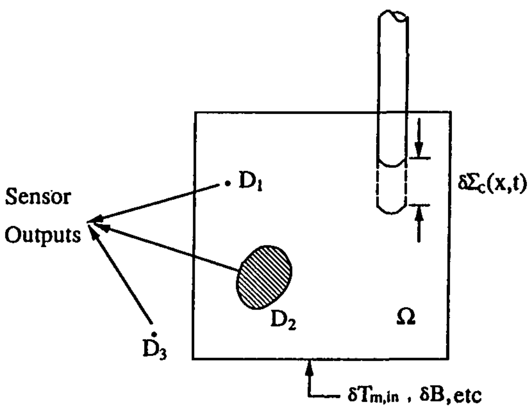
요 약

본 논문에서는 제한된 중성자속 측정치로부터 중성자속과 프리커서의 공간분포를 추정하는 방법을 기술하였다. Luenberger옵저버 이론에 근거하여, 이를 시간과 공간에 의존하는 원자로 동특성 방정식과 같은 분산변수계통에 확장하였다. 간단한 원자로 모형에 대해서 이 방법을 응용한 결과, 소수의 계측기만으로 얻은 정보를 이용하여 중성자속과 프리커서의 공간분포를 효과적으로 추정할 수 있음을 알았다.

1. Introduction

There has been strong need for studies of spatial reactor control due, on the one hand, to the economic necessity of large core design trend resulting in decreased spatial stability and, on the other hand, to the growing demand for operational flexibility such as load-follow operation. The modern control theory could be beneficially used for these studies, although practical implementation of the theoretical results to nuclear reactor control has been very limited so far [1-3].

A nuclear reactor is a distributed parameter system (DPS) since its state variables such as the neutron flux and temperature should be described by partial differential equations. State feedback control provides many advantages such as stabilization and improved transient response. Several theoretical results are available in the literature for feedback stabilization of certain classes of unstable distributed parameter system, which could be used to design a state feedback controller of a nuclear reactor. However, the synthesis of the stabilizing feedback controls frequently calls for complete knowledge of the system state in space and time. But not all the state variables of concern are measurable, (*e.g.*, delayed neutron precursors, iodine, and xenon are not measurable) and, even if measurable, they are measured only at a finite number of localized zones due to practical reasons (*e.g.*, neutron flux is measured at a few positions in the reactor). Moreover, because of the distributed nature of the problem, the spatial locations of the sensors and controllers are in general different, as depicted in Figure 1. Thus determining the states of a DPS from output measurement data is of fundamental importance.



Ω : reactor core

D_i : sensing domain (i = 1, 2, 3)

1 - internal point sensor

2 - internal regional sensor

3 - external sensor

$\delta\Sigma_c$: control rod input

$\delta T_{m,in}$: inlet temperature input

δB : Boron concentration input

Fig.1. Sensor and Controller Configuration of a Nuclear Reactor

As summarized in Section 2, the optimal control is obtained as a linear feedback form in states. Thus, in order to implement the state feedback control, the states that are not measurable should be estimated or reconstructed. In modern control theory, the state reconstruction using dynamic observer has been well developed since the pioneering work of Luenberger [4] and the extension to infinite dimensional systems such as DPS has attracted particular attention and has widened its scope [5,6].

This paper describes the observer theory in the distributed parameter system and presents its applications to reactor problems : estimation of the flux and precursor distributions using flux measurements by a finite number of sensors. In Section 2 we present the model system to be solved and discuss the optimal control theory and the necessity of the state estimator. In Section 3 we describe the observer theory in DPS and in Section 4 we present its applications to the reactor problems, followed by concluding remarks in Section 5.

2. Problem Formulation

2.1 System Description

Consider a nuclear reactor which can be described by the following multigroup state-space model :

$$\Lambda(z,r,t)\frac{d}{dt}z=F[z,r,t], \quad r \in \Omega,$$

$$\Gamma[z,r,t]=0, \quad r \in \partial\Omega, \qquad (2.1)$$

where $z(r,t)$ is the state vector function, $\Lambda(z,r,t)$ is a matrix-valued function of its arguments of appropriate dimensions, $F[\cdots]$ is a spatial operator over Ω, Ω and $\partial\Omega$ are the spatial domain of interest and its boundary surbace, and $\Gamma[\cdots]$ is a spatial operator over $\partial\Omega$, compatible with $F[\cdots]$.

The nonlinear DPS state-space model (2.1) is quite general. The model is also valid for "multiregion" reactors which have some parameters with step discontinuities on the interfaces of the various regions. As an example, a reactor with one fast-, one slow-group flux, and one-group delayed neutron precursor can be represented as

$$z=[\Phi_F,\ \Phi_S,\ C]^T,$$

$$A=\mathrm{diag}[v_F^{-1},\ v_S^{-1},\ a]$$

$$F=[\nabla\cdot D_F\nabla-(\Sigma_{aF}+\Sigma_R)+(1-\beta)\upsilon\Sigma_{fF}]\Phi_F+(1-\beta)\upsilon\Sigma_{fS}\Phi_S+\lambda C,$$
$$\Sigma_R\Phi_F+(\nabla\cdot D_S\nabla-\Sigma_{aS})\Phi_S$$
$$\beta\upsilon\Sigma_{fF}\Phi_F+\beta\upsilon\Sigma_{fS}\Phi_S-\lambda C]^T \qquad (2.2)$$

where Φ_F, Φ_S, and C are the fast−, slow-group neutron flux, precursor concentration, respectively. Here Φ_F, Φ_S, D_F, $\nabla\Phi_F$, and $D_S\nabla\Phi_S$ are continuous at all region interfaces, and z vanishes at the extrapolated boundaries.

