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Generalized Nyquist Criterion for the Stability of Xenon Oscillation

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일반화된 Nyquist 요건에 의한 제논진동의 안전성 분석

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Abstract

The Xenon spatial oscillation may give rise to operational difficulties in a nuclear power plant. In this study, in order to investigate the Xenon instability for a PWR, the frequency-domain technique is adopted by using Generalized Nyquist Criterion, which is more general and suitable for the multi-input/multi-output system. Also linearized modal fluxes are obtained by a modal expansion. This model has been implemented to test the axial Xenon stability of YGN-1 unit against the changes in plant operating parameters; power level, control rod position, and core average burnup. The results show that the increase of power level and the deeper insertion of control rod have the destabilizing effect, and that the burnup progress makes the core less stable. Also the results show that the overestimation due to modal interaction was found not to be significant.

요 약

제논의 공간적인 진동은 원자로의 운전에 지장을 초래할 수 있다. 본 연구에서는 가압경수로에서의 제논에 의한 불안정성을 분석하기 위하여, 보다 일반적이고 다중입력/다중출력 계통에 적합한 일반화된 Nyquist 조건을 사용하는 진동수위주의 기술을 적용하였다. 또한 모드 전개 방법에 의하여 선형화된 중성자속을 구했다. 이 모형으로 출력 준위, 제어봉 위치, 그리고 평균 노심 연소도 등의 노물리 변수의 변화에 따른 영광 1호기의 제논에 대한 축방향 안정성을 조사하였다. 결과로는, 출력 준위의 증가나 제어봉 삽입의 증가는 안정성을 저해하는 효과를 가져오고, 연소도가 증가할 수록 불안정한 것으로 나타났다.

$D(z)$	: neutron diffusion coefficient
$\phi(z, t)$	: one group neutron flux
$\Sigma_a(z)$	: macroscopic neutron absorption cross-section
$\Sigma_f(z)$	: macroscopic neutron fission cross-section
$k_\infty(z, t)$	: infinite multiplication factor
υ_N	: average number of neutrons generated per fission
$I(z, t)$	: iodine number density
$X(z, t)$	: xenon number density
λ_I	: decay constant of I–135
λ_X	: decay constant of Xe–135
$\gamma_I(z)$	: yield fraction of I–135
$\gamma_X(z)$	: yield fraction of Xe–135
$\sigma_X(z)$	: microscopic absorption cross-section of Xe–135
υ	: neutron velocity
$\phi(z)$	: steady state neutron flux
$I_0(z)$	: steady state iodine number density
$X_0(z)$	: steady state xenon number density

1. Introduction

A various kinds of the study of the stability against xenon oscillation have been developed. For the space and time dependent reactor stability problem of neutron flux shape variations due to xenon oscillation, the modal expansion method with or without an eigenvalue type method is often used.[1,2,3]

The advantages of modal expansion method are to approximate the unknown functions of space and time by a linear combination of known space functions with time dependent coefficients and not to find all the solutions which can be the reactor parameters. Through this method, the finite set of partial differential equations describing the problem is replaced by an infinite set of ordinary differential equations.[1,10]

Another attention of this paper is drawn to the most developed eigenvalue type stability investigation. This is a part of frequency–domain stability method using Generalized Nyquist Criterion.[1,2,3] This method forms a characteristic transfer function matrix and calculates the determinant of the matrix to investigate the stability. The eigenvalue type method cannot be solved, unless the transfer function matrix has non zero determinant and has the complexity to find all its eigenvalues. If, however, the system could be described as multi–input/multi–output system with unity–feedback, the Generalized Nyquist Criterion is introduced to see the effect of the space–dependent reactor kinetics problem.[8] This criterion only requires diagonally dominant transfer function matrix as an option. Then the system is reduced to m single–input/single–output system which is simpler and more convenient.

From this point of view, this paper deals with the stability against xenon spatial oscillation using modal expansion method and the Generalized Nyquist Criterion.

To examine the adaptability, we study the axial stability in a PWR core to the changes in core physical parameters such as power level, control rod position, and core average burnup. Through this study, Yonggwang 1 core is analyzed against the axial xenon oscillation. The data corresponding to core parameters were achieved from referrence 12.