The control rod model can be added to (2.2) by expressing the control rod movement as a movement of the rod tip. A convenient representation of the control rods is that Σ_{aS}, the thermal absorption cross-section, is given by

$$\Sigma_{aS}=\Sigma_{aS0}(r)+\sum_{N}^{k\ 1}u_k(t)\,\delta\,(r-r_k), \qquad (2.3)$$

where $\Sigma_{aS0}(r)$ is some average value, and $u_k(t)$ is the strength of a thermal absorption source or sink at the tip r_k of the k–th control rod. Equation (2.3) represents the effect of slightly displacing the control rods. If we consider various feedback effects such as temperature, xenon and fuel burnup, the more general description for Σ_{aS} can be obtained, which is left to a further work.

We use linearized models by considering small deviations of the state variables from their nominal(operating-point) ones. In this case the controller is usually designed such that the operating-point performance is optimal in the sense of making the deviations as small as possible. We assume that the operating-point is the "steady-state," which means that the control frods at the steady-state position keep the reactor critical. The steady-state $\bar{z}(r)$ is determined by setting $\dot{z}=0$, *i.e.*, by solving the equation

$$F[\bar{z}(r),\ r]=0 \qquad (2.4)$$

Denoting by δz the deviation of the state vector from its steady state $\bar{z}(r)$, we can get the linearized model by neglecting higher-order terms. Without loss of generality, we can put the equation into the following form

$$\frac{d}{dt}z(r,t)=A(r,t)z(r,t)+B(r,t)u(r,t),\ r\in\Omega, t\geq t_0, \qquad (2.5)$$

where the symbol z is used for the deviation of the state vector, δz, and u(r,t) for the control inputs. Here A is a linear spatial differential operator and B is a known matrix, which can be time-dependent as in burnup problems.

This is the linear DPS nuclear reactor model which is to be studied here, and holds only for small control movements about some steady-steady-state operating point $\bar{z}$. Also the boundary conditions associated with (2.5) have the form

$$a(r)z(r,t)=0,\ r\in\partial\Omega,\ t\geq t_0, \qquad (2.6)$$

where a(r) is a spatial operator over $\partial\Omega$ compatible with A.

2.2 Optimal Control Theory and Dynamic Observer

We want to apply the results of optimal control theory of DPS to the systems described by the equations (2.5) and (2.6). The control objectives are expressed by the objective function and constraint sets. The objective function is essentially based on the desired flux and precursor distributions. These distributions are time-varying, and the reactor is controlled so as to be close to the desired distributions in an integral sense. In mathematical notations, the objective function is an integral over the control period $[t_0, T]$ and the reactor core volume Ω

$$L=\int_{t_0}^{T}dt\int_{\Omega}d\Omega\Big(W_\phi(r,t)[\phi(r,t)-\phi_d(r,t)]^2$$
$$+W_C(r,t)[C(r,t)-C_d(r,t)]^2$$

$$+ \sum_{k=1}^{N} [W_{u,k}(t)[u_k(t) - u_{k,d}]^2$$
$$+ W_{\dot{u},k}(t)\dot{u}_k^2(t)] \Big), \qquad (2.7)$$

where u_k and $\dot{u}_k$ are values of the control inputs and their time rates of change, and $W(r,t)$ are weighting functions to be freely chosen. The subscript d is used to indicate the desired values of the variables.

What we really want to obtain is the optimal input which drive the system into the desired state in such a way that the objective function L is minimized. This is the well-known linear-quadratic optimal control problem. The *optimal control law* derived in modern optimal control theory is of the form

$$u^0(r,t) = -K(r,t)z(r,t), \qquad (2.8)$$

where K is a matrix operator, the explicit form of which can be found as a solution of the Riccati equation.

Note that the optimal input in (2.8) can be realized only if we know all the states as functions of both space and time. All versions of the optimal control problems have in common the basic assumption that all the state vector $z(r,t)$ is completely available. However, not all the information of the states can be obtained due to various practical reasons as discussed in Section 1. For lumped parameter systems, Kalman filters or Luenberger observers have been developed to obtain a suitable estimation or reconstruction of the states. Recently, Woo and Cho [7] applied the observer theory to the optimal control of xenon concentration in a nuclear reactor, where the observer was used to estimate the iodine and xenon concentrations in point reactor kinetics.

In the present work, the dynamic observer in distributed parameter systems is developed in order to reconstruct the missing space-dependent state variables. These reconstructed states are to be used in (2.8) to provide the optimal input.

3. Observer Thory

3.1 Mathematical Formulations

We consider the system described by the following abstract partial differential equation

$$(S) \begin{cases} \dot{z} = Az + Bu\,; \\ z(0) = z_0, \end{cases} \qquad (3.1)$$

with output function

$$y = Cz, \qquad (3.2)$$

where the state variable z is defined on an appropriate separable Hilbert space Z and the control input u and the output y are defined similarly on separable Hibert spaces U and Y, respectively.