The prescedent investigators[1,2,5] found that omega method does not have the property of finality. So the modal coefficients are coupled with higher modes, which leads to affecting the degree of stability.

2. Model Description

2.1. Reactor Kinetics State Equations

For simplicity, one group neutron flux with the averaged delayed neutron is assumed.

The most important fission product poison is Xe–135, which is formed as the result of the decay of I–135. The I–135 is also formed in fission and by the decay of Te–135. In view of fact that Te–135 decays so rapidly to I–135, it is possible to assume that all I–135 are produced directly in fission.

An input is taken as the perturbation of infinite multiplication factor, which then becomes the negative feedback influenced by the change of xenon number density.

$$\begin{aligned}
\frac{1}{\upsilon}\frac{\partial \phi(z,t)}{\partial t} &= \nabla \cdot D(z)\nabla \phi(z,t) - \Sigma_a(z)\,\upsilon(z,t) + \upsilon\Sigma_f(z)\,\phi(z,t), \\
\frac{\partial I(z,t)}{\partial t} &= -\lambda_I(z,t) + \gamma_I(z)\Sigma_f(z)\phi(z,t), \\
\frac{\partial X(z,t)}{\partial t} &= -\lambda_X X(z,t) - \sigma_X(z)\,\phi(z,t)\,X(z,t) + \lambda_I I(z,t) + \gamma_X(z)\,\Sigma_f(z)\,\phi(z,t), \\
k_\infty(z,t) &= \frac{\upsilon\,\Sigma_f(z)}{\Sigma_a(z) + \sigma_X(z)\,X(z,t)}\,.
\end{aligned} \tag{1}$$

2.2. Linearization

Let the initial flux be known as Φ_0. From the equations (1), the initial values of xenon number density and infinite multiplication factor can be verified. Then, the steady state equations for neutron balance, iodine–xenon number densities, and infinite multiplication factor are as follows;

$$\begin{aligned}
0 &= \nabla \cdot D(z)\ \nabla\phi_0(z) - (\Sigma_a(z) + \sigma_X(z)\ X_0(z) + \upsilon\Sigma_f(z))\ \phi_0(z), \\
0 &= -\lambda_I I_0(z) + \gamma_I(z)\ \Sigma_f(z)\ \phi_0(z), \\
0 &= -\lambda_X\ X_0(z) + \lambda_I\ I_0(z) - (\sigma_X(z)\ X_0(z) + \gamma_X(z)\ \Sigma_f(z))\ \phi_0(z), \\
k_{\infty,0}(z) &= \frac{-\upsilon\ \Sigma_f(z)}{\Sigma_a(z) + \sigma_X(z)\ X_0(z)}\,.
\end{aligned} \tag{2}$$

From Eqs.(2), the steady state of xenon number density is given as

$$X_0(z) = \frac{(\gamma_I(z) + \gamma_X(z))\ \Sigma_f(z)}{\lambda_X + \sigma_X(z)\ \phi_0(z)}\ \phi_0(z). \tag{3}$$

Now, if the steady state is known, all time–dependent variables will be expressed with the perturbed quantities as

$$\begin{aligned}
\phi(z,t) &= \phi_0(z) + \delta\phi(z,t), \\
I(z,t) &= I_0(z) + \delta I(z,t), \\
X(z,t) &= X_0(z) + \delta X(z,t), \\
X(z,t) &= X_0(z) + \delta X(z,t), \\
k_\infty(z,t) &= k_{\infty,0}(z) + \delta k(z,t),
\end{aligned} \tag{4}$$

Inserting Eqs.(4) into Eqs.(1) and subtracting the steady state part, the system equation then becomes.

$$\begin{aligned}
\frac{1}{\upsilon}\frac{\partial}{\partial t}\delta\phi(z,t) = {} & \nabla \cdot D(z)\nabla\,\delta\phi(z,t) - \Sigma_a(z)\ \delta\phi(z,t) \\
& - \sigma_X(z)\ (X_0(z))\ \delta\phi(z,t) + \phi_{(}z)\ \delta X(z,t)) \\
& + (\Sigma_a(z) + \sigma_X(z)\ X_0(z))\ k_{\infty,0}(z)\ \delta\phi(z,t) \\
& + (\Sigma_a(z) + \sigma_X(z)\ X_0(z))\ \phi_0(z)\ \delta k(z,t),
\end{aligned} \tag{5}$$