Then, if we assume that A generates a strongly continuous semigroup $(S(t))_{t \geq 0}$ on Z, the system (S) admits a solution

$$z(t) = S(t)z_0 + \int_0^t S(t-s)Bu(s)ds\,; \qquad (3.3)$$

$$y(t) = Cz(t).$$

Note that the assumption on the spatial operator A is generally satisfied by the parabolic distributed systems, an example of which is the neutron diffusion equation. Curtain [8] considers that the spatial operator A satisfies the following three fundamental assumptions concerning the spectrum of the operator :

A1) spectrum determined growth assumption

$$\sup Re\ \sigma(A) = \lim_{t \to \infty} \frac{\log \| S(t) \|}{t} = w_0, \qquad (3.4)$$

where $\sigma(A)$ is the spectrum of A.

A2) spectrum decomposition assumption

For every $\delta > 0$, define

$$\sigma_u(A) = \sigma(A) \cap \{\lambda : Re\,\lambda \geq -\delta\},$$
$$\sigma_s(A) = \sigma(A) \cap \{\lambda : Re\,\lambda < -\delta\}. \qquad (3.5)$$

The assumption is that $\sigma_u(A)$ is bounded and is separated from $\sigma_s(A)$, so that a rectifiable simple, closed curve can be drawn so as to enclose an open set containing $\sigma_u(A)$ in its interior and $\sigma_s(A)$ in its exterior.

A3) analytic spectrum assumption

A is normal with compact resolvent and there are only finitely many eigenvalues in $\{\lambda : Re\ \lambda \geq -\delta\}$ for any $\delta > 0$ and each eigenvalue has a finite dimensional eigenmanifold.

Then, we can decompose the space Z with Z^u finite dimensional and Z^s infinite dimensional satisfying A1) in the following manner:

$$Z = Z^u \oplus Z^s \,;\ Z^u = PZ,\ Z^s = (I-P)Z,$$

$$P = \frac{1}{2\pi i} \oint_{\Gamma} (\lambda I - A)^{-1} d\lambda, \tag{3.6}$$

where Γ is a curve enclosing $\sigma_u(A)$. A^u is the restriction of A to Z^u and is bounded, so is A^s to Z^s but unbounded.

In order to design the state observers, El jai and Pritchard [9] introduced the concept of "strategic" sensors by defining the following properties:

D1) weak observability

The system (S) is *weakly observable* on $[0, T]$ if

$$CS(t)z_0 = 0,\ 0 \leq t \leq T. \Rightarrow z_0 = 0. \tag{3.7}$$

D2) strategic sensors

If we define the sensors as the couples $(D_i, f_i)_{1 \leq i \leq q}$ where D_i is the support of the sensor, f_i is the spatial distribution of the sensor and q is the total number of sensors, the sensors $(D_i, f_i)_{1 \leq i \leq q}$ are said to be "strategic" if the system (S) is weakly observable on $[0, T]$ for any $T > 0$. In other words, $(D_i, f_i)_{1 \leq i \leq q}$ are strategic if and only if

(i) $q \geq \sup\ \gamma_n$,

(ii) rank $H_n = \gamma_n,\ n = 1,2,\cdots,J$ (3.8)

$$H_{n,ij} = \langle f_i, \Psi_{nj} \rangle_{L^2(D_i)},\ i = 1,2,\cdots,q\,;\ j = 1,2,\cdots,\gamma_n$$

where γ_n is the multiplicity of the eigenvalue λ_n, Ψ_{nj} are the associated eigenfunctions, and J is the number of the unstable eigenvalues.

Then, the following dynamics system

$$(O)\begin{cases} \dot{\hat{z}} = A\hat{z} + Bu + G(y - C\hat{z})\,; \\ \hat{z}(0) = \hat{z}_0, \text{ arbitrary} \end{cases} \tag{3.9}$$

is an identity state observer (estimator) for the system (S) if the sensors $(D_i, f_i)_{1 \leq i \leq q}$ are strategic for the unstable system of (S), or equivalently the operator $(A - GC)$ generates a strongly continuous semigroup $(S_G(t))_{t \geq 0}$ which is exponentially stable, *i.e.*,

$$\| S_G(t)z_0 \| \leq M exp(-w_0 t) \| z_0 \|. \tag{3.10}$$

Thus, the error dynamics

$$\dot{e} = (A - GC)_e, \tag{3.11}$$

will be asymptotically stable, where $e = (A - GC)_e$, will be asymptotically stable, where $e = z - \hat{z}$ is an estimation error.

3.2 Pole Assignment in Finite Dimensional Systems

It is natural to ask what form of the observer gain G should be if we want the operator$(A - GC)$ to be exponentially stable. We first determine the unstable eigenvalues $\lambda_1, \lambda_3, \cdots, \lambda_J$ with corresponding eigenfunctions $\{\Psi_{nj},\ n = 1,\cdots,J,\ j = 1,\cdots, r_n\}$, all assumed continuous on the spatial domain. Thus, relative to the basis $\{\Psi_{nj},\ n = 1,\cdots,J,\ j = 1,\cdots, r_n\}$ for Z^u, we have

$$A^u = \mathrm{diag}[\lambda_1 I r_1, \cdots, \lambda_J I r_J]. \tag{3.12}$$

If C^u is the restriction of C to Z^u, C^u has the form

$$C^u = [C_1, C_2, \cdots, C_J], \tag{3.13}$$

where

$$C_n = \begin{pmatrix} C^1_{n1} & C^1_{n2} & \cdots & C^1_{nr_n} \\ C^2_{n1} & C^2_{n2} & \cdots & C^2_{nr_n} \\ \\ C^q_{n1} & C^q_{n2} & \cdots & C^q_{nr_n} \end{pmatrix} n = 1,\cdots,J. \tag{3.14}$$

Note that the elements of C_n are identical to those of H_n in (3.8). Rewriting the elements of C^u we have

$$C_{nj} = H_{n,ij} = \langle f_i, \Psi_{nj} \rangle_{L^2(D_i)}. \tag{3.15}$$

The rank condition in (3.8) means that the sensors should be suitablely located so as to satisfy the properties of the strategic sensors.