$$\begin{aligned}
\frac{\partial}{\partial t}\delta I(z,t) = {} & -\lambda_I(z,t) \\
& + \frac{\gamma_I(z)}{\upsilon}[\sigma_X(z)k_{\infty,0}(z)\ \phi_0(z)\ \delta X(z,t) \\
& + (\Sigma_a(z) + \sigma_X(z)\ X_0(z))\ k_{\infty,0}(z)\ \delta\phi(z,t) \\
& + (\Sigma_a(z) + \sigma_X(z)\ X_0(z))\ \phi_0(z)\ \delta k(z,t)],
\end{aligned} \tag{6}$$

$$\begin{aligned}
\frac{\partial}{\partial t}\delta X(z,t) = {} & -\lambda_X\delta X(z,t) + \lambda_I\ \delta I(z,t) \\
& - \sigma_X(z)\ (X_0(z)\ \delta\phi(z,t) + \phi_0(z)\ \delta X(z,t)) \\
& + \frac{\gamma_I(z)}{\upsilon}[\sigma_X(z)\ k_0(z)\ \phi_0(z)\ \delta X(z,t)
\end{aligned} \tag{7}$$

$$+(\Sigma_a(z)+\sigma_X(z))k_{\infty,0}(z)\ \delta\phi(z,t)$$
$$+(\Sigma_a(z)+\sigma_X(z)\ X_0(z))\ \phi_0(z)\ \delta_k(z,t)],$$
$$\delta k(z,t)=\frac{\upsilon\ \Sigma_f(z)\,\sigma_X(z)}{(\Sigma_a(z)+\sigma_X(z)\ X(z,t))^2}\,\delta X(z,t) \tag{8}$$

Here the nonlinear terms are ignored since their contributions are too little in comparison with the linear terms.

2.3. Modal Expansion

In order to solve the above equation, the perturbed quantities are modal-expanded as,

$$\delta\phi(z,t)=\sum_n a_n(t)\ \psi_n(z),$$
$$\delta I(z,t)=\sum_n b_n(t)\ \psi_n(z), \tag{9}$$
$$\delta X(z,t)=\sum_n b_n(t)\ \psi_n(z).$$

In Eq.(9) ψ_n and ω_n are the eigenvectors and the eigenvalues, respectively in the omega-mode[1], which satisfy

$$\nabla\cdot D(z)\nabla\psi_n(z)-\Sigma_a(z)\ \psi_n(z)$$
$$+\ \upsilon\,\Sigma_f(z)=\frac{\omega_n}{\upsilon}\psi_n(z). \tag{10}$$

In combination with the eigenvalues of the adjoint equation,

$$-\nabla\cdot D(z)\nabla\psi^*_n(z)-\Sigma_a(z)\psi_n(z)^*$$
$$+\ \upsilon\,\Sigma_f(z)=\frac{\omega_n^*}{\upsilon}\psi_n^*(z). \tag{11}$$

these eigenvectors have the biorthogonality properties as

$$\int_H\left|\frac{1}{\upsilon}\right| dz\ \psi_n(z)\cdot\psi_m{}^*(z)$$
$$=\begin{cases}1 & \text{if } \omega_n=\omega_m{}^*\\ 0 & \text{if } \omega_n\neq\omega_m{}^*\end{cases} \tag{12}$$

Although the omega-mode method does not have the property of finality, the interactions of the expansion coefficients can be reduced in xenon stability to have the property of finality.[2,11] If we assume that the set of eigenvalues $\omega_n{}^*$ is the same as the set ω_n, and the eigenvectors ψ_n form a complete set, then the expansion coefficients are given explicitly in terms of the unknown $\delta\phi(z,t)$, in view of Eq.(12), as

$$a_n(t)\int_H\left|\frac{1}{\upsilon}\right| dz\ \psi^*_m(z)\cdot\psi_n(z)$$
$$=\int_H\left|\frac{1}{\upsilon}\right| dz\ \psi^*_m(z)\cdot\delta\phi(z,t). \tag{13}$$

The biorthogonality of Eq. (12) leads to some simplification in modal equations, but does not decouple the different modal equations, which means that this does not lead to finality.[5] This finality can be achieved if we let the time scale of delayed neutrons equal to prompt neutrons.[2,11]

3. Generalized Nyquist Criterion

An attention is restricted to the case of a unity-negative feedback system with m-input/m-output which interacts with each other through m×m transfer function matrix Q(s).