Since we want to design the observer gain G in the decomposed finite dimensional space such that $(A-GC)$ is exponentially stable, the pair of (A^u, C^u) should be observable in the finite dimensional space in order for the stabilizing observer gain G^u to exist, where G^u is the restriction of G to Z^u. So the problem is reduced to the finite dimensional pole placement problem, namely, to the problem of determining the observer gain G so that $(A^u-G^uC^u)$ has the desired stabilizing eigenvalues. Pole assignment algorithms for such finite dimensional problems are well developed in modern control theory [10]. The next section presents the results of the application to three reactor model problems.

4. Applications

4.1 Simple Examples

Let us consider a hypothetical simple reactor which is described by the one-group diffusion equation without feedback. We apply the theory in Section 3 to the one-and two-dimensional cases where the multiplicity is 1 and 2, respectively. The diffusion equation for such a reactor is

$$\frac{1}{\upsilon}\frac{\partial \phi}{\partial t}=\nabla\cdot D\nabla\phi+(\upsilon\epsilon\Sigma_f-\Sigma_a)\phi$$

$$\phi(0,t)=\phi(H,t)=1,\ t\in[0,T], \tag{4.1}$$

where the parameters have their usual meanings. Initially small perturbation is introduced to the flux, but the initial condition is not completely known since the flux is measured only at a few points. Thus, the model equation cannot be solved. This is one reason why we want to design the state observer. Since the state observer gives us the information of the states irrespective of their initial conditions, we can reconstruct the states from the plant output and arbitrary (usually zero) initial conditions (see equation (3.9)).

For numerical experiments in this study, we used the material constants given in Table I from Ref. 1 and used as the plant output y in (3.9) the simulation results of the model plant (4.1) with assumed initial conditions on the flux perturbation. The control input is not included here to see how the state observer works in control-free unstable systems.

Table I. Material Constants Used in Section 4.1

Constants	Value
$l=\frac{1}{\Sigma_a\upsilon}$	0.1 *sec*
$L^2=\frac{D}{\Sigma_a}$	160 cm^2
$k_\infty=\frac{\upsilon\epsilon\Sigma_f}{\Sigma_a}$	1.0256

4.1.1 *One-dimensional case : Slab reactor with H $=500$ cm*

Following the standard procedure we first determine the eigensets for the system

$$A\psi=\lambda\psi, \tag{4.2}$$

where

$$A\equiv\frac{L^2\partial^2}{l\partial x^2}+\frac{(k_\infty-1)}{l}, \tag{4.3}$$

$$\psi(0)=\psi(H)=0.$$

It is well known that in this case the multiplicity is 1, so only one sensor is enough to design an observer. The eigensets are

$$\lambda_n=\frac{(k_\infty-1)}{l}-\frac{L^2}{l}\left(\frac{n\pi}{H}\right)^2,\ n=1,2,\cdots \tag{4.4}$$

$$\Psi_n(x)=\sqrt{\frac{2}{H}}\sin\left(\frac{n\pi x}{H}\right).$$

It is clear that the operator A satisfies the assumptions A1), A2) and A3) and the unstable eigenvalues are λ_1 and λ_2. If we locate a sensor at the point x_1, then A^u and C^u have the form

$$A^u=\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix},\quad C^u=[\Psi_1(x_1)\ \Psi_2(x_1)]. \tag{4.5}$$

The rank condition in (3.8) gives us the permissible sensor locations as

$$\text{rank } H_1=\Psi_1(x_1)\neq 0, \tag{4.6}$$

$$\text{rank } H_2=\Psi_2(x_1)\neq 0.$$

Hence, the sensor should not be located at the reactor boundaries or at the midpoint.

It is well known in finite–dimensional control theory [10] that if the pair of (A^u, C^u) is completely observable, then it is always possible to find a G^u which will yield any set of desired eigenvalues for $(Au-G^uC^u)$. In this case if we assign the eigenvalues $\{\lambda_1, \lambda_2\}$ to $\{\gamma_1, \gamma_2\}$ all of which hve negative real parts, we obtain the values of the gain $\{g_1, g_2\}$ that are the elements of G^u using the standard pole assignment technique. Recalling that G^u is the restriction of G to Z^u, it is easy to find the infinite–dimensional observer gain G, which is in reality the sum of the eigenfunctions weighted by the elements of G^u.

Then, the infinite–dimensional observer equation is of the form

$$\frac{\partial \hat{\phi}}{\partial t}=\frac{L^2}{l}\frac{\partial^2 \hat{\phi}}{\partial x^2}+\frac{(k_\infty-1)}{l}\hat{\phi}$$
$$+G(x)(\phi(x_1)-\hat{\phi}(x_1)), \qquad (4.7)$$
$$\hat{\phi}(0,t)=\hat{\phi}(H,t)=0,$$

where

$$G(x)=g_1\Psi_1(x)+g_2\Psi_2(x). \qquad (4.8)$$

For numerical results, we introduced a perturbation of magnitude $1\times10^{13}/cm^2$ *sec* in the flux at $x=\frac{H}{4}$. The sensor is located at $x_1=\frac{3H}{4}$. For $\{\gamma_1, \gamma_2\}=\{-0.1, -0.2\}$, Figures 2a and 2b display as functions of time the two flux distributions : real and estimated by the observer. Figure 2c shows the estimation error at two points. It is shown that the flux distribution estimated by the observer approaches the real flux very quickly.

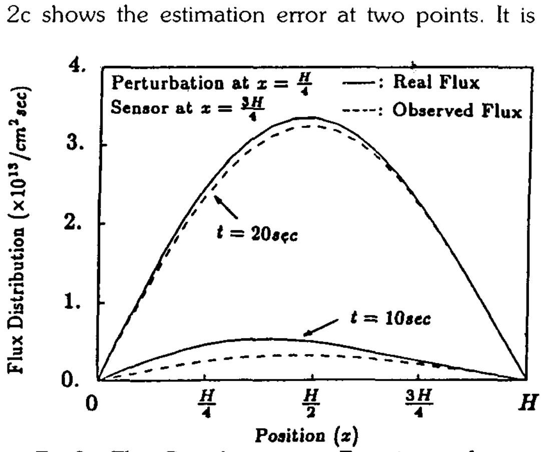


Fig.2a Flux Distributions as Functions of Time for the Slab Reactor

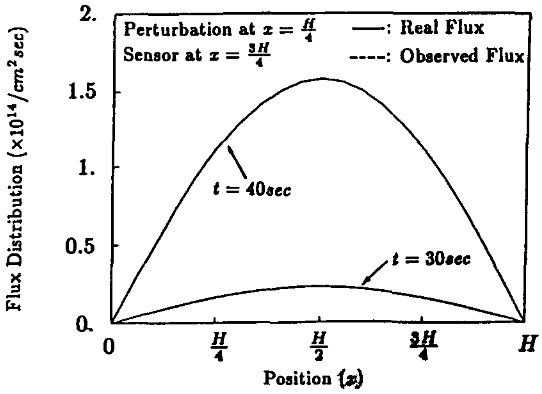


Fig.2b Flux Distribuions as Functions of Time for the Slab Reactor

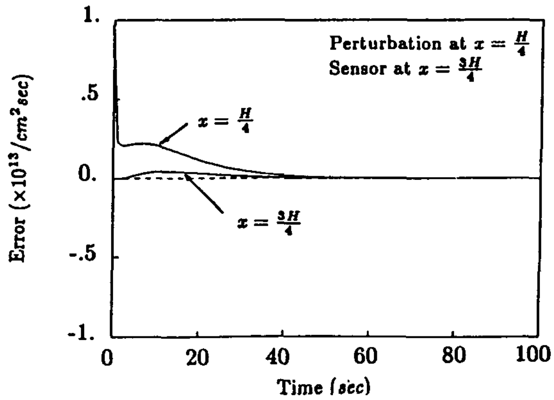


Fig.2c Estimation Error Dynamics($\phi-\hat{\phi}$) at $x=\frac{H}{4}$ and $\frac{3H}{4}$ for the Slab Reactor

4.1.2 *Two-dimensional case : Square reactor with $H_x=H_y=H=600cm$*

Since in this case the second unstable eigenvalue is of multiplicity 2, the minimum number of sensors required is 2. Let us order the unstable eigenvalues as $\{\lambda_1, \lambda_2, \lambda_3\}=\{\lambda_{11}, \lambda_{12}, \lambda_{21}\}$ and the corresponding eigenfunctions as

$$\Psi_1(x,y)=\frac{2}{H}\sin\left(\frac{\pi x}{H}\right)\sin\left(\frac{\pi y}{H}\right),$$

$$\Psi_2(x,y)=\frac{2}{H}\sin\left(\frac{2\pi x}{H}\right)\sin\left(\frac{\pi y}{H}\right), \qquad (4.9)$$

$$\Psi_3(x,y)=\frac{2}{H}\sin\left(\frac{\pi x}{H}\right)\sin\left(\frac{2\pi y}{H}\right).$$

A^u is a 3×3 matrix and C^u is a 2×3 matrix. The impermissible sensor locations should be determined carefully, *i.e.*, if we locate two sensors at points (x_1, y_1), (x_2, y_2), the rank condition is

$$\text{rank } H_1=\text{rank}\begin{pmatrix}\Psi_1(x_1,y_1)\\ \Psi_1(x_2,y_2)\end{pmatrix}=1,$$

$$\text{rank } H_2=\text{rank}\begin{pmatrix}\Psi_2(x_1,y_1) & \Psi_3(x_1,y_1)\\ \Psi_2(x_2,y_2) & \Psi_3(x_2,y_2)\end{pmatrix}=2. \qquad (4.10)$$

The matrix H_2 has rank of order 2 in the case that the determinant of H_2 is not zero, Hence,

$$\Psi_2(x_1,y_1)\Psi_3(x_2,y_2)-\Psi_2(x_2,y_2)\Psi_3(x_1,y_1)\neq0. \qquad (4.11)$$