The stability of the closed loop system is described by the return difference determinant

$$\rho_c(s)/_0(s)=|T(s)|=|I_m+Q(s)| \tag{14}$$

Where $\rho_c(s)$ and $\rho_0(s)$ are the closed loop and open loop characteristic polynomials, respectively.[8] To obtain a relationship between closed-loop stability and the characteristic transfer functions, it is necessary to move into the realm of a multivariable equivalent to the Nyquist stability criterion.

The symbol D will be used to denote the usual Nyquist contour in the complex plane consisting of the imaginary axis $s=iw$, $|w|<R$ and the semicircle $|s|=R$ in the right half complex plane, where R is large enough to ensure that all the right half plane zeros of $\rho_c(S)$ and $\rho_o(s)$ lie within D. Suppose that $\rho_c(s)$ has n_c zeros in the interior of D and $\rho_0(s)$ has n_o zeros in the interior of D.

If Γ is the closed-contour in the complex plane generated by $|T(s)|$ when s varies on D in a clockwise manner, it follows directly from the identity Eq.(4),

$$n_c-n_o=n_T. \tag{15}$$

where n_T is the number of clockwise encirclements of about the origin of the complex plane. Noting that n_c and n_o represent the number of unstable poles of the closed-loop and open-loop system,[8] respectively, then the closed-loop system is asymptotically stable if, and only if, $n_o = 0$, i.e,

$$n_o + n_T = 0 \tag{16}$$

To avoid the complexity of the eigenvalue-type methods, we need the structural constraintment on Q(s) in the form of diagonal dominance conditions. This can be easily verified. Let r_i (or r_j) be the sum of off-diagonal erms in ith column (or row). This r_i (or r_j) is the radius of Gershgorin circle. The Gershgorin circle is very useful to find the diagonal dominance of frequency dominated transfer function matrix.

$$r_i = \sum_{i=0, j \neq i}^{m} | Q_{ij}(s) | , \tag{17}$$

or

$$r_j = \sum_{i=0, i \neq i}^{m} | Q_{ij}(s) | .$$

The transfer function matrix Q(s) is diagonally dominant at every point $|s|$ on the D contour, if, and only if, the Gershgorin circle[8] of radius r_i (or r_j) does not encircle the point $(-1, 0)$ on the polar plot as seen in Fig. 2. Let the diagonal transfer functions $Q_{jj}(s)$ map D onto closed-contours C_j encircling the $(-1, 0)$ point of the complex plane n_j times, $1 < j < m$, in a clockwise manner, then

$$n_T = \sum_{j=1}^{m} n_j, \tag{18}$$

and, from Eq.(15), the closed-loop system is asymptotically stable if, and only if,

$$n_o + \sum_{j=1}^{m} n_j = 0. \tag{19}$$

4. Numerical Scheme.

4.1. Finite Difference Approximation.

The diffusion equation, Eq.(5) has the boundary conditions expressed by a general form[7] as

$$\frac{d}{dz}\psi_n(z) + \gamma\,\psi_n(z) = \sigma . \tag{20}$$

Here we assume that $\sigma = 0$ and $\psi_n(\pm H/2) = 0$, where H is the length of core. One general technique for obtaining finite difference solutions is the usage of box integration method.[9] If we take the integration of Eq.(5), the diffusion term is of the form

$$\int_{z_i - \frac{\triangle_i}{2}}^{z_i + \frac{\triangle_i + 1}{2}} dz\ \nabla \cdot D(z) \nabla \psi(z)$$

$$\approx D_i \frac{\psi_{i-1} - \psi_i}{\triangle_i} + D_{i+1} \frac{\psi_{i+1} - \psi_i}{\triangle_{i+1}} . \tag{21}$$

The non-diffusion terms are of the form

$$\int_{z_i - \frac{\triangle_i}{2}}^{z_i + \frac{\triangle_i + 1}{2}} dz\ \Sigma(z) \approx \Sigma_i \psi_i \frac{\triangle_i}{2} . \tag{22}$$