Using function-product relations of trigonometry [11], and after some algebraic manipulations, we obtain the relations on the impermissible sensor locations as

$$x_1+x_2=y_1+y_2. \qquad (4.12)$$

The gain matrix G^u is 3×2, and the observer equation has the following form

$$\frac{\partial\hat{\phi}}{\partial t}=\frac{L^2}{l}\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+\frac{(k_\infty-1)}{l}\hat{\phi}$$
$$+G_1(x,y)(\phi(x_1,y_1)-\hat{\phi}(x_1,y_1))$$
$$+G_2(x,y)(\phi(x_2,y_2)-\hat{\phi}(x_2,y_2)),$$
$$\hat{\phi}(0,y,t)=\hat{\phi}(H,y,t)\neq\hat{\phi}(x,0,t)=\hat{\phi}(x,H,t)\neq0, \qquad (4.14)$$

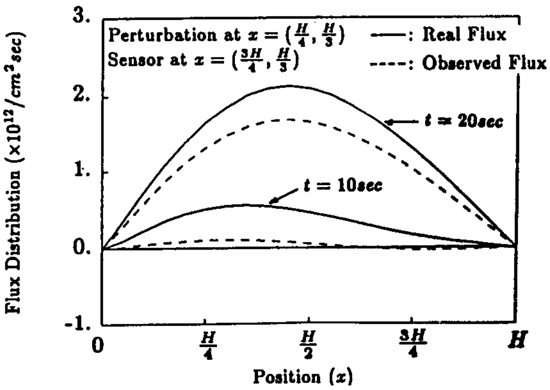


Fig.3a Flux Distributions as Functions of Time for the Square Reactor at Nodal Line $y=\frac{H}{3}$

where

$$G_i(x,y)=g_{1i}\Psi_1(x,y)+g_{2i}\Psi_2(x,y)+g_{3i}\Psi_3(x,y),$$
$$i=1,2. \qquad (4.15)$$

The perturbation was introduced at $(\frac{H}{f}, \frac{H}{3})$ and the two sensors are at $(\frac{3H}{4}, \frac{2H}{3})$ and $(\frac{H}{2}, \frac{2H}{3})$.

Figures 3a,3b, and 3c show the results for $\{\gamma_1, \gamma_2, \gamma_3\}=\{-0.1, -0.2, -0.3\}$.

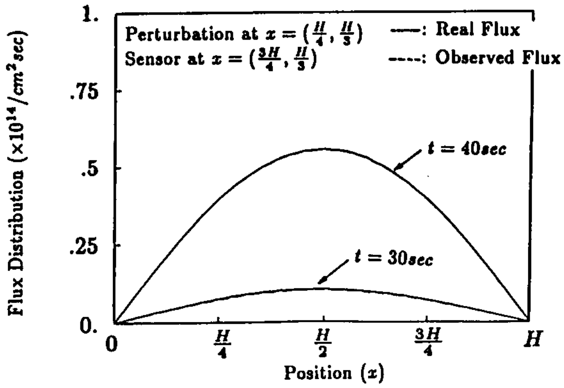


Fig.3b Flux Distributions as Functions of Time for the Square Reactor at Nodal Line $y=\frac{H}{3}$

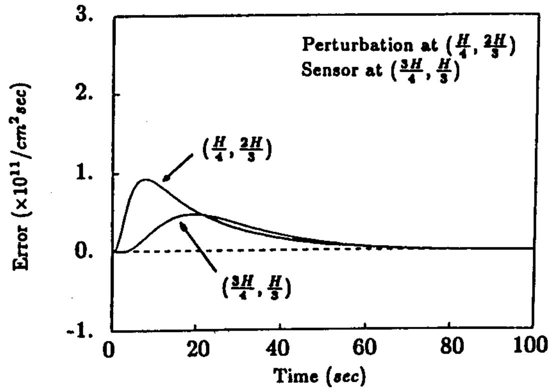


Fig.3c Estimation Error Dynamics$(\phi-\hat{\phi})$ at Points $x=(\frac{H}{4}, \frac{2H}{3})$ and $(3\frac{H}{4}, \frac{H}{3})$ for the Square Reactor at Nodal Line $y=\frac{H}{3}$

4.2 With One-Group Delayed Neutron Precursor

Now let us apply the present theory to the system with more than one distributed state variables. We consider an infinite bare slab reactor with extrapolated thickness H, and approximate its dynamics by the one-group diffusion equation with one-group delayed neutron precursor:

$$\frac{1}{\upsilon}\frac{\partial \phi(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(D(x)\frac{\partial \phi(x,t)}{\partial x}\right) - \Sigma_a(x)\phi(x,t)+(1-\beta)\upsilon\Sigma_f\phi(x,t) + \lambda C(x,t)-\Sigma_c(x,t)\phi(x,t), \tag{4.16}$$

$$\frac{\partial C(x,t)}{\partial t}=\beta\upsilon\Sigma_f\phi(x,t)-\lambda C(x,t),$$

where $\Sigma_c(x,t)$, the absorption cross section of a control rod, is used for a distributed control variable but will be omitted as before to see how the designed observer works in control-free systems. The spatially varying parameters are assumed, following Iwazumi and Koga [12], to be spatially independent. Since physically the neutron precursor concentration is not measurable, is the required that the dynamic observer be designed to reconstruct the precursor distribution as well as the flux distribution, if the feedback control system is desired.