Where the continuity of current at z_i has been employed to cancel the derivatives evaluated at z_i, and central difference approximations have been made for derivatives evaluated between $z_i - \frac{\triangle_i}{2}$ and $z_i + \frac{\triangle_i + 1}{2}$

4.2. Eigenvalue Problem.

In need of finding all the eigenvalues and the corresponding eigenvectors, various iterative methods are suggested. Among them, the inverse power method is adapted to solve the eigenvalue problem.[6] This can be done with the inverse iteration in the power method. The inverse iteration requires the construction of the matrix (B–pI) where B is of the matrix from finite difference diffusion equation.

$$B\psi_n = \omega_n \psi_n, \tag{23}$$

and p is a value quite close to one of the eigenvalues $\omega_0, \cdots \omega_n$, then from

$$(B-pI)\psi_n^{(m-1)} = \psi_n^{(m)}. \tag{24}$$

Once we have obtained a PLU factorization for the matrix (B−pI), we obtain $\psi_n^{(m)}$ from $\psi_n^{(m-1)}$ by forward–backward substitution.

4.3. Formulation of Transfer Function.

With all set of eigenvalues and eigenvectors known, Eqs.(5) are then multiplied by the adjoint eigenvector set of ψ_n^*, which are just self adjoint, and then integrated over the entire volume using the biorthogonality Eq.(12), which are then Laplace transformed.

By Lapalace transforming, the linearized expansion coefficients have transfer functions of the form

$$\{a_n(s)\} = \{F(s)\} \; \{\delta k_{ext}(s)\} \tag{25}$$

From

$$\delta X(s) = \frac{\dfrac{\gamma_I \lambda_I \Sigma_f}{s+\lambda_I} + \gamma_X \Sigma_f - \sigma_X X_0}{s+\lambda_X+\sigma_X \phi_0}\, \delta\phi(s) \tag{26}$$

and Eq.(9), the xenon expansion coefficient $d_n(s)$ can be expressed in terms of $\delta k_{ext}(s)$.

$$\{d_n(s)\} = \{G(s)\} \; \{a_n(s)\} \tag{27}$$
$$= \{G(s)\} \; \{F(s)\} \; \{\delta k_{ext}(s)\}$$

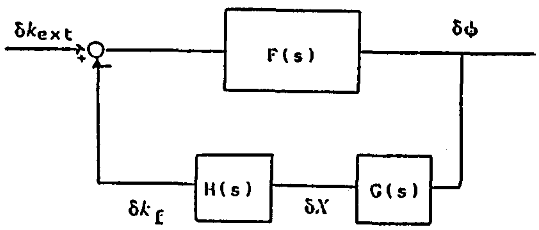

Fig.1. Block Diagram of Transfer Function

and the negative feedback, $\delta k_f(s)$ becomes

$$\{\delta k_f(s)\} = \{H(s)\} \; \{G(s)\} \; \{F(s)\} \; \{\delta k_{ext}(s)\} \tag{28}$$
$$= \{Q(s)\} \; \{\delta k_{ext}(s)\}$$

Then the diagonal terms $Q_{jj}(s)$ in the transfer function matrix Q(s) are investigated to the stability along the varying frequency already given.

5. Results

5.1. Effect of mesh size on stability

An optimal node number should be used for this study because the variation of mesh size can affect the loop stability. Form the Table 1., by increasing the number of nodes from 6 to 20, stability decreases gradually, afterwards, however, there's a little change, less than 1% difference on stability. The number of nodes for further study is assumed to be 20 to see the worst case.

Table 1. Effect of mesh size on Stability

number of node	power(%)	mesh size(in)	COP
6	100	24	-.3271
12	100	12	-.3826
18	100	8	-.4133
20	100	7.2	-.4262
24	100	6	-.4257
48	100	3	-.4258

COP–Cross–Over Point on the real axis on Nyquist Plot.

5.2. Diagonal Dominace

First of all, if the transfer function matrix Q(s) does not have a diagonal dominance, the Generalized Nyquist Criterion cannot be used. So, the diagonal dominance of Q(s) was investigated for the extreme case of r_{10} (10th column or row), 100% power.

As shown in Fig. 2., the dotted Gershgorin circles at various frequencies do not encircle the point (–1, 0), and even, are getting smaller as frquency increases. Then, the transfer function matrix Q(s) is found diagonally dominant, which allows to use the Generalized Nyquist Criterion.

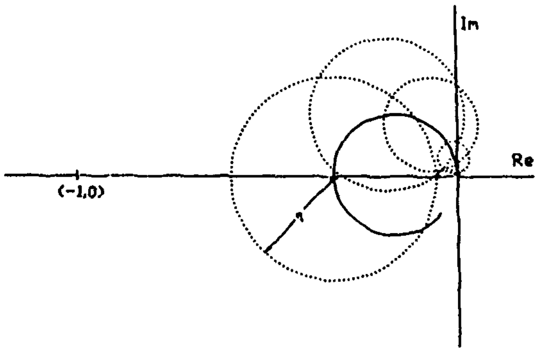

Fig.2. The Gershgorin circles of Nyquist contour

5.3. Effect of Power Level Variation on Stability.

With cosine shaped burnup distribution the maximum COP occurs at the middle of the core which can be the result of the initial power shape. As shown in Table 2. and Fig. 3. an increase in power level causes decrease in stability.

Table 2. Effect of Power·Level Variation on Stability

power level	loop number	COP
50%	10,11	-.3741
80%	10,11	-.4047
100%	10,11	-.4262

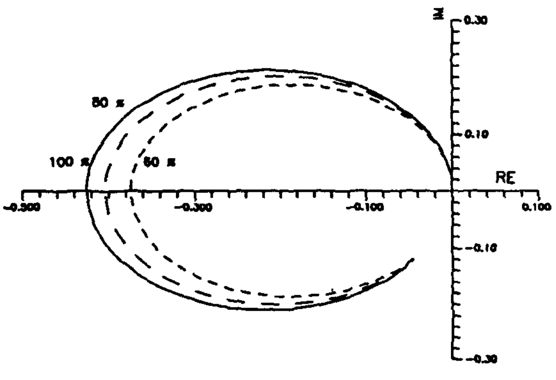

Fig.3. Nyquist Plot for Power Level Variation

5.4. Effect of Control Rod Position on Stability.

Fig. 5. and Fig. 6. illustrate the power distribution at total power level 100% and xenon concentration distribution, respectively. From Fig. 4, and Table 3, the maximum COP occurrs at peak power and xenon number density.

The comparison of these COP values indicates that deeper the control rod position is, less stable the core is.

Table 3. Effect of Control Rod Position on Stability.

rod position (% from top)	power (%)	loop number	COP
20	100	11	-.5012
40	100	13	-.6573
60	100	17	-.9211
80	100	18	-1.225