A linearized state equation can be obtained by considering small deviations from the steady state :

$$\frac{\partial}{\partial t}\Phi(x,t)=S_x\Phi(x,t), \tag{4.17}$$

where

$$\Phi^T(x,t)=(\delta\phi(x,t)\ \ \delta C(x,t)),$$

$$S_x=\begin{pmatrix} a_1\frac{\partial^2}{\partial x^2}+a_2 & a_3 \\ a_4 & a_5 \end{pmatrix} \tag{4.18}$$

$$a_1=\upsilon D,\ a_2=\upsilon[(1-\beta)\upsilon\Sigma_f-\Sigma_a],$$

$$a_3=\upsilon\lambda,\ a_4=\beta\upsilon\Sigma_f,\ a_5=-\lambda.$$

It can be shown that the operator S_x satisfies the assumptions A1), A2) and A3) by proving that the evolution equation (4.17) has the unique solution. The detailed proof is worked out by Kuroda and Makino [13]. The eigenvalues λ_{ij} and vector eigenfunctions $\Psi_{ij}(x)$ of the operator S_x are found as

$$\lambda_{i1,2}=\frac{1}{2}\left(a_2+a_5-a_1\left(\frac{i\pi}{H}\right)^2\pm\left[(a_2+a_5-a_1(\frac{i\pi}{H})^2)^2-4(a_2a_5-a_3a_4-a_1a_5(\frac{i\pi}{H})^2)\right]^{1/2}\right), \tag{4.19}$$

$$\vec{\Psi}_{ij}(x)=\eta_{ij}\vec{\varphi}_{ij}\sin\frac{i\pi}{H}x;\ i=1,2,\cdots,\infty,\ j=1,2, \tag{4.20}$$

where

$$\varphi_{ij}=\begin{pmatrix}\varphi_{ij1}\\ \varphi_{ij2}\end{pmatrix}=\begin{pmatrix}1\\ a_4/(\lambda_{ij}-a_5)\end{pmatrix}, \tag{4.21}$$

$$\eta_{ij}=\sqrt{\frac{2(\lambda_{ij}-a_5)^2}{[(\lambda_{ij}-a_5)^2+a_3a_4]H}}.$$

The coefficients η_{ij} are normalization factors defined in such a manner as to satisfy the biorthonormal relation

$$\int_0^H \Psi_{kl}^{T*}(x)\Psi_{ij}(x)dx=\delta_{ij}^{kl}. \tag{4.22}$$

The adjoint eigenfunctions $\Psi_{ij}(x)$ are given by

$$\vec{\Psi}^*_{ij}(x)=\eta_{ij}\vec{\varphi}^*_{ij}\sin\frac{i\pi}{H}x, \tag{4.23}$$

where

$$\Psi_{ij}=\begin{pmatrix}\varphi_{ij1}\\ \varphi_{ij2}\end{pmatrix}=\begin{pmatrix}1\\ a_3/(\lambda_{ij}-a_5)\end{pmatrix} \tag{4.24}$$

Table II. Material Constants Used in Section 4.2

Constants	Value
λ	$0.078\ sec^{-1}$
$\upsilon\Sigma_f$	$0.0505\ cm^{-1}$
Σ_a	$0.05\ cm^{-1}$
D	$0.5071\ cm$
υ	$2.2\times10^5 cm/sec$
β	7.5×10^{-3}
H	$200\ cm$

The data used in this example are listed in Table II. We know that the unstable eigenvalue is

λ_{11} and its multiplicity 1, so only one sensor is enough, which is located at $x=x_1$. The observer equation we get is written as

$$\frac{\partial}{\partial t}\hat{\Phi}(x,t)=S_x\hat{\Phi}(x,t)+G[\delta\phi(x_1,t)-\delta\hat{\phi}(x_1,t)], \qquad (4.25)$$

where

$$\hat{\Phi}(x,t)=\begin{pmatrix}\delta\hat{\phi}\\ \delta\hat{C}\end{pmatrix},$$

$$G=\begin{pmatrix}g(x)\\ 0\end{pmatrix}, \qquad (4.26)$$

$$g(x)=\left[(\gamma_1-\lambda_{11})/\sin\left(\frac{\pi x_1}{H}\right)\right]\sin\left(\frac{\pi x}{H}\right).$$

It should be noted that the time scale between neutron flux and precursor equations is very large, approximately of the order of 10^7, and this causes the so-called stiffness, which results in the restriction of very small time step increments in numerical solutions of the dynamics equations. Since the major concern in this example is, for the moment, to see how the designed observer works in the system with more than one state variables, the explicit method for parabolic equations is used using extremely small time step increment, *e.g.*, $\sim 10^{-4}$ sec. The pole chosen is -10^4, and the results are shown in Figure 4. Figures 4a and 4b display the real and observed flux distributions, and Figures 4c and 4d the distributions of the two precursor concentrations: calculated from the model and estimated by the observer. Figures 4e and 4f show the estimation error of fluxed and precursor concentrations at two points. It is shown that the precursor concentration as well as the flux can be reconstructed by the observer very quickly.

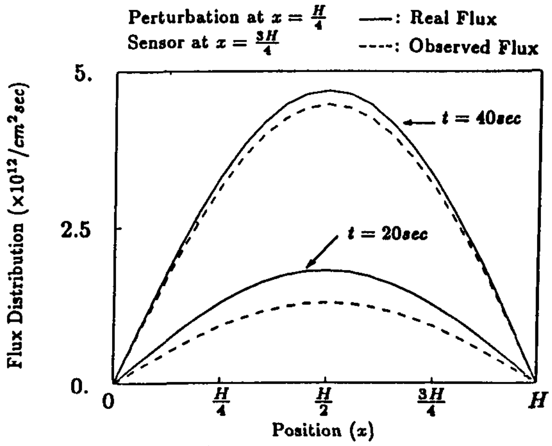


Fig.4a Flux Distributions as Function of Time for the Slab Reactor with Delayed Neutron

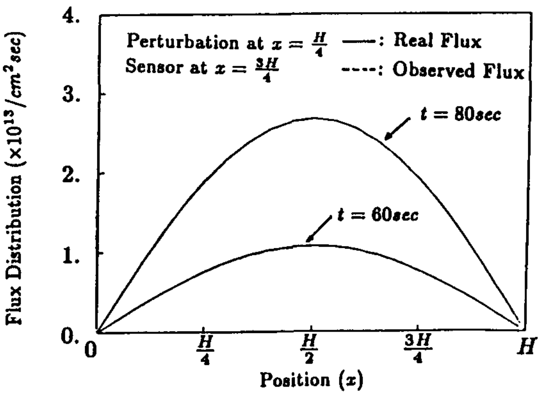


Fig.4b Flux Distributions as Functions of Time for the Slab Reactor with Delayed Neutron

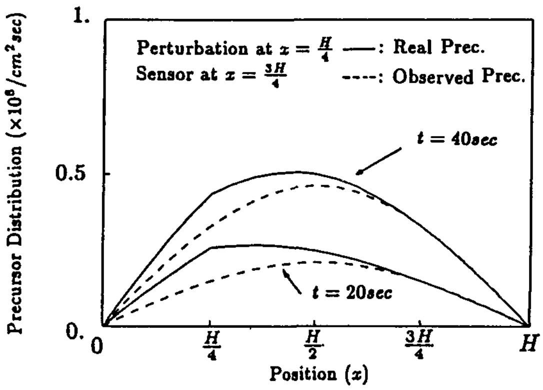


Fig.4c Precursor Distributions as Functions of Time for the Slab Reactor with Delayed Neutron

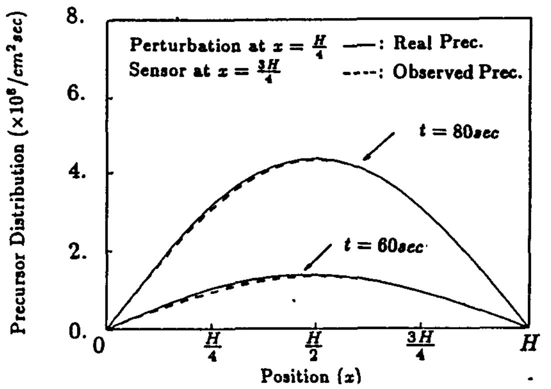


Fig.4d Precursor Distributions as Functions of Time for the Slab Reactor with Delayed Neutron

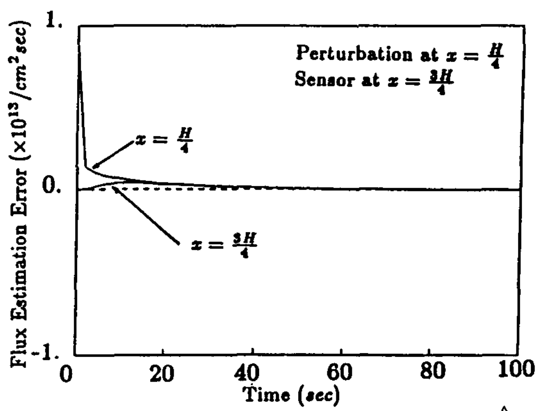


Fig.4e Flux Estimation Error Dynamics($\phi - \hat{\phi}$) at $x=\frac{H}{4}$ and $\frac{3H}{4}$ for the Slab Reactor with Delayed Neutron

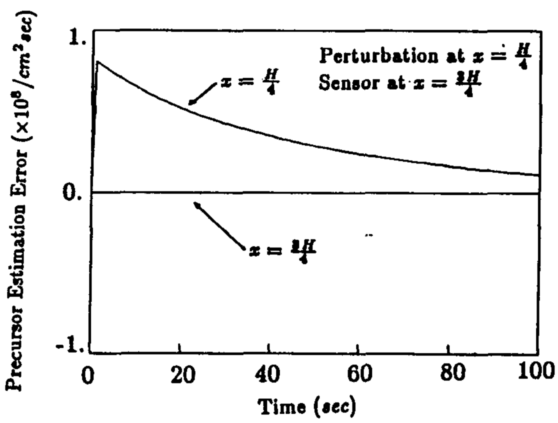


Fig.4f Precursor Estimation Error Dynamics at $x=\frac{H}{4}$ and $\frac{3H}{4}$ for the Slab Reactor with Delayed Neutron

5. Concluding Remarks

Spatial control of a nuclear reactor calls for complete of the system state variables when state feedback control is considered. We described a method for reconstructing the measurable and unmeasurable state variables, which is based on the observer theory for distributed parameter systems. If the properties of the eigensets of the spatial operator are known, the modal decomposition of the state variables enables us to use the pole assignment algorithms developed in finite dimensional systems to obtain the stabilizing observer gain.

The results of the application of the method to simple reactor models showed that the flux and precursor distributions are estimated by the observer very efficiently using information from only a few flux sensors. Although we applied the method to simple reactor models in the present paper, we believe that it is possible to extend it to more realistic reactor models with temperature and xenon feedbacks. Work is in progress in this area.

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