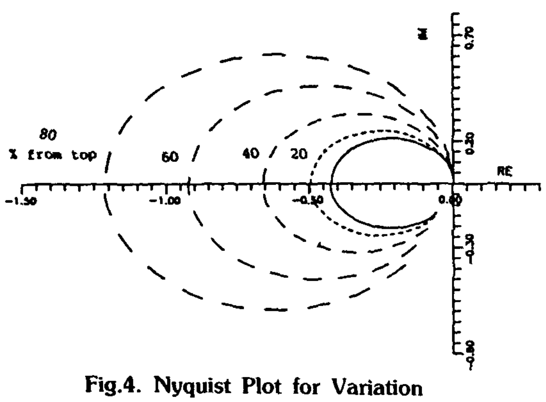

Fig.4. Nyquist Plot for Variation of Control Rod Position

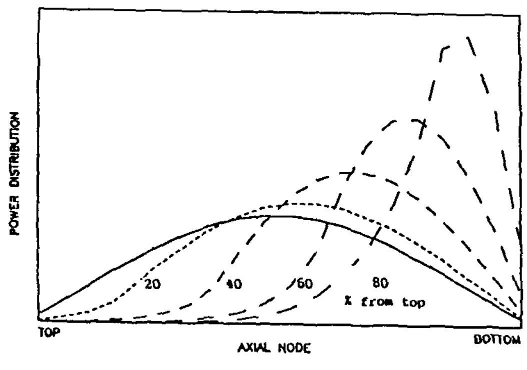

Fig.5. Power Distribution due to Various Control Rod Position

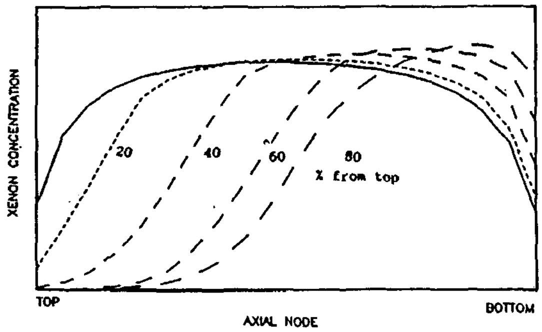

Fig.6. Xenon Concentration Distribution due to Yarious Control Rod Position

5.5. Effect of Burnup Distrbution on Stability.

At the constant 100%, the power shape differs from the cosine-shape as fuel burnup progresses. When 150 MWD/MTU was regarded enough as BOL, 6000 MWD/MTU and 14000 MWD/MTU were regarded as MOL and EOL, respectively, the stability at EOL was found more unstable than that of BOL, from Table 4. Because the peak power shifts from node number 10 to node number 7, the most unstable loop number shifts as well. This leads to less stable core against xenon oscillation.

Table 4. Effect of Burnup Distribution on Stability

Burnup(MWD/MTU)	power(%)	loop number	COP
150	100	10,11	-.4262
6000	100	8	-.5431
14000	100	7	-.7900

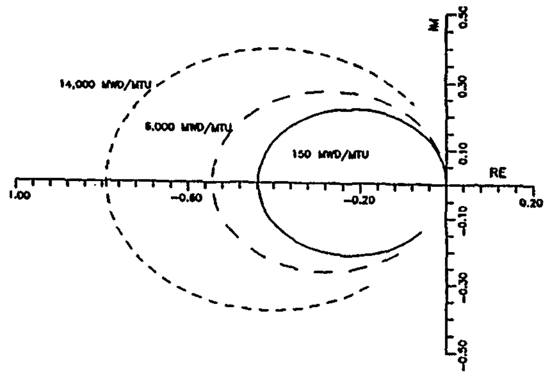

Fig.7. Nyquist Plot for Burnup Distribution

5.6. Effect of Mode Coupling on Stability.

Table 5. and Fig 8. represent the effect of interaction between different modes. Analyzed are variations of modal expansion limit. Up to now, calculation has been carried out with expansion into two modes on Eqs.(9). From Table 5., modal expansion into three or four modes leads to more stable in 100% power level with all rod out situation. The difference, however, was found slight from the two mode expansion. Since the omega mode does not have the property of finality, this overestimation of stability is taken for granted.

Table 5. Effect of Mode Coupling on Stability

Modes	power(%)	node number	COP
TWO	100	10,11	–.4262
THREE	100	10,11	–.4143
FOUR	100	10,11	–.4040

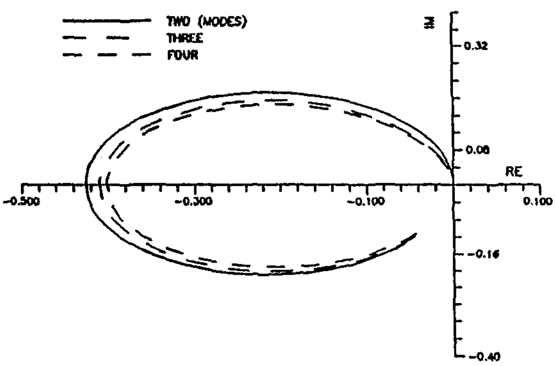

Fig.8. Nyquist Plot for Mode Coupling

6. Discussion and Conclusion

The generalized Nyquist Criterion is suitable for the stability analysis of the space and time dependent reactor kinetics problem such as xenon oscillation. The prescedent study dealt only with the stability on overall system, but the Generalized Nyquist Criterion makes it possible to see the *effect of each node on stability.*

A study using the Generalized Nyquist Criterion was done for YGN 1 core against xenon oscillation with the changes of core parameters. The power level increase leads to less stable core. Also the control rod position and the burnup distribution are able to influence the stability. The modal method was found exact to represent the kinetics variables.

This work was done just to see the behaviour of reactor kinetics parameters, so further study should be taken as adding the effect of Temperature and Void coefficient and for other reactor parameters.

